6th Mandelstam Theoretical Physics School & Workshop 2024
Recent developments in Large N, Holography and Complexity

Dilaton-Gauss-Bonnet gravity and Cosmology

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JCAP08 (2023) 023
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Contents

1. Modified Gravity beyond Einstein - Is it needed?

I. Gravity : Theoretical Aspect

- GR is an **effective theory** valid below UV cut-off, $M_{Pl} \sim 10^{19} GeV$

  Ex) String theory $\xrightarrow[\text{Low Energy}]{}$ Einstein Gravity + higher curvature terms (α'-expansion)

- Holography : (asymptotic) AdS (BH) in d+1 dim. ↔ Quantum in d dim. (AdS/QCD, CMT etc.)

II. Standard Model of Cosmology (Λ CDM) - Observational Aspect

Extreme fine-tuning ($\Lambda = 2,9\text{x}10^{-122}\ell_P^{-2}$) needed
for Present accelerating Expansion (c.c. or dark energy)

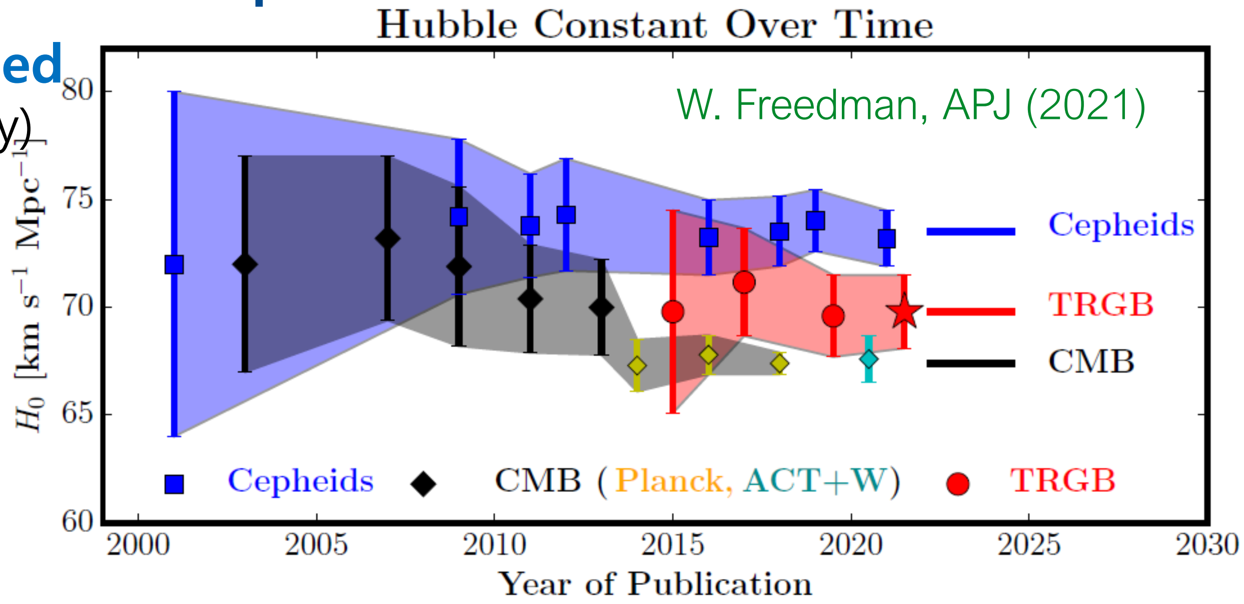
Observational Cosmic Tension

1) H_0 tension

H_0 = 67.4 ± 0.5 km/s/Mpc (CMB),
= 73.5 ± 1.4 km/s/Mpc (SN & Cepheids)

2) Cosmic Birefringence (3σ significance)

3) $\sigma_8(S_8)$ etc.



III. Modified Gravity beyond Einstein :- Alternatives to Λ CDM

Q : Is it (the dilaton-Einstein-Gauss-Bonnet (dEGB) theory) better working?
We investigate 1) the Black Hole properties & 2) the implication to the cosmology.

2. Dilaton Einstein Gauss-Bonnet(DEGB) theory

Lovelock theory (dim. $D = 2t + 1$ or $2t$)

Lagrangian with only **1) metric** **2)2nd order e.o.m** (for no ghosts and instabilities) will be in the following form

$$\mathcal{L}_D = \sqrt{-g} \sum_{n=0}^{t} \alpha_n L^n$$

L^n : Lovelock term, topological in 2n D

Ex) $L^1 = R$ Einstein-Hilbert term topol in 2 D

$L^2 = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$
$= R_{GB}^2$ Gauss-Bonnet term. topol in 4 D

$$L^m = \frac{1}{2^m}\delta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_m\nu_m}_{a_1b_1a_2b_2\cdots a_mb_m} R_{a_1b_1}{}^{\mu_1\nu_1} R_{a_2b_2}{}^{\mu_2\nu_2} \cdots R_{a_mb_m}{}^{\mu_m\nu_m}$$

Euler characteristic of dim 2m topol in $2m$ D

$$\delta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_m\nu_m}_{a_1b_1a_2b_2\cdots a_mb_m} = (2m)!\,\delta^{\mu_1}_{[a_1}\delta^{\nu_1}_{b_1}\cdots\delta^{\mu_m}_{a_m}\delta^{\nu_m}_{b_m}$$

Ex) D-dim

$\mathcal{L}_2 = L^1 = \sqrt{-g}\,R$ topological

$\mathcal{L}_3 = L^1 = \sqrt{-g}\,R$

$\mathcal{L}_4 = L^1 + L^2 = \sqrt{-g}(R + R_{GB}^2) \approx \sqrt{-g}\,R$

$\mathcal{L}_5 = L^1 + L^2 = \sqrt{-g}(R + R_{GB}^2)$

$\mathcal{L}_6 = L^1 + L^2 + L^3 = \sqrt{-g}(R + R_{GB}^2 + R_{E.C}^3) \approx \sqrt{-g}(R + R_{GB}^2)$

$\mathcal{L}_7 = L^1 + L^2 + L^3 = \sqrt{-g}(R + R_{GB}^2 + R_{E.C}^3)$

2-1)Lovelock's theorem (in dim =4 (& 3)

The Einstein eqns (w/ c.c.) are the only possible 2nd-order eqns derived in 4 dim. solely from the metric.

Modification of GR needs to relax the assumptions of Lovelock's theorem.
→ Adding a **new degree of freedom (such as scalars)** other than the metric

2-2) Horndeski Theory - the most general scalar-tensor theory w/ 2nd-order field eqn in 4D

$$\mathcal{L} = G_2(\phi, X) \; - G_3(\phi, X)\Box\phi \; + G_4(\phi, X)R \; + G_{4X}[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\nu\mu}]$$
$$+ G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} - \frac{G_{5X}}{6}[(\Box\phi)^3 - 3\Box\phi\phi_{\mu\nu}\phi^{\nu\mu} + 2\phi_{\mu\nu}\phi^{\nu\lambda}\phi_\lambda{}^\mu]$$

higher derivative theories may have ghosts and Ostrogradsky instability :

Note : Horndeski theory is classified by 4 arbitrary functions $\{G_i(\phi, X),\; i = 2,3,4,5\}$.

