Toward QCD on Quantum Computer: Orbifold Lattice Approach

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11 Jan 2024 @ Johannesburg (remotely)

Classical Mechanics

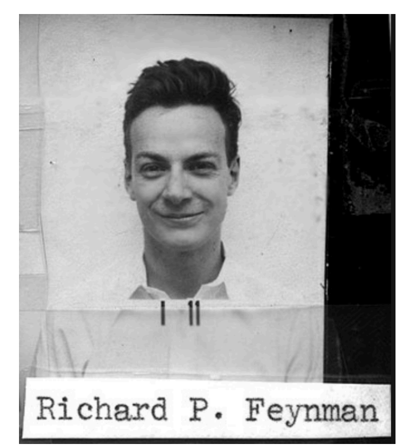
Quantum Mechanics

$H(x, p)$

Hamilton Formulation

Lagrange Formulation

$$L(x, \dot{x}), \quad \dot{x} = \frac{dx}{dt}$$



Path Integral

Path integral + Monte Carlo simulation ✔

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Confinement of quarks

Kenneth G. Wilson
Phys. Rev. D **10**, 2445 – Published 15 October 1974

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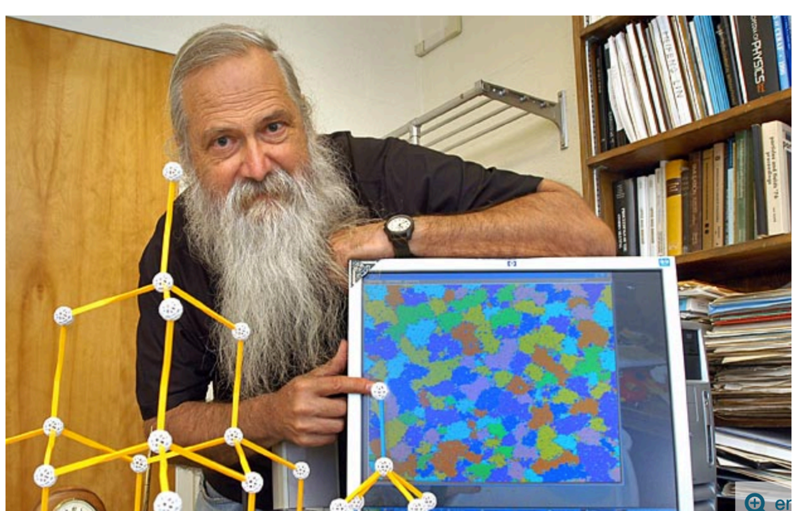

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Milestone

Monte Carlo study of quantized SU(2) gauge theory

Michael Creutz
Phys. Rev. D **21**, 2308 – Published 15 April 1980

An article within the collection: *Physical Review D* 50th Anniversary Milestones

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... if there is no **sign problem**

$$\mathrm{Tr}(e^{-\hat{H}/T}) = \int [dx]\, e^{-\int_0^{T^{-1}} dt\tilde{L}(x,\dot{x})}$$

Positive path integral weight ("probability") is required

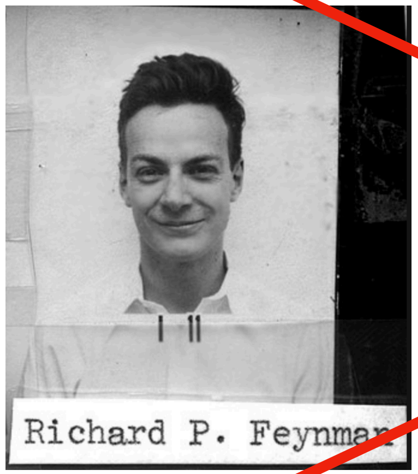
$$\langle x_f |\, e^{-i\hat{H}t}\, | x_i \rangle = \int_{x_i}^{x_f} [dx]\, e^{i\int dt L(x,\dot{x})}$$

Complex number → not "probability"

Sign Problem

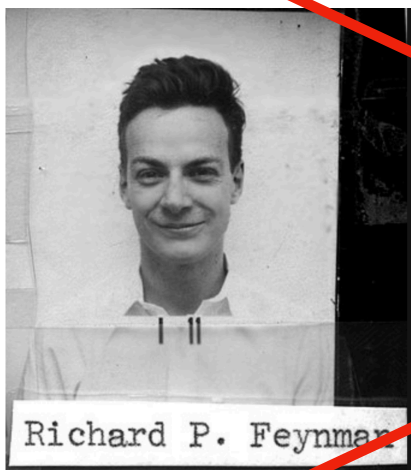
(or Phase Problem)

i is important but difficult to handle.



