Gravitational edge mode in $\mathcal{N} = 1$ super Jackiw-Teitelboim (JT) gravity

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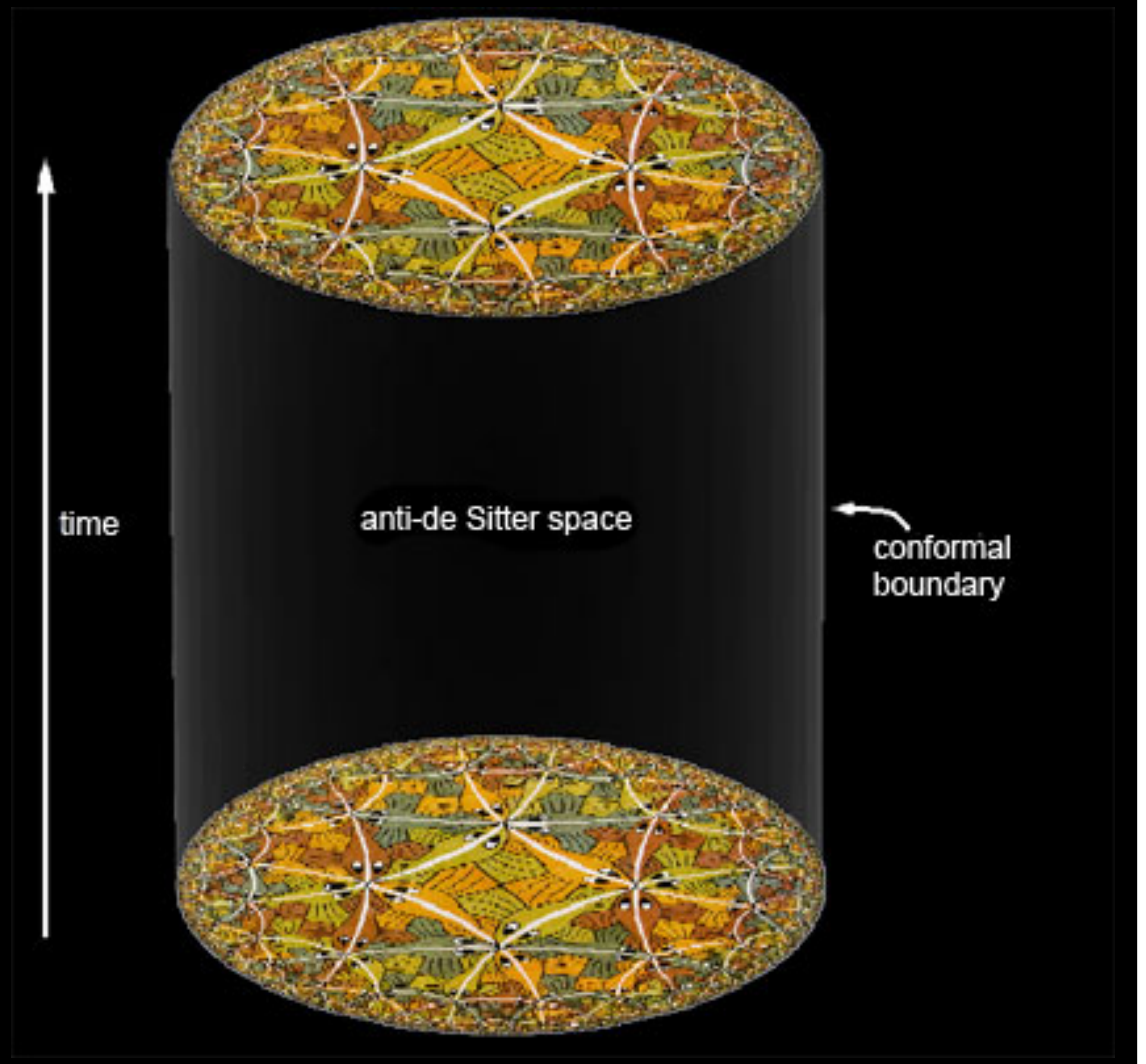
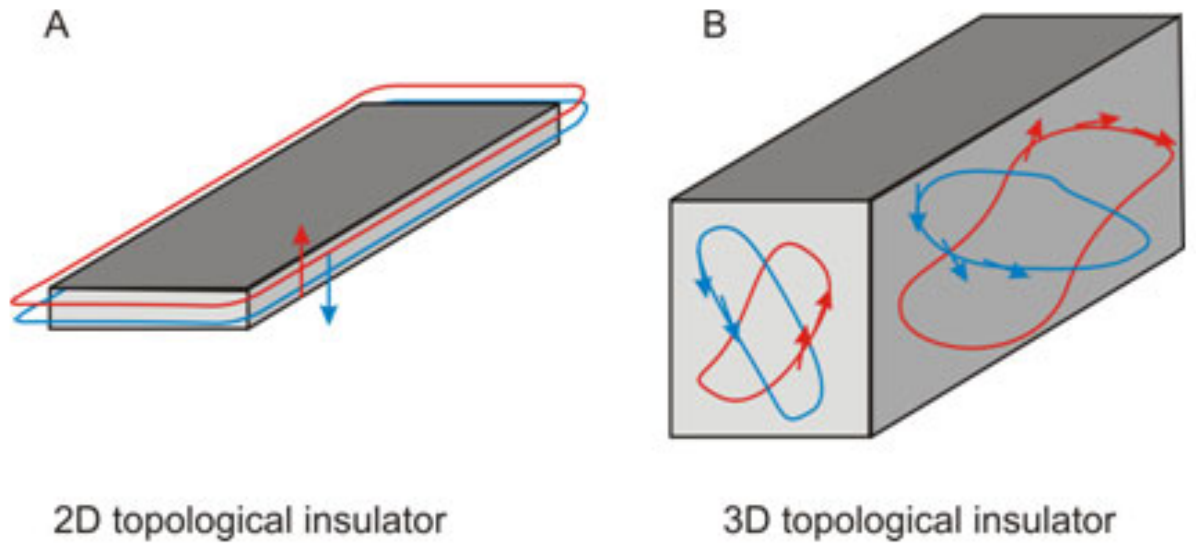
Recent developments in Large N, Holography and Complexity