Examples:

(i) Einstein Gravity is obtained by taking $G_4 = \frac{M_P{}^2}{2}$ (other $G_i = 0$)

$$S = \int d^4x \sqrt{-g}\frac{M_P{}^2}{2}R$$ Linear in curvature scalar

(ii) Brans-Dicke $f(R)$, k-inflation/k-essence, Quintessence gravity, etc

(iii) Gauss-Bonnet Term $S = -\frac{1}{2}\int d^4x \sqrt{-g}\,\xi(\phi)R_{GB}^2$ where $R_{GB}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

can be shown to be realized (at the level of the e.o.m)

Horndeski, *Int. J. Theor. Phys.* **10** 363–84 (1974)
Charmousis, Copeland, Padilla & Saffin *Phys. Rev. Lett.* **108** 051101 (2012)

2-3) the Dilaton-Einstein-Gauss-Bonnet (DEGB) Gravity $f(\phi) = \alpha e^{\gamma\phi}$ polynomial etc.

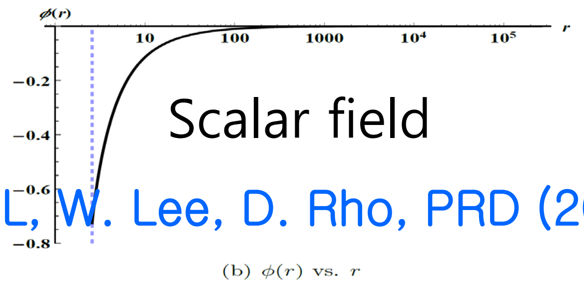
$$S_{dEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R - \frac{\Lambda e^{\lambda\phi(r)}}{2\kappa} + f(\phi)R_{GB}^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m^{matt}\right]$$

Goal : To understand the physics due to the main parameters

3. dEGB theory - Black Holes

New Properties of the Black Holes

Scalar Hair,

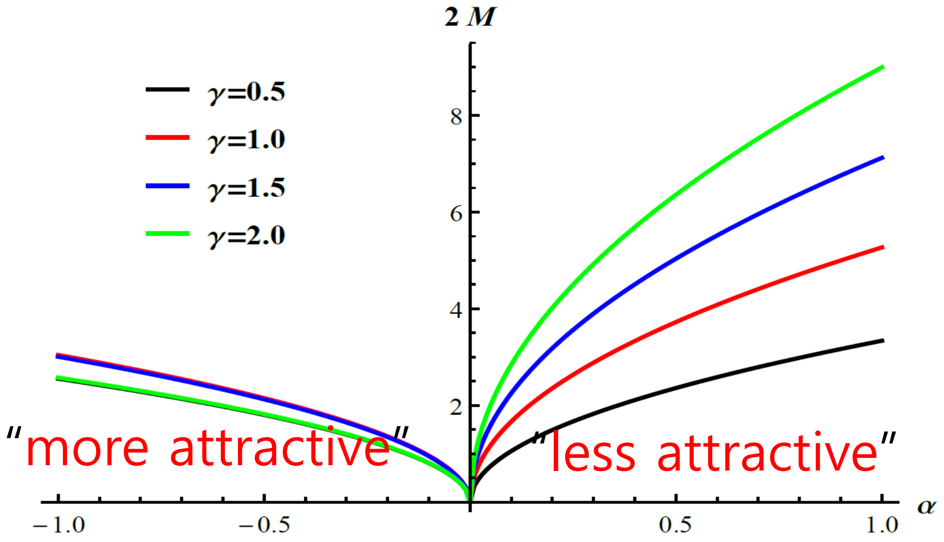


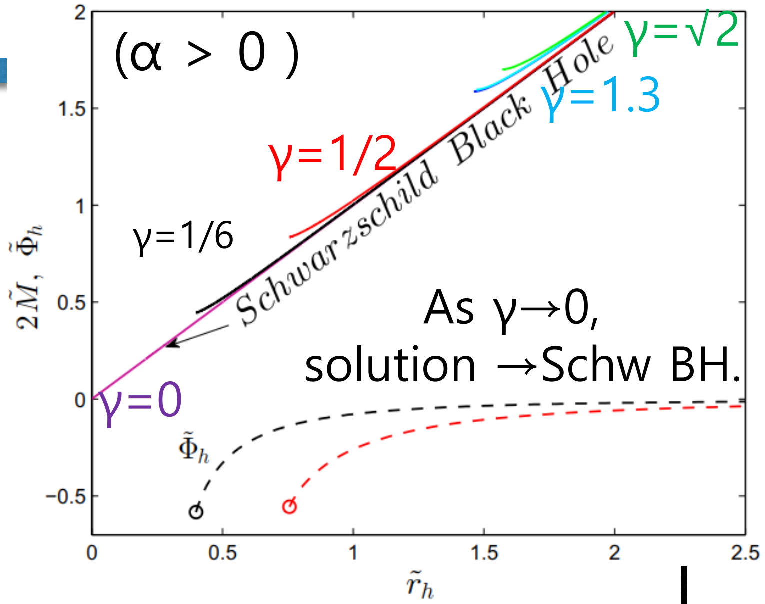
BHL, W. Lee, D. Rho, PRD (2019)

minimum mass → New Phase?