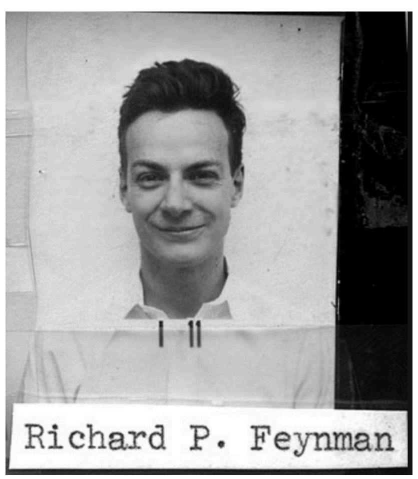
Feynman path integral

sign problem



Feynman path integral

sign problem



Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

Quantum Simulation

Qubit-based universal quantum computer

(Assuming bright future)

Rule of the game

- Many, but finitely many, qubits with error correction
- Unitary time evolution is realized by using simple quantum gates

Our task

- Truncate Hilbert space to finite dimensions
- Write down Hamiltonian explicitly using simple quantum gates

Hamiltonian Lattice Gauge Theory

- Kogut-Susskind formulation
  - Popular because it was the only known option.
  - Complicated.

PHYSICAL REVIEW D VOLUME 11, NUMBER 2 15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Leonard Susskind†
Belfer Graduate School of Science, Yeshiva University, New York, New York and Tel Aviv University, Ramat Aviv, Israel and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received 9 July 1974)

- Orbifold lattice construction
  - Invented to build supersymmetric lattice gauge theory.

    (Kaplan, Katz, Unsal, 2003)
  - Convenient for quantum simulation.

(Buser, Gharibyan, MH, Honda, Liu, 2020; Bergner, MH, Rinaldi, Schafer, in preparation)

PHYSICAL REVIEW D VOLUME 11, NUMBER 2 15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories → Unitary link variable

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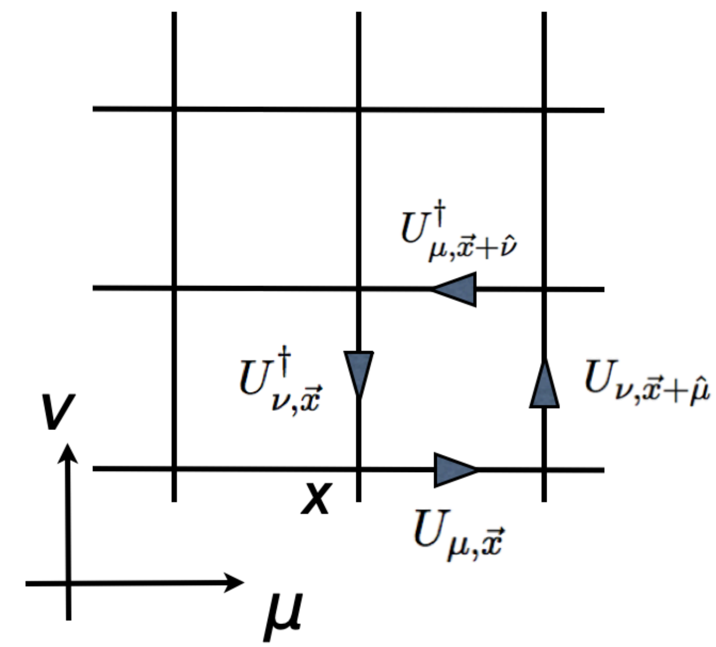
(Received 9 July 1974)

"Wilson's Lattice Gauge Theory"

$$U_{\mu,\vec{x}} = e^{iaA_\mu(x)}$$

$$U_{\mu,\vec{x}} \to \Omega(x) U_{\mu,\vec{x}} \Omega(x+\hat{\mu})^\dagger$$

$$S = -\beta N \sum_{\vec{x}} \sum_{\mu \neq \nu} Tr \left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^\dagger_{\mu,\vec{x}+\hat{\nu}} U^\dagger_{\nu,\vec{x}} \right)$$



Make time direction continuous, and go to operator formalism

$$\hat{H} = \hat{H}_{\mathrm{E}} + \hat{H}_{\mathrm{B}}$$

Electric $$\hat{H}_{\mathrm{E}} = \frac{a^3}{2} \sum_{\vec{n}} \sum_{\mu=1}^{3} \sum_{\alpha=1}^{k^2} \left(\hat{E}^{\alpha}_{\mu,\vec{n}} \right)^2$$