Physical systems with boundary

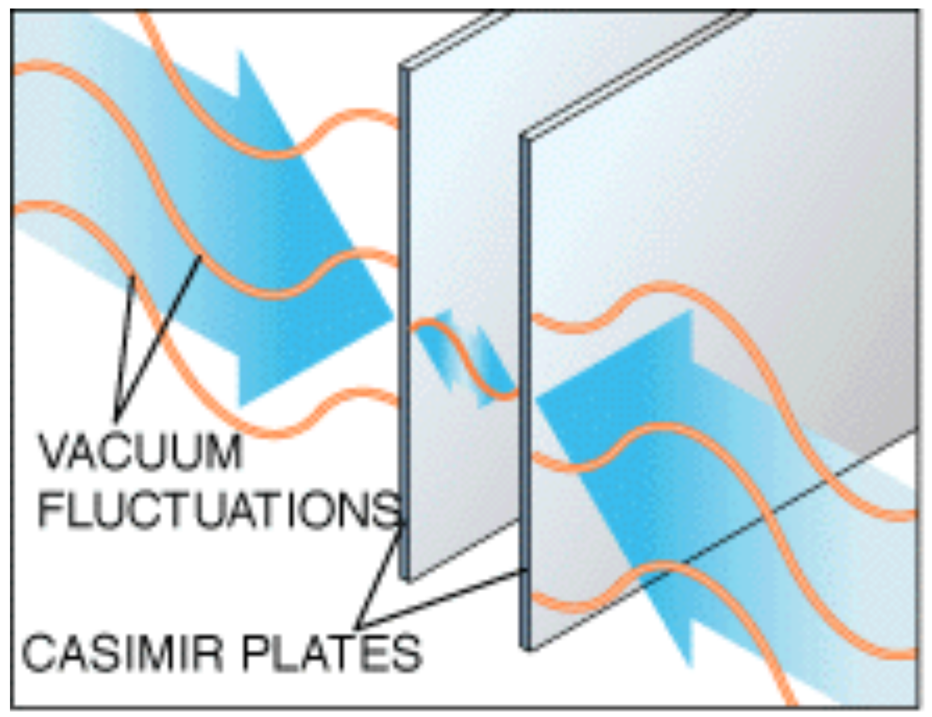
AdS/CFT



Topological insulator



Casimir effect …



"God made the bulk; surfaces were invented by the devil" - W.Pauli

Gravitational Edge mode

Bulk

Boundary

Where are the rest of the symmetry? Do they gone for good?

How boundary affects the gauge symmetry?

Edge mode is the advent of **physical degrees of freedom** on the ***boundary*** which was originally the gauge redundancy in the ***bulk***.