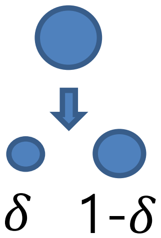
Soliton Star? Black Holes

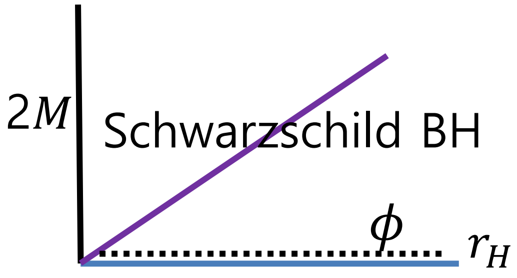
M=0 M_{min} → ∞





(in)stability under fragmentation

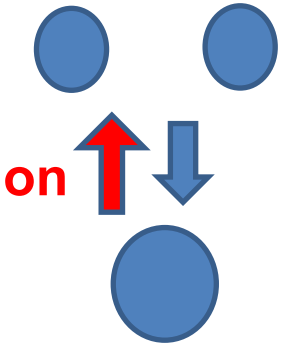




Stability of the DEGB Blackholes under fragmentation

B. Gwak and BHL, PRD91 (2015) 6, 064020.
B.Gwak, BHL, D. Rho, PL.B761 (2016)

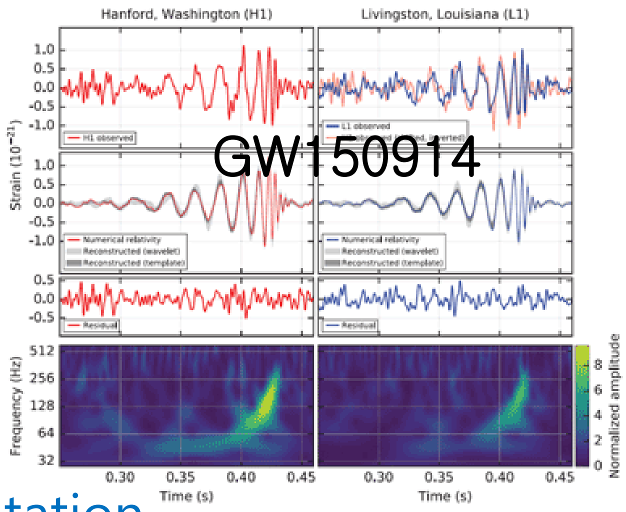
B) fragmentation



A) Merging
+ Gravitational Wave

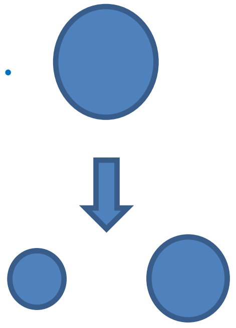
Observed!

?



B) Fragmentation Process : one BHs → two BH ?

Schwarzschild black holes are always stable under the fragmentation,
is marginally stable under the fragmentation of shooting off the infinitesimal mass BH .



$$\frac{S_f}{S_i} = \frac{M_1^2+M_2^2}{(M_1+M_2)^2} = \frac{(\delta r_h)^2+((1-\delta)r_h)^2}{r_h^2} = \delta^2 + (1-\delta)^2 \leq 1$$

(equality only when $M_2 \cdot M_2 = 0$)

These phenomena could happen in the theory with the higher order of curvature term with appropriate parameters.

4. dEGB Cosmology

- Effects to Inflation

  The duration of inflation gets shorter

S. Koh, BHL, Tumurtushaa
PRD98 (2018) 10, 103511

S. Koh, BHL, Tumurtushaa
PRD 95 (2017) 12, 123509

S. Koh, BHL, W. Lee, Tumurtushaa
PRD90 (2014) no.6, 063527

- Reheating parameters in Gauss-Bonnet inflation Models

  the reheating parameters are highly sensitive to the presence of the GB term

- DE dominant era : Doesn't help in resolving the H0 tension

Constraints from Nuclear & Particle Physics

- constraints from the GW signals from BBN merger events

  WIMP indirect detection

B-HL,W.Lee, Colgáin, Sheikh-Jabbari, Thakur
JCAP (2022)

- New Cosmological Phases and implication to the GW

NEW PHASEs⟶ | ⟵Rad Dom⟶ | ⟵ Matt ⟶ | ⟵ Λ(DE) ⟶

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How to test?

1) Astroparticle physics 2)Grav. Waves, etc.

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4. Dilaton-Einstein-Gauss-Bonnet (dEGB) Cosmology

A. Biswas, A. Kar, **BHL**, H. Lee, W. Lee, **S. Scopel**, L. Yin **JCAP08 (2023) 023**

Action

$$S_{dEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R + f(\phi)R^2_{GB} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m\right]$$

$R^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ Gauss-Bonnet term

$(\kappa \equiv 8\pi G = \frac{1}{M^2_{PL}})$

$\mathcal{L}_m = \mathcal{L}_{SM} + \mathcal{L}_{CDM} - \frac{1}{\kappa}\Lambda$

1) If $f(\phi) =$ const, the theory is reduced to a **quintessence model.**

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}^{rad}_m + \mathcal{L}_m\right]$$

(dEGB $\xrightarrow{\text{GB term dropped}}$ **Quintessence**)

2) If $f(\phi) =$ const & $\phi =$ const, it is reduced to **Standard ΛCDM**.

$$S = \int d^4x \sqrt{-g}\,[\frac{1}{2\kappa}R + \mathcal{L}_{m;}] \quad \mathcal{L}_m = \mathcal{L}_{rad} + \mathcal{L}_{matt} + \mathcal{L}_{CDM} - \frac{1}{\kappa}\Lambda$$

Gravity : Einstein Gravity

Matter : Standard Model +(C)DM +DE

(dEGB $\xrightarrow{\text{GB } and\ \phi \text{ dropped}}$ **ΛCDM**)

The Einstein and scalar Eqs.

$$H^2 = \frac{\kappa}{3}(\rho_{\{\phi+GB\}} + \rho_m)$$

$$\dot{H} = -\frac{\kappa}{2}[(\rho_{\{\phi+GB\}} + p_{\{\phi+GB\}}) + (\rho_m + p_m)]$$

$$= -\frac{\kappa}{2}\rho_{tot}(1 + w_{tot}) \qquad (w = \frac{p}{\rho} \text{ Eq of state})$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + V'_{GB} = 0$$

The deceleration parameter $q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}(1 + 3w_{tot})$

ρ_{GB}&p_{GB} are NOT necessarily positive.

acceleration⟶ ⟵ deceleration

w_I:	-1	$-1/3$	0	$+\frac{1}{3}$	$+1$
$\rho_I \sim a^{-3(1+w_I)}$	a^{0-}	a^{-2}	a^{-3}	a^{-4}	a^{-6}
$a(t) \sim t^{\frac{2}{3(1+w)}}$	$t^\infty \sim e^{Ht}$	t	$t^{2/3}$	$t^{1/2}$	$t^{1/3}$
$H \sim a^{-3(1+w)/2}$	a^{0-}	a^{-1}	$a^{-3/2}$	a^{-2}	a^{-3}

2-2) Overview – Effects of GB

- Effects to the ΛCDM through the different evolution of $a(t)$

(1) the Standard ΛCDM Cosmology

Particle & Nuclear Physics in Cosmology Ex) WIMPs

t= 0				10^{-5}sec	1sec	3 min.	50,000yr	370,000yrs	10^{10}yrs	1.38x10^{10}
T=∞				3x10^{12}K	10^{10}K	10^{9} K	8,700 K	3,000K	3.8K	2.73K
	TeV	150GeV	GeV	300MeV	1MeV	0.1MeV	0.8 eV			