Magnetic

$$\hat{H}_{\mathrm{B}} = -\frac{1}{2ag^2} \sum_{\vec{n}} \sum_{\mu<\nu} \left(\mathrm{Tr} \left(\hat{U}_{\mu,\vec{n}} \hat{U}_{\nu,\vec{n}+\hat{\mu}} \hat{U}^{\dagger}_{\mu,\vec{n}+\hat{\nu}} \hat{U}^{\dagger}_{\nu,\vec{n}} \right) + \mathrm{h.c.} \right)$$

$$\left[\hat{E}^{\alpha}_{\mu,\vec{n}}, \hat{U}_{\nu,\vec{n}'} \right] = a^{-2} g \delta_{\mu\nu} \delta_{\vec{n}\vec{n}'} \tau_{\alpha} \hat{U}_{\nu,\vec{n}'}$$

Why complicated?

$$\left[\hat{E}^{\alpha}_{\mu,\vec{n}}, \hat{E}^{\beta}_{\nu,\vec{n}'} \right] = -i f^{\alpha\beta\gamma} a^{-2} g \delta_{\mu\nu} \delta_{\vec{n}\vec{n}'} \hat{E}^{\gamma}_{\nu,\vec{n}'}$$

$$\hat{U}\,|U\rangle = U\,|U\rangle$$

$$U \in \mathrm{SU}(N)$$

complicated

$$\hat{x}\,|x\rangle = x\,|x\rangle$$

$$x \in \mathbb{R}$$

simple

$$\hat{U}\,|U\rangle = U\,|U\rangle$$

$$U \in \mathrm{SU}(N)$$

complicated

Electric field
~ momentum

$$\hat{E}$$

$$\langle U|R, ij\rangle = \rho^{(R)}_{ij}(U)$$

'plane wave' = irreducible representation

$$\hat{x}\,|x\rangle = x\,|x\rangle$$

$$x \in \mathbb{R}$$

simple

momentum

$$\hat{p}$$

$$\langle x|p\rangle = e^{ipx}$$

$$[\hat{x}, \hat{p}] = i$$

$\hat{x}$ and $\hat{p}$ are simpler

- Fock basis truncation

$$\hat{a} = \frac{\hat{x} + i\hat{p}}{\sqrt{2}} \qquad \hat{a}\,|0\rangle = 0$$

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}\,|0\rangle \qquad 0 \le n < \Lambda$$

- Coordinate basis truncation

$$\hat{x}\,|x\rangle = x\,|x\rangle \qquad -R \le x \le R$$

$$x_n = -R + n\delta_x, \qquad \delta_x = \frac{2R}{\Lambda - 1}$$

$$n = 0, 1, \cdots, \Lambda - 1$$

$$\boxed{\hat{x} \text{ and } \hat{p} \text{ are simpler}}$$

→ Scalar QFT is simpler

$$\hat{x}, \hat{p} \leftrightarrow \hat{\phi}_{\vec{n}}, \hat{\pi}_{\vec{n}}$$

(e.g., Jordan, Lee, Preskill, 2011)

$$[\hat{\phi}_{\vec{n}}, \hat{\pi}_{\vec{n}'}] = i\delta_{\vec{n}\vec{n}'}$$

→ Matrix model is simpler

$$\hat{x}, \hat{p} \leftrightarrow \hat{X}_{M,ij}, \hat{P}_{M,ij}$$

(e.g., Gharibyan, MH, Honda, Liu, 2011
Maldacena, 2023)

Orbifold lattice construction

(Kaplan, Katz, Unsal, 2003)

Original motivation = supersymmetric lattice

SUSY Matrix model —orbifold projection→ SUSY Lattice

$$X_1 + iX_2 = Z = c \cdot W \cdot U = c \cdot e^{as} \cdot e^{iaA}$$

scalar field (s)

gauge field (A)

W: positive definite Hermitian

U: unitary

$$L = \int d^3x \mathrm{Tr}\left(-\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}(D_\mu s_I)^2 + \frac{g_{4\mathrm{d}}^2}{4}[s_I, s_J]^2\right)$$

Forget about SUSY

Orbifold lattice for QCD

(Gharibyan, MH, Honda, Liu, 2020; Bergner, MH, Rinaldi, Schafer, in preparation)