Thus, it is also called as **would-be-gauge mode**.

Gravitational edge mode in JT gravities

Part I. **Bosonic** JT gravity

Part II. $\mathcal{N} = 1$ **supersymmetric** JT gravity

KL, Akhil Sivakumar (APCTP), Junggi Yoon (APCTP)

Part I. **Bosonic** JT gravity

Gravitational edge mode in JT gravity

Gravitational Edge Mode in Asymptotically AdS_2
: JT Gravity Revisited

Euihun Joung[a] **Prithvi Narayan**[b] **Junggi Yoon**[c,d,e]

arXiv: **2304.06088**

- Revisit the derivation of the **Schwarzian action**
- Alternative description for the gravitational edge mode from the **metric fluctuation** with the fixed boundary
- Make connection with the description with **wiggling boundary**

Jackiw-Teitelboim (JT) 2D dilaton gravity

Dilaton

Extrinsic curvature

$$I_{\rm JT} = \int_{\mathcal{M}} d^2x \sqrt{g}\phi(R+2) + 2\int_{\partial\mathcal{M}} du\sqrt{h}\phi(K-K_0)$$

Teitelboim ’83, Jackiw ’85

- Exactly solvable and non-perturbative contributions are also comprehensible
- Describes low-energy sector of Sachdev-Ye-Kitaev (SYK) model: **Schwarzian theory**
- Describes near-horizon region of near-extremal higher-dimensional black holes

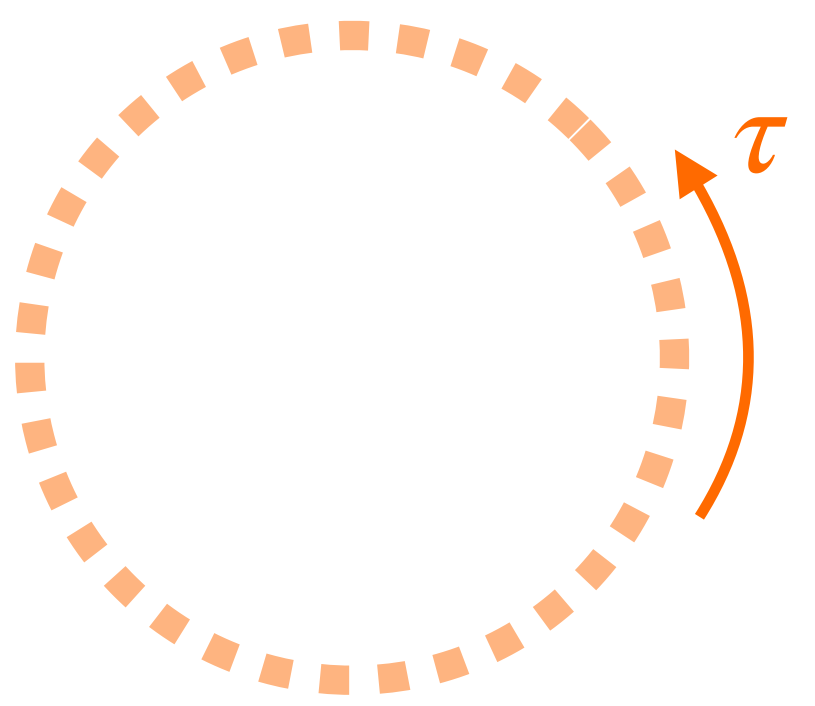
Classical solution of JT gravity

Dilaton

$$I_{\mathrm{JT}} = \int_{\mathcal{M}} d^2x\sqrt{g}\phi(R+2) + 2\int_{\partial\mathcal{M}} du\sqrt{h}\phi(K-K_0)$$

Classical solution

$$ds^2 = (r^2-1)d\tau^2 + \frac{dr^2}{r^2-1} \qquad \phi = r$$



Integrate out $\phi \Rightarrow R = -2$

The remaining degrees of freedom are all on the boundary.