← | ←---**WIMPs**---→ | Radiation Dominated → | ← Mat Dom→ | ←DE →

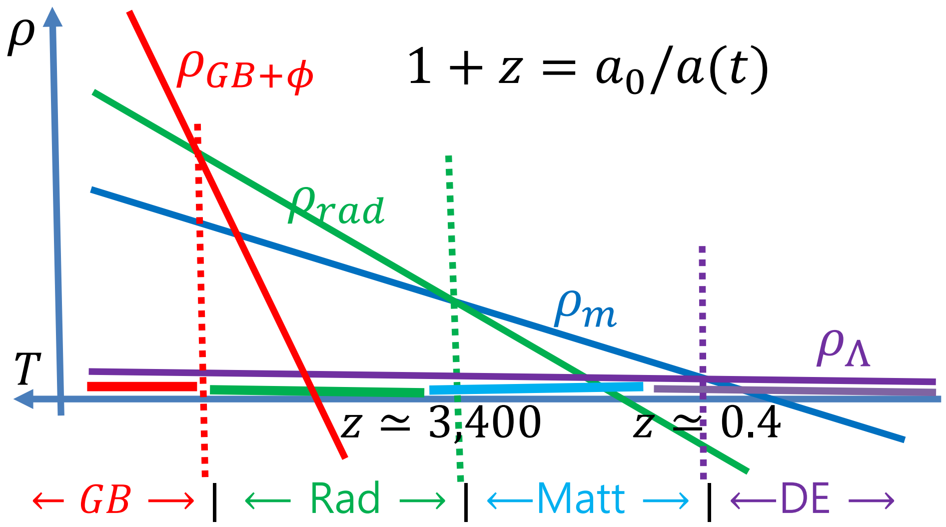
z=∞ | ←---$T_f \simeq m_\chi/20$---→ | ν-decpl **BBN** z=3,400 1,100 z=0.66 0

EW PhTr QCD PhTr e^+e^-annihil $\rho_{\rm rad} = \rho_{\rm Matt}$ Recom $\rho_{\rm Matt} = \rho_\Lambda$

quarks,	quarks,	baryons(p,n)	p,n	Nucleus (p,n,**He**,)	atoms(H,He)	**DE 70%,**
leptons	leptons	leptons(e,ν)	$e^\pm$,ν	ν	ν	**DM 25%,**
gauge bosons	gluon, $W^\pm$, Z, γ	γ	γ	γ	γ (CMB)	**Baryon 5%**

(2) New Phases

←**GB PHASE**→	← Radiation →	←Matt	←DE →
w_I=-1/3,+1,7/3,etc.	1/3	0	-1
$\rho_I \sim a^{-3(1+w_I)}$	a^{-4}	a^{-3}	a^0
$a(t) \sim t^{\frac{2}{3(1+w)}}$	$t^{1/2}$	$t^{2/3}$	t^∞
$H \sim a^{-3(1+w)/2}$	a^{-2}	$a^{-3/2}$	a^0



A. Biswas, A. Kar, **BHL**, H. Lee, W. Lee,
S. Scopel, L. Yin **JCAP08 (2023) 023**

4-4) WIMPs in DEGB cosmology

$$S_{DEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R + f(\phi)R^2_{GB} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m^{rad} + \mathcal{L}_{DM}^{WIMP}\right]$$

WIMPs (Weakly Interacting Massive Particle) (GeV $\lesssim m_\chi \lesssim$ TeV) are the most popular candidates of the Cold Dark Matter (CDM).

WIMP thermal decoupling scenario - thermal relic density

$\Gamma > H \longrightarrow | \longleftarrow \Gamma < H$

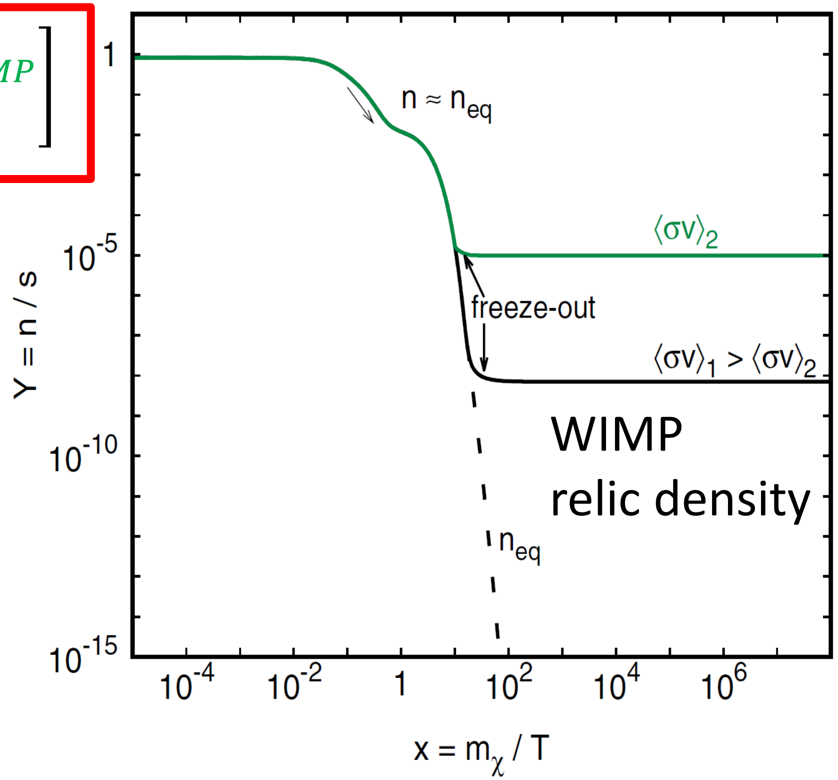
thermal equil $T > T_f \longrightarrow | \longleftarrow T < T_f$ Decoupled, freezing

T

m_χ $\quad T_f \simeq m_\chi/20$

the observed **DM relic density** at present,

$\Omega_\chi h^2 = 0.12$ (assuming all DM are WIMPs)



dEGB cosmological scenario

Higher $A(T)$ **or** $H(T)$ ⇝ **larger** $< \sigma v >_f \; (=< \sigma v >_{relic})$.