Bosonic part

$$\hat{H} = \sum_{\vec{n}} \mathrm{Tr}\left(\sum_{j=1}^{3} \hat{P}_{j,\vec{n}} \hat{\bar{P}}_{j,\vec{n}} + \frac{g_{4\mathrm{d}}^2}{2a^3} \left| \sum_{j=1}^{3} \left(\hat{Z}_{j,\vec{n}} \hat{\bar{Z}}_{j,\vec{n}} - \hat{\bar{Z}}_{j,\vec{n}-\hat{j}} \hat{Z}_{j,\vec{n}-\hat{j}} \right) \right|^2 + \frac{2g_{4\mathrm{d}}^2}{a^3} \sum_{j<k} \left| \hat{Z}_{j,\vec{n}} \hat{Z}_{k,\vec{n}+\hat{j}} - \hat{Z}_{k,\vec{n}} \hat{Z}_{j,\vec{n}+\hat{k}} \right|^2 \right) + \Delta \hat{H} \, .$$

$$\Delta \hat{H} \equiv \frac{m^2 g_{4\mathrm{d}}^2}{2a} \sum_{\vec{n}} \sum_{j=1}^{3} \mathrm{Tr} \left| \hat{Z}_{j,\vec{n}} \hat{\bar{Z}}_{j,\vec{n}} - \frac{a}{2g_{4\mathrm{d}}^2} \right|^2 + \frac{\mu^2 g_{4\mathrm{d}}^2}{2a} \sum_{\vec{n}} \sum_{j=1}^{3} \left| \frac{1}{N} \mathrm{Tr}(\hat{Z}_{j,\vec{n}} \hat{\bar{Z}}_{j,\vec{n}}) - \frac{a}{2g_{4\mathrm{d}}^2} \right|^2$$

Scalar mass term
→YM at low energy

$$[\hat{Z}_{j,\vec{n},pq}, \hat{\bar{P}}_{k\vec{n}',rs}] = i\delta_{jk}\delta_{\vec{n}\vec{n}'}\delta_{ps}\delta_{qr}$$

Forget about SUSY

Orbifold lattice for QCD

(Gharibyan, MH, Honda, Liu, 2020; Bergner, MH, Rinaldi, Schafer, in preparation)

Fermionic part

$$\hat{H}_{\text{naive}} = a^3 \sum_{\vec{n}} \left\{ \frac{i}{2a} \sum_{j=1}^{3} \left(\hat{\bar{\psi}}_{\vec{n}} \gamma^j \hat{U}_{j,\vec{n}} \hat{\psi}_{\vec{n}+\hat{j}} - \hat{\bar{\psi}}_{\vec{n}+\hat{j}} \gamma^j \hat{U}^{\dagger}_{j,\vec{n}} \hat{\psi}_{\vec{n}} \right) + m \hat{\bar{\psi}}_{\vec{n}} \hat{\psi}_{\vec{n}} \right\}$$

$$\hat{H}_{\text{naive}} = a^3 \sum_{\vec{n}} \left\{ \frac{i}{2a} \sqrt{\frac{2g_{4\text{d}}^2}{a}} \sum_{j=1}^{3} \left(\hat{\bar{\psi}}_{\vec{n}} \gamma^j \hat{Z}_{j,\vec{n}} \hat{\psi}_{\vec{n}+\hat{j}} - \hat{\bar{\psi}}_{\vec{n}+\hat{j}} \gamma^j \hat{\bar{Z}}_{j,\vec{n}} \hat{\psi}_{\vec{n}} \right) + m \hat{\bar{\psi}}_{\vec{n}} \hat{\psi}_{\vec{n}} \right\}$$

Forget about SUSY

Orbifold lattice for QCD

(Gharibyan, MH, Honda, Liu, 2020; Bergner, MH, Rinaldi, Schafer, in preparation)

Fermionic part

$$\hat{H}_{\rm Wilson} = a^3 \cdot \frac{i}{2a} \sum_{j=1}^{3} \sum_{\vec{n}} \left(\hat{\bar{\psi}}_{\vec{n}} - \hat{\bar{\psi}}_{\vec{n}+\hat{j}} \hat{U}^{\dagger}_{j,\vec{n}} \right) \left(\hat{\psi}_{\vec{n}} - \hat{U}_{j,\vec{n}} \hat{\psi}_{\vec{n}+\hat{j}} \right)$$

$$\Downarrow$$

$$\hat{H}_{\rm Wilson} = a^3 \cdot \frac{i}{2a} \sum_{j=1}^{3} \sum_{\vec{n}} \left(\hat{\bar{\psi}}_{\vec{n}} - \sqrt{\frac{2g_{\rm 4d}^2}{a}} \hat{\bar{\psi}}_{\vec{n}+\hat{j}} \hat{\bar{Z}}_{j,\vec{n}} \right) \left(\hat{\psi}_{\vec{n}} - \sqrt{\frac{2g_{\rm 4d}^2}{a}} \hat{Z}_{j,\vec{n}} \hat{\psi}_{\vec{n}+\hat{j}} \right)$$

Forget about SUSY

Orbifold lattice for QCD

U(N) → SU(N)

$$Z = c \cdot W \cdot U$$

Complex U(N)

- No difference in the IR if U(1) is not asymptotically free at UV
- To remove U(1) explicitly, we can add $\Delta \hat{H} \propto |\mathrm{Im}\,\det \hat{Z}|^2$

Easy for SU(2) and SU(3).
Harder for larger N.