Two descriptions for the edge mode in JT gravity

I. Wiggling boundary

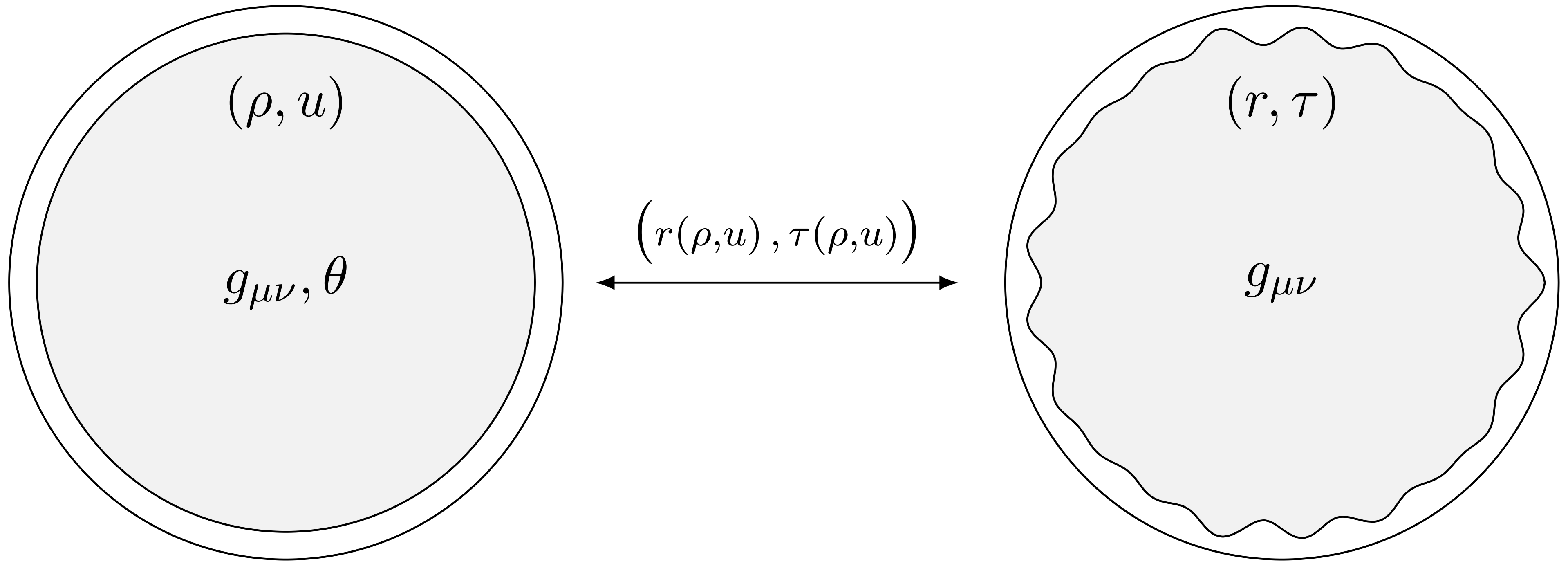
*With fixed **proper boundary length***

II. Metric fluctuation

*With fixed **asymptotic boundary condition***

I. Wiggling boundary

$$ds^2 = (\rho^2 - 1)du^2 + \frac{d\rho^2}{\rho^2 - 1}$$



$$ds^2 = (r^2 - 1)d\tau^2 + \frac{dr^2}{r^2 - 1}$$

$$\rho = \epsilon^{-1}$$

$$r(u) \equiv r(\epsilon^{-1}, u)$$
$$\tau(u) \equiv \tau(\epsilon^{-1}, u)$$

Fixing the proper boundary length

$$\epsilon^{-2} - 1 = \frac{r'(u)^2}{r(u)^2 - 1} + \big(r(u)^2 - 1\big)\tau'(u)^2$$

J.Maldacena, D.Stanford, and Z.Yang '16

Dilaton

$$\phi(u) = r(u) = \frac{1}{\epsilon\tau'(u)} + \mathcal{O}(\epsilon)$$

Extrinsic curvature

$$K = \frac{(t, \nabla_t n)}{(t, t)} = 1 + \epsilon^2 \left(\frac{\tau'''}{\tau'} - \frac{3}{2}\frac{\tau''^2}{\tau'^2} + \frac{1}{2}\tau'^2 \right) + \mathcal{O}(\epsilon^4)$$

Schwarzian action..?

$$K - K_0 = \epsilon^2 \left(\frac{\tau'''}{\tau'} - \frac{3}{2}\frac{\tau''^2}{\tau'^2} + \frac{1}{2}\tau'^2 - \frac{1}{2} \right) + \mathcal{O}(\epsilon^4)$$

$$\phi(u) = r(u) = \frac{1}{\epsilon\tau'(u)} + \mathcal{O}(\epsilon)$$

$$I_{\mathrm{JT}} = \int_{\mathcal{M}} d^2x\sqrt{g}\phi(R+2) + 2\int_{\partial\mathcal{M}} du\sqrt{h}\phi(K-K_0)$$

$$= -2\int du \left(-\frac{1}{\tau'}\mathrm{Sch}[\tau,u] - \frac{1}{2}\tau' + \frac{1}{2\tau'} \right)$$ **..?**

$$\mathrm{Sch}[\tau,u] = \frac{\tau'''}{\tau'} - \frac{3}{2}\left(\frac{\tau''}{\tau'}\right)^2$$