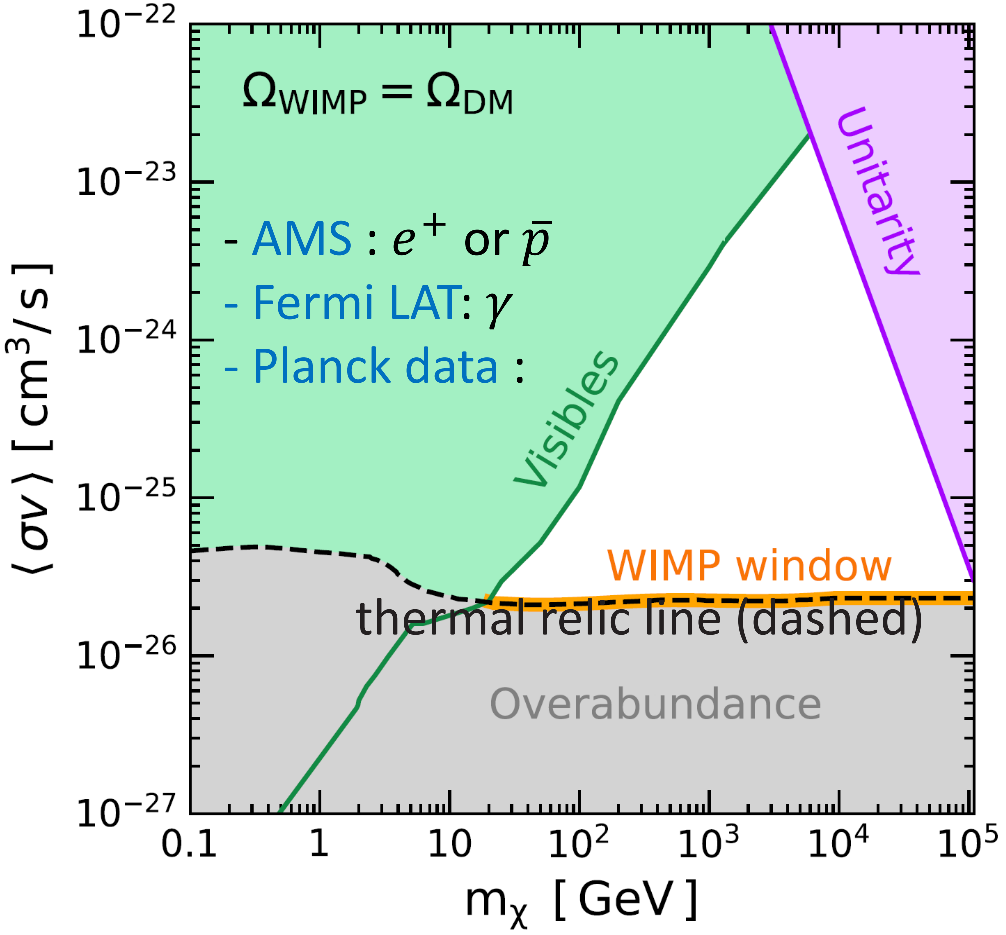
Key point) If $< \sigma v >_f = < \sigma v >_{relic}$ is too large, the decay signal in our Galaxy should be detected.

Goal : Constrain the **Modified Gravity (dEGB)** based on the physics of **WIMPs decoupling**

Indirect detection bounds on WIMP annihilation

- find $< \sigma v >_{relic}$ of $< \sigma v >_f$ (fn m_χ) for $\Omega_\chi h^2 \simeq 0.12$
- Nonobservation (of an excess over b.g.) of the WIMP annihilation in the Galaxy today to $\gamma, e^\pm, p, \bar{p}$ ⇝ an upper bound $< \sigma v >_{ID}$ on $< \sigma v >_{gal}$ (fn of m_χ).
- **The favoured parameters** are those satisfying

$$< \boldsymbol{\sigma v} >_{gal} / < \boldsymbol{\sigma v} >_{ID} \lesssim \mathbf{1}$$



Thermal Relic line

If $< \sigma v >_f$ larger, then decouples later
⇝ smaller $n_\chi(T_f)$ & relic density.

Region below the dotted line is disfavored **(Overabundance)**

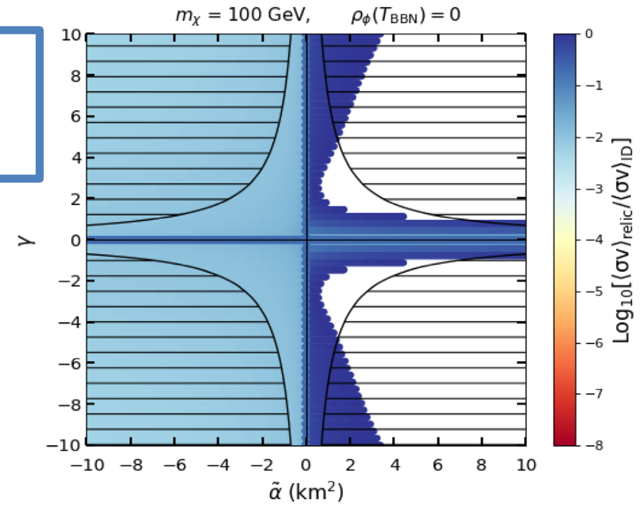
Constraints)

1) Indirect detection bounds on WIMP annihilation :

If $< \sigma v >_f$ too high, then the annihilation signal should be observed. No such signal observed means :

Only parameters producing smaller $< \sigma v >$ than the indirect detection upper bound **(solid green curve)**, are favored.

The **white regions** are excluded by **WIMP indirect searches,**



2) the GW forms from Binary BH merger events

the constraints

$$|f'(\phi(T_L))| \leq \sqrt{8\pi}\alpha_{GB}^{max} \text{ with } \alpha_{GB}^{max} = (1.18)^2 \text{km}^2$$

- If $\dot{\phi}\ (T_{BBN}) = 0,$ then $|\tilde{\alpha}\gamma| \leq \sqrt{8\pi}\alpha_{GB}^{max}$
- If $\dot{\phi}\ (T_{BBN}) \neq 0,$ then $|\tilde{\alpha}\gamma e^{\gamma \frac{\dot{\phi}_{BBN}}{H_{BBN}}}| \leq \sqrt{8\pi}\alpha_{GB}^{max}$

Hatched areas of the $\tilde{\alpha}$-γ parameter space are disallowed by the constraint

Note

1) As m_χ increases for fixed ϵ, $\frac{<\sigma v>_f}{<\sigma v>_{ID}}$ decreases (more favored).