How easy?

$$\hat{H} = \sum_{\vec{n}} \mathrm{Tr}\left(\sum_{j=1}^{3} \hat{P}_{j,\vec{n}} \hat{\bar{\bar{P}}}_{j,\vec{n}} + \frac{g_{4\mathrm{d}}^2}{2a^3} \left| \sum_{j=1}^{3} \left(\hat{Z}_{j,\vec{n}} \hat{\bar{\bar{Z}}}_{j,\vec{n}} - \hat{\bar{\bar{Z}}}_{j,\vec{n}-\hat{j}} \hat{Z}_{j,\vec{n}-\hat{j}} \right) \right|^2 \right.$$

$$\left. + \frac{2g_{4\mathrm{d}}^2}{a^3} \sum_{j<k} \left| \hat{Z}_{j,\vec{n}} \hat{Z}_{k,\vec{n}+\hat{j}} - \hat{Z}_{k,\vec{n}} \hat{Z}_{j,\vec{n}+\hat{k}} \right|^2 \right) + \Delta\hat{H} \, .$$

$$\Delta\hat{H} \equiv \frac{m^2 g_{4\mathrm{d}}^2}{2a} \sum_{\vec{n}} \sum_{j=1}^{3} \mathrm{Tr} \left| \hat{Z}_{j,\vec{n}} \hat{\bar{\bar{Z}}}_{j,\vec{n}} - \frac{a}{2g_{4\mathrm{d}}^2} \right|^2 \, .$$

$$+ \frac{\mu^2 g_{4\mathrm{d}}^2}{2a} \sum_{\vec{n}} \sum_{j=1}^{3} \left| \frac{1}{N} \mathrm{Tr}(\hat{Z}_{j,\vec{n}} \hat{\bar{\bar{Z}}}_{j,\vec{n}}) - \frac{a}{2g_{4\mathrm{d}}^2} \right|^2$$

4-boson interaction $\hat{x}_1 \hat{x}_2 \hat{x}_3 \hat{x}_4$

$$x_n = -R + n\delta_x, \qquad \delta_x = \frac{2R}{\Lambda - 1}$$

$$n = 0, 1, \cdots, \Lambda - 1$$

$$\hat{x} = \sum_{n=0}^{\Lambda-1} x_n \left| n \right\rangle \left\langle n \right| = -R \cdot \mathbf{1} + \delta_x \cdot \hat{n}$$

$$\left| n \right\rangle = \left| b_1 \right\rangle \left| b_2 \right\rangle \cdots \left| b_K \right\rangle\ , \qquad b_i = 0 \text{ or } 1\,, \qquad n = b_1 + 2b_2 \cdots + 2^{K-1} b_K$$

$$\hat{n} = \frac{\hat{\sigma}_{z,1} + 1}{2} + 2 \cdot \frac{\hat{\sigma}_{z,2} + 1}{2} + \cdots + 2^{K-1} \cdot \frac{\hat{\sigma}_{z,K} + 1}{2}$$

$$\boxed{\hat{x}_1 \hat{x}_2 \hat{x}_3 \hat{x}_4 \sim \sum \hat{\sigma}_z \otimes \hat{\sigma}_z \otimes \hat{\sigma}_z \otimes \hat{\sigma}_z}$$

$$\hat{p}^2 = \frac{1}{\delta_X^2} \sum_{n=0}^{\Lambda-1} \{2 |n\rangle \langle n| - |n+1\rangle \langle n| - |n\rangle \langle n+1|\}$$

identity

just add or subtract 1

$$\Delta \hat{H} \propto |\mathrm{Im}\,\det \hat{Z}|^2 \quad \leftarrow \quad \hat{\sigma}_z^{\otimes 6}$$ for SU(3)

Future Directions

- Study of QCD via orbifold lattice
    - How large scalar mass? RG flow? Better lattice fermions?
    - Euclidean lattice simulation is enough
- Resource estimate for quantum simulation
- Quantum algorithm
- Quantum simulation of matrix model (before QCD)
    - Quantum Gravity in the Lab
    - SYK and spin models are technically similar to orbifold lattice