II. Metric fluctuation

$$ds^2 = \frac{d\rho^2}{\rho^2 - r_h^2} + G_{uu}(\rho, u)du^2$$

Fefferman-Graham (FG) gauge

$$G_{uu}(\rho, u) = \rho^2 + G(u) + \dots$$

$$ds^2 = \frac{dr^2}{r^2 - r_h^2} + g_{\tau\tau}(r, \tau)d\tau^2$$

Which coordinate transformation map still satisfy FG gauge condition?

$$g_{\tau\tau}(r, \tau) = r^2 + g(\tau)$$

↓

$$r = \frac{\rho}{\tau'(u)} + \frac{1}{4\rho}\left(\frac{\tau''(u)^2}{\tau'(u)^3} - \frac{1}{\tau'(u)} + \tau'(u)\right) + \mathcal{O}(\rho^{-3})$$

$$\tau = \tau(u) - \frac{1}{2\rho^2}\tau''(u) + \mathcal{O}(\rho^{-4})$$

Metric fluctuation

$$G_{uu} = \rho^2 - \frac{1}{2}(\tau'^2 + 1) - \frac{\tau'''}{\tau'} + \frac{3\tau''^2}{2\tau'^2} + \mathcal{O}(\rho^{-2})$$

One more step for the Schwarzian action

From two descriptions of gravitational edge mode, we equivalently have

$$I_{JT} = -2\int du\tau'\left(-\frac{1}{\tau'^2}\mathrm{Sch}[\tau, u] - \frac{1}{2} + \frac{1}{2\tau'^2}\right)$$

Inversion $\tau(u) \to u(\tau)$

$$\tau'(u) \to \frac{1}{u'(\tau)} \qquad\qquad \mathrm{Sch}[\tau; u] \to -\tau'^2\mathrm{Sch}[u; \tau]$$

Schwarzian action at finite temperature

$$I_{JT} \to -2\int d\tau\left(\mathrm{Sch}[u;\tau] + \frac{1}{2}u'^2 - \frac{1}{2}\right)$$

Path integral measure

$$\mathcal{D}\tau(u) \to \frac{\mathcal{D}u(\tau)}{u'(\tau)}$$

E.Witten, D.Stanford '17

Two descriptions?