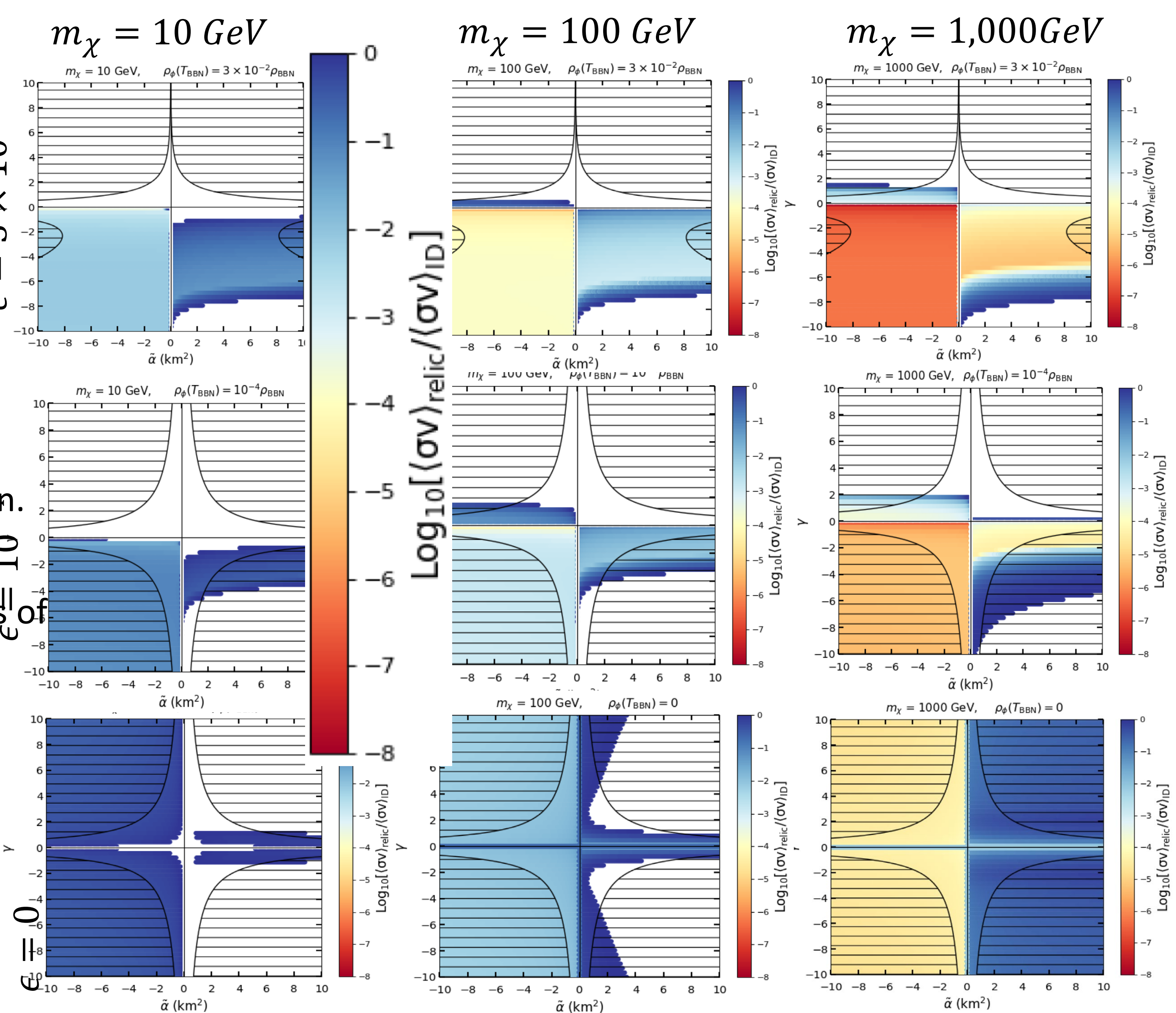
2) As ϵ increases for fixed m_χ, $< \sigma v >_f / < \sigma v >_{ID}$ usually increase. However, can also decrease in some parameter region.

- White regions($\frac{<\sigma v>_{relic}}{<\sigma v>_{ID}} > 1$) are disfavoured by WIMP indirect detection.

- Hatched areas of the $\tilde{\alpha}$-γ parameter space are disallowed by the constraints of GWs in BBH merge .

The **white regions** are excluded by **WIMP indirect searches,**

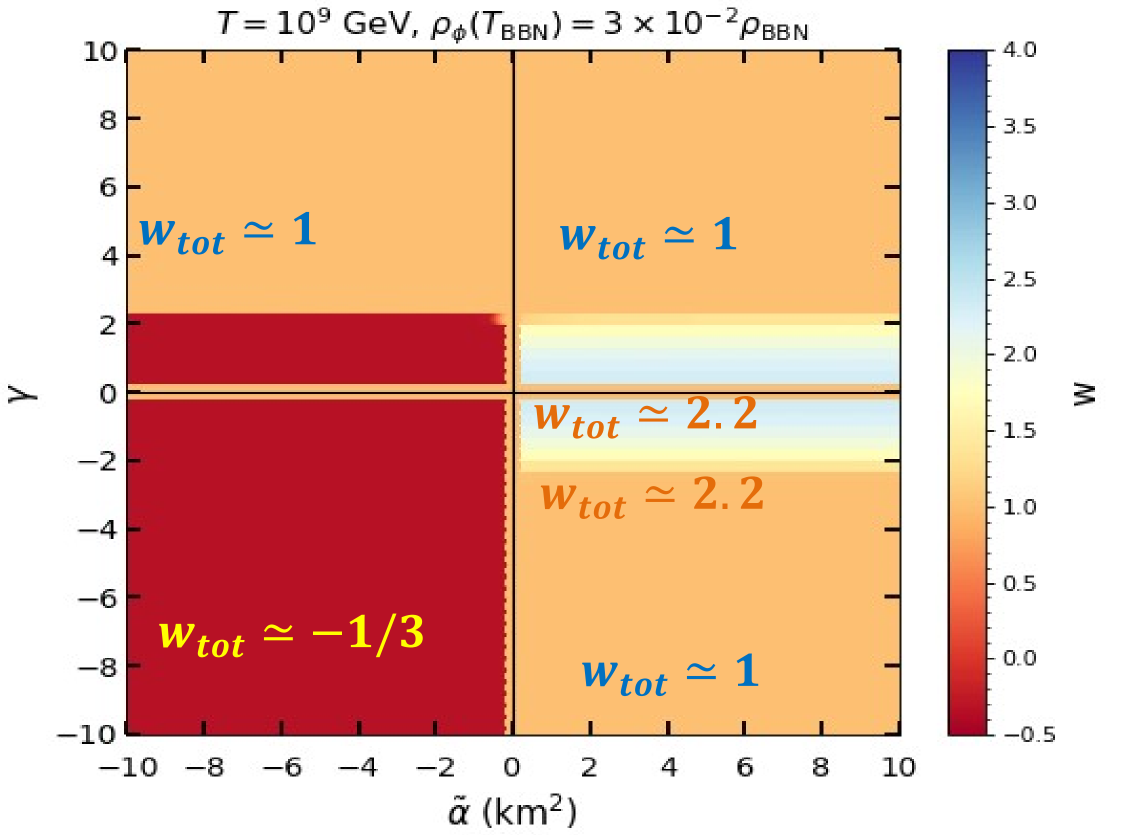
the **hatched** ones are ruled out by the **GW detection from compact binary mergers**.

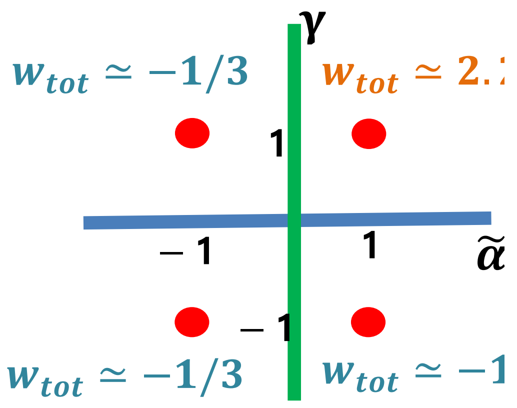


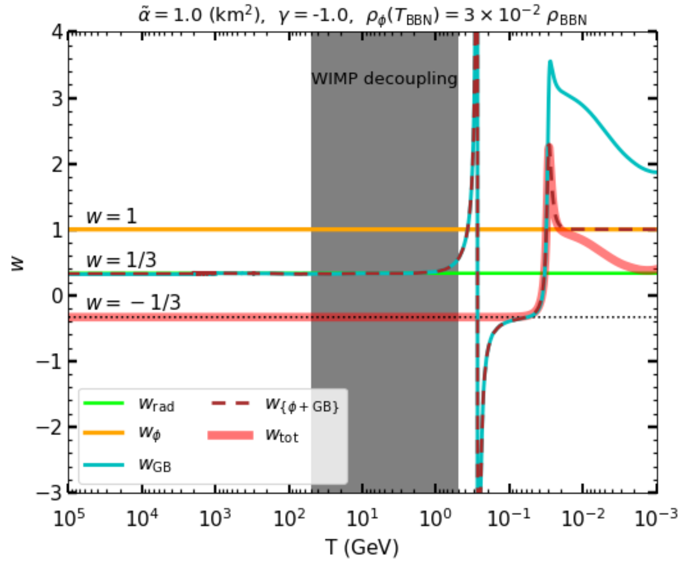
High T behavior of dEGB cosmology

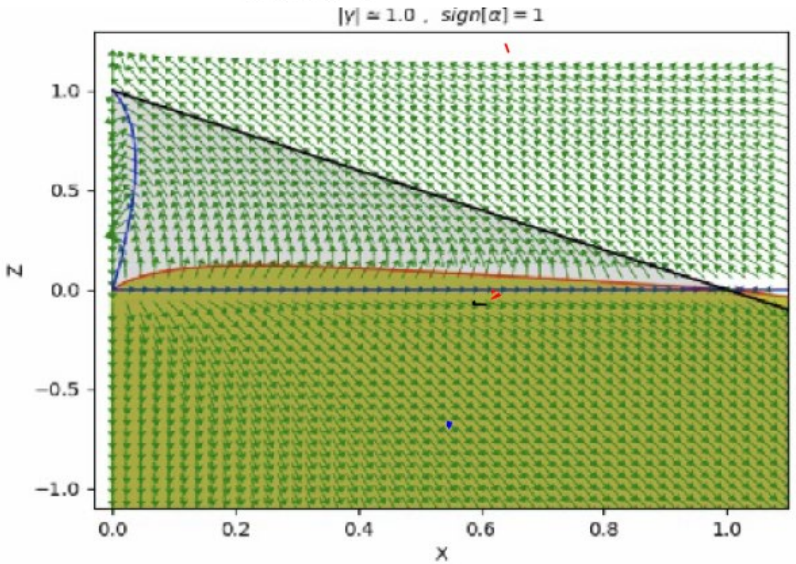
NEW PHASEs⟶ | ⟵Rad Dom⟶ | ⟵ Matt ⟶ | ⟵ Λ(DE) ⟶

1) New Phases appear
   Ex) Super Kination phase ($w > 1$)
   Kination Phase ($w = 1$)
   Slow rolling phase ($w \approx -1/3$)
2) These are attractor/fixed point solutions)
3) May affect observation -New Physics
   Ex) GWs









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5.Summary

Ex) The String theory at low Energy

→ Einstein Grav + higher curvature terms

Modified Gravity beyond Einstein needed?

Theoretical Aspect

- an **effective theory** below UV cut-off, $M_{Pl} \sim 10^{19} GeV \rightarrow$ Einstein Grav + **higher curvature** terms
- Is Standard Cosmology (***Λ*CDM**) satisfactory? extremely fine-tuned ($\Lambda = 2{,}888\text{x}10^{-122} \ell_P^{-2}$)
- Holography

Observational Aspect - $\boldsymbol{H_0}$ **tension**, Cosmological Birefringence etc.

Modification of GR - needs to introduce additional d.o.f.

- higher than 2nd order theories have generically, ghosts & Ostrogradsky instability :

Horndeski theory is the most general scalar-tensor theory w/ 2nd-order field eqn in 4 dim. (no ghost or instability, as a result), classified by 4 arbitrary functions $\{G_i(\phi, X),\ i = 2,3,4,5\}$.

the Dilaton-Einstein-Gauss-Bonnet (dEGB) Gravity belongs to Horndeski theory

$$S_{dEGB} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R + f(\phi) R_{GB}^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \mathcal{L}_m \right]$$

We chose

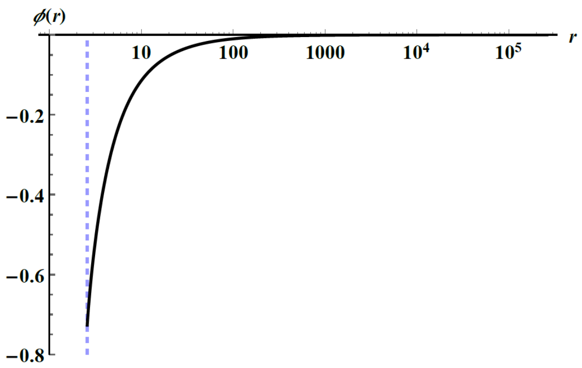
$$f(\phi) = \alpha e^{\gamma\phi}$$

5.Summary (continued)

DEGB hairy Black Hole

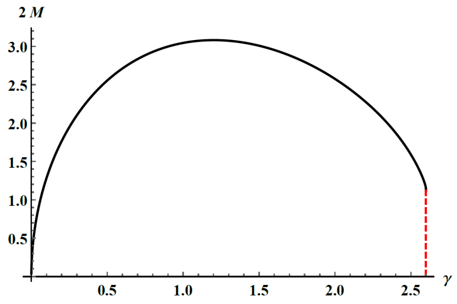
(with Dilaton, Gauss-Bonnet term and cosmological constant)

- There exists **minimum mass**.
- BHs have hairs (shown to be consistent with the no hair theorem).

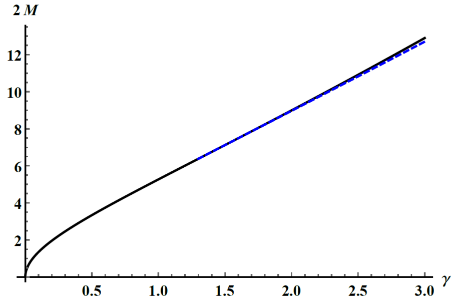


(b) $\phi(r)$ vs. r

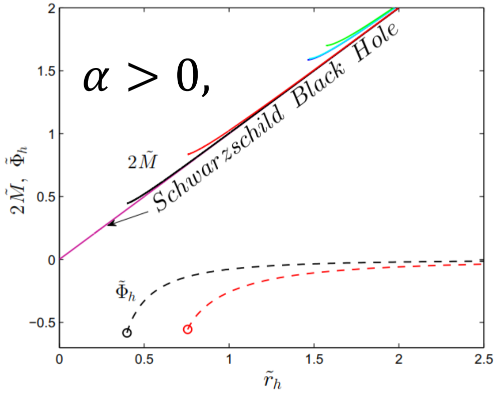
- The BH solution & its properties are strongly dependent on the signature of the Gauss-Bonnet term (as well as Λ).