I. Wiggling boundary

Exact AdS_2 metric

with **fluctuating boundary**

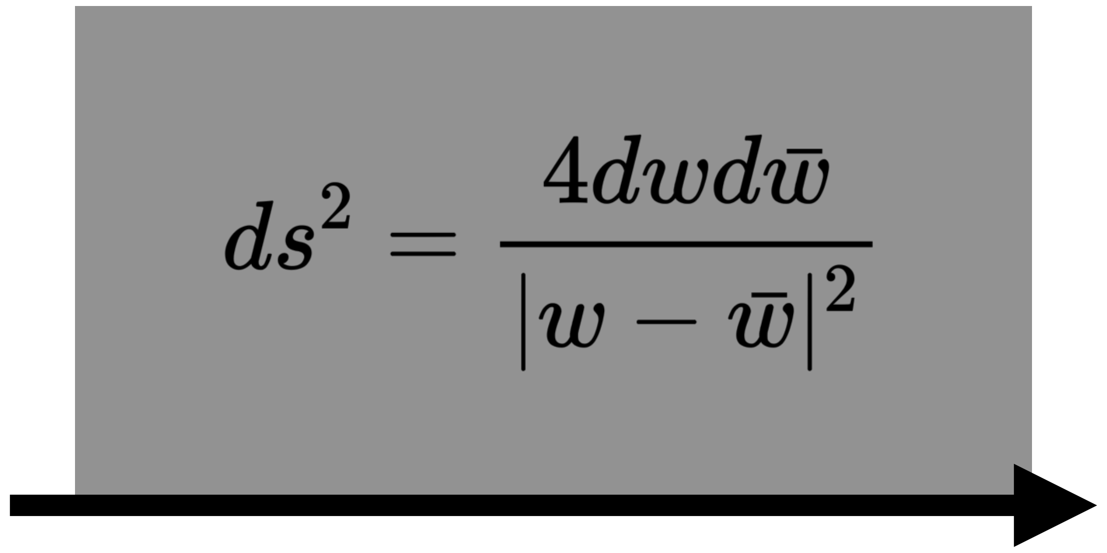
II. Metric fluctuation

Fluctuating AdS_2 metric

with fixed boundary

Isometry of exact Poincaré disk metric

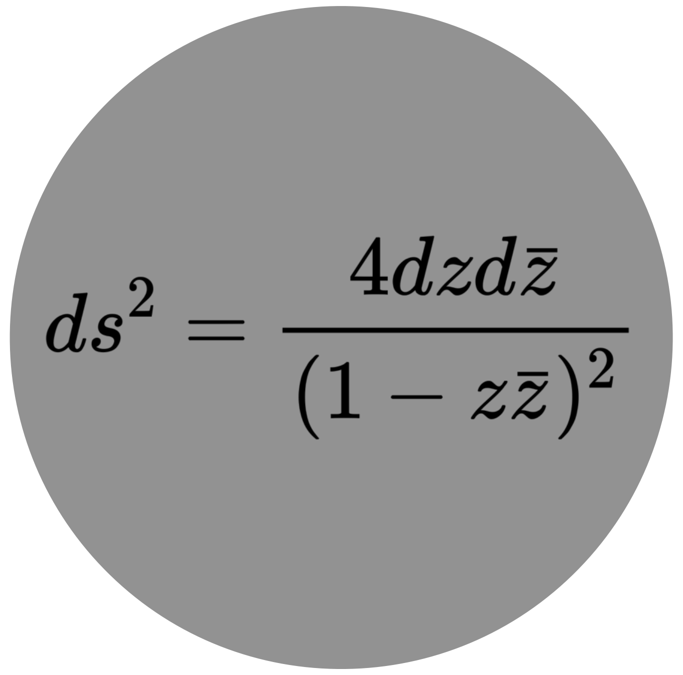
Poincaré upper half plane



$$ds^2 = \frac{4dwd\bar{w}}{|w-\bar{w}|^2}$$

$$w = \frac{-iz+1}{z-i}$$

Poincaré disk



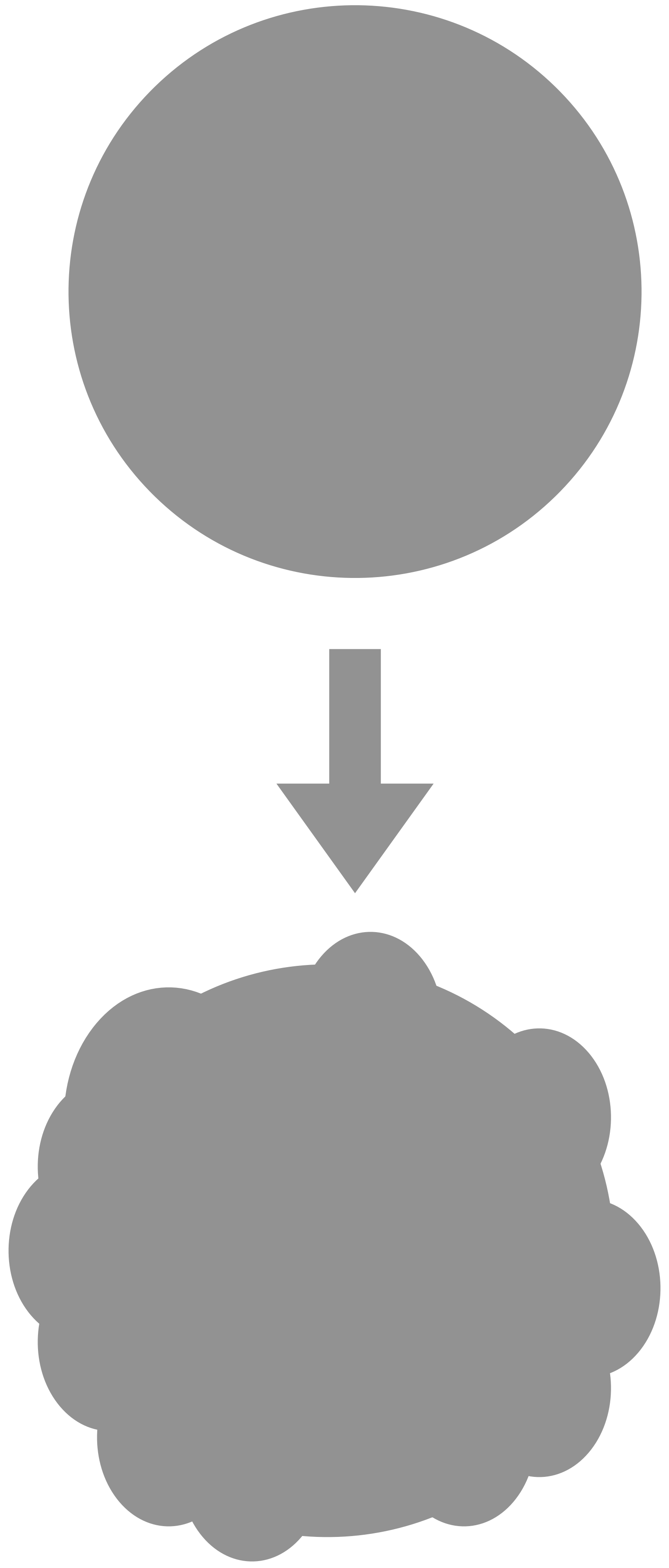
$$ds^2 = \frac{4dzd\bar{z}}{(1-z\bar{z})^2}$$