(a) The black hole mass vs. γ with $\alpha = -1$.



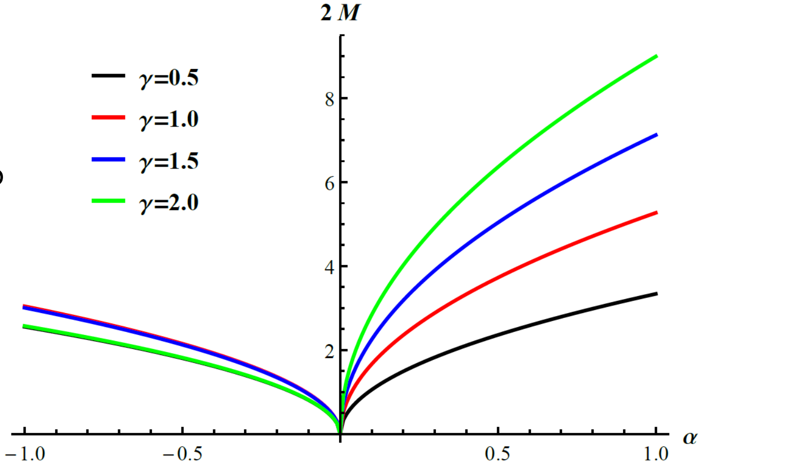
(b) The black hole mass vs. γ with $\alpha = 1$.



When the scalar field on the horizon is the maximum, the DGB black hole solution has the minimum horizon size.

The amount of black hole hair decreases as the DGB black hole mass increases.

"attractive" "repulsive"



- With Cos. Const :

$\Phi(r) \rightarrow \Phi_\infty$(Constant) $\quad only\ when\ \gamma + \lambda = 0,\ \&\ \Lambda = \frac{3\lambda}{8\kappa\alpha\gamma} e^{-(\gamma+\lambda)\Phi_\infty}$

Fragmentation instability of black holes:

For some parameter range, the dEGB BH is unstable under fragmentation, even if these phases are stable under perturbation.

5. Summary (continued)

Cosmological implications-Inflation, reheating, rad-dom period, etc

- Inflation : Smaller e-folding; The blue-tilt of the spectrum could be realized
- Reconstruction of the Scalar Field Potential in Inflationary Models : “inverse scattering method”
- Reheating parameters are sensitive to the Gauss-Bonnet term
- **WIMPs** put some constraints to the DEGB cosmology parameters

WIMP indirect detection

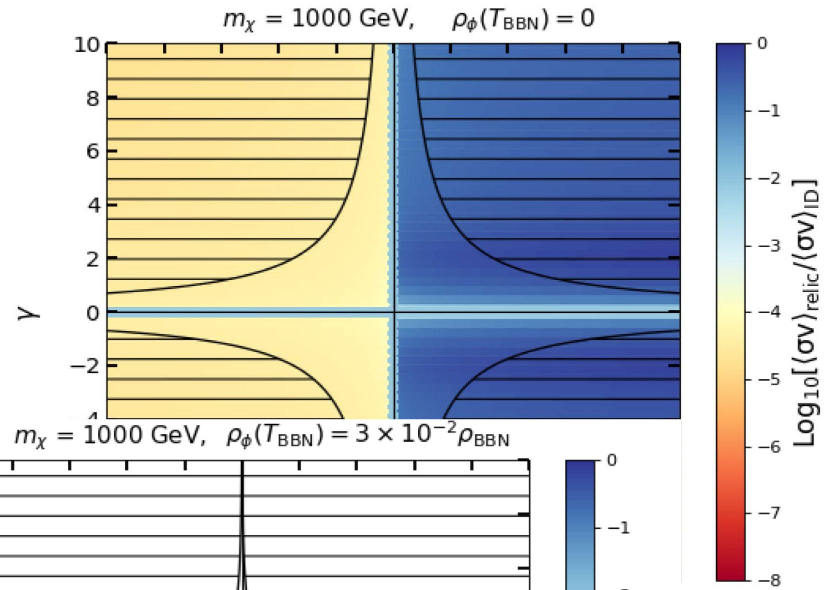
The favoured parameters of the dEGB cosmology by WIMP indirect detection are those satisfying $<\sigma v>_{gal}/<\sigma v>_{ID} \lesssim 1$

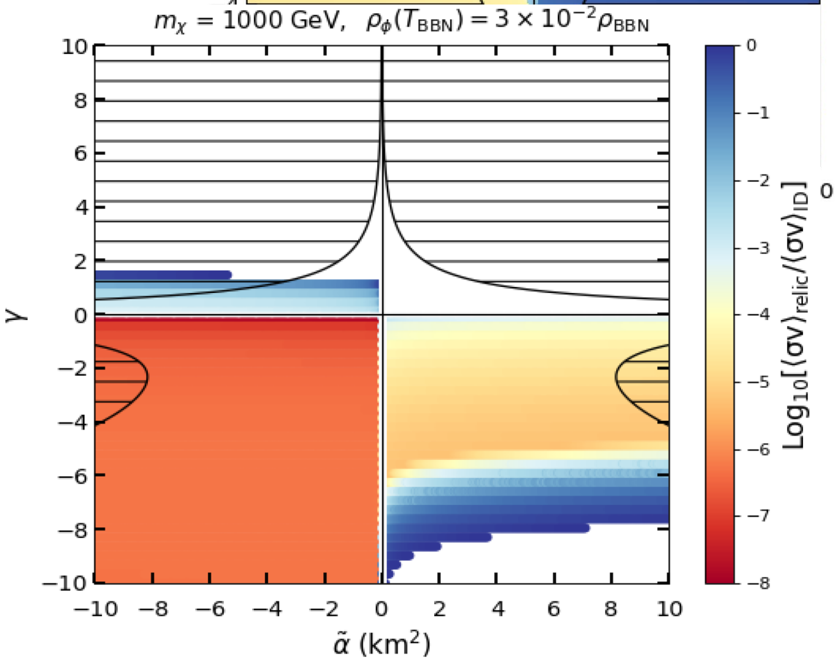
where $<\sigma v>_{ID}$, the upper bound on the present annih σ in the Milky Way

White regions in the figures are disfavored.

Bounds from GWs of BH-BH & BH-NS mergers

Hatched areas of the $\tilde{\alpha}$-γ space are disallowed by the BBHs





NEW PHASEs⟶ | ⟵Rad Dom⟶ | ⟵ Matt ⟶ | ⟵ Λ(DE) ⟶

New Phases exists at high enough temperature

Thank you!