$$w \longrightarrow \frac{aw+b}{cw+d}$$

$$z \longrightarrow \frac{z(ia-b+c+id)-a+i(b+c)+d}{z(a+i(b+c)-d)+ia+b-c+id}$$

Gauge symmetry of Schwarzian

Exact Poincaré disk



Bulk isometry $z \longrightarrow \frac{z(ia-b+c+id)-a+i(b+c)+d}{z(a+i(b+c)-d)+ia+b-c+id}$

Boundary isometry $\tan\frac{u}{2} \longrightarrow \frac{a\tan\frac{u}{2}+b}{c\tan\frac{u}{2}+d}$

Boundary isometry is redundant one that maps into the $\mathrm{SL}(2,\mathbb{R})$ **gauge symmetry** of Schwarzian theory

Part II. $\mathcal{N} = 1$ supersymmetric JT gravity

Supersymmetric JT gravity

$$I_{sJT} = i\int_{\mathcal{M}} dz d\bar{z}\, d\theta d\bar{\theta}\, \mathsf{E}\, \Phi\, (\mathcal{R}_{\theta\bar{\theta}} - 2) + 2\int_{\partial\mathcal{M}} du d\vartheta\, e\, \Phi(\mathcal{K} - \mathcal{K}_0)$$

Space	$(z,\bar{z})$ u	$\longrightarrow$ $\longrightarrow$	$(z,\theta,\bar{z},\bar{\theta})$ (u,ϑ)	**Superspace**
Metric	$g_{\mu\nu}$	$\longrightarrow$	$\mathsf{E}_M{}^A$	**Supervielbein**
Curvatures	$R,\ K$	$\longrightarrow$	$\mathcal{R}_{\theta\bar{\theta}},\ \mathcal{K}$	**Supercurvatures**

Classical solutions

$$I_{sJT} = i\int_{\mathcal{M}} dz d\bar{z}\, d\theta d\bar{\theta}\, \mathsf{E}\, \Phi\, (\mathcal{R}_{\theta\bar{\theta}} - 2) + 2\int_{\partial\mathcal{M}} du d\vartheta\, e\, \Phi(\mathcal{K} - \mathcal{K}_0)$$

Superconformal gauge

$${\mathsf{E}_N}^B = \begin{pmatrix} \mathsf{E}_{\text{holo}} & 0 \\ 0 & \mathsf{E}_{\text{anti-holo}} \end{pmatrix} \quad \text{where} \quad \mathsf{E}_{\text{holo}} = \begin{pmatrix} e^{\Sigma} & De^{\Sigma/2} \\ -\eta e^{\Sigma} & D(\eta e^{\Sigma/2}) \end{pmatrix}$$

(Bosonic) conformal gauge and classical solution

$$ds^2 = 4e^{2\Sigma} dz d\bar{z}$$

$$e^{\Sigma} = \frac{1}{1 - z\bar{z}}, \quad \Phi = \frac{1 + z\bar{z}}{1 - z\bar{z}}$$

$$ds^2 = 4dy^M {\mathsf{E}_M}^{\mathbb{z}} dy^N {\mathsf{E}_N}^{\bar{\mathbb{z}}}$$

$$e^{\Sigma} = \frac{1}{1 - z\bar{z} + i\theta\bar{\theta}}, \quad \Phi = \frac{1 + z\bar{z}}{1 - z\bar{z} + i\theta\bar{\theta}}$$

Wiggling boundary

Matching the proper boundary length

Bosonic

$$\frac{du^2}{\epsilon^2} = ds^2 \quad \left(= 4dy^m e_m{}^{z} dy^n e_n{}^{\bar{z}}\right)$$

$$z = -ie^{i\tau(u)}(1 - \epsilon\tau'(u) + \mathcal{O}(\epsilon^2))$$

Supersymmetric

$$\frac{\left(du^2 + \vartheta d\vartheta\right)^2}{\epsilon^2} = 4dy^M \mathsf{E}_M{}^{z} dy^N \mathsf{E}_N{}^{\bar{z}}$$

$$z = -ie^{i\tau(u)}\left(1 - \epsilon\left((\mathcal{D}\xi)^2 - \xi\mathcal{D}^2\xi\right) + \mathcal{O}(\epsilon^2)\right)$$

$$\theta = e^{\frac{i\tau(u)}{2}}\left(\xi - \epsilon\left(\xi(\mathcal{D}\xi)^2/2 - i\mathcal{D}^2\xi\right) + \mathcal{O}(\epsilon^2)\right)$$

$$\mathcal{T}^{z} = \partial_u y^N \mathsf{E}_N{}^{z} = \frac{1}{2\epsilon}\frac{\mathcal{D}\theta}{\mathcal{D}\bar{\theta}} \qquad \mathcal{N}_{z} = \frac{i}{2}\frac{\mathcal{D}\bar{\theta}}{\mathcal{D}\theta}$$

super-Schwarzian

Super extrinsic curvature

$$\mathcal{K} = 4\epsilon^2\left(\mathrm{sSch}[\tau,\xi;u,\vartheta] + \frac{1}{2}\xi(\mathcal{D}\xi)^3\right) + \mathcal{O}(\epsilon^3) \qquad \mathrm{sSch}[\tau,\xi;u,\vartheta] = \frac{\mathcal{D}^4\xi}{\mathcal{D}\xi} - 2\frac{\mathcal{D}^2\xi\mathcal{D}^3\xi}{(\mathcal{D}\xi)^2}$$

$$\mathcal{D}\tau = \xi\mathcal{D}\xi \quad \Rightarrow \quad \xi = \sqrt{\tau'(u)}\left(\vartheta + \psi(u) + \frac{1}{2}\vartheta\psi(u)\psi'(u)\right)$$

In terms of component fields $\tau(u), \psi(u)$

$$\mathrm{sSch}[\tau,\xi;u,\vartheta] + \frac{1}{2}\xi(\mathcal{D}\xi)^3 = \frac{\vartheta}{2}\left[(1-\psi\psi')\left(\mathrm{Sch}[\tau,u] + \frac{1}{2}\tau'^2\right) + \psi\psi''' + 3\psi'\psi''\right] + \left(1 + \frac{1}{2}\psi\psi'\right)\psi'' + \frac{1}{2}\psi\left(\mathrm{Sch}[\tau,u] + \frac{1}{2}\tau'^2\right)$$

Metric fluctuation

$$(w, \eta, \bar{w}, \bar{\eta}) \longleftrightarrow (z, \theta, \bar{z}, \bar{\theta})$$

$$\mathcal{E}_M{}^A \longleftrightarrow \mathsf{E}_N{}^B$$

Asymptotic boundary condition (~FG gauge condition)

$$\mathcal{E}_M{}^A = \begin{pmatrix} \mathcal{E}_{\text{holo}} & 0 \\ 0 & \mathcal{E}_{\text{anti-holo}} \end{pmatrix} \qquad \mathsf{E}_N{}^B = \begin{pmatrix} \mathsf{E}_{\text{holo}} & 0 \\ 0 & \mathsf{E}_{\text{anti-holo}} \end{pmatrix}$$

⬇

Superconformal transformation

$$z \to w(z,\theta), \quad \theta \to \eta(z,\theta), \quad \bar{z} \to \bar{w}(\bar{z},\bar{\theta}), \quad \bar{\theta} \to \bar{\eta}(\bar{z},\bar{\theta})$$

$$\mathsf{D}w = \eta \mathsf{D}\eta \,, \qquad \bar{\mathsf{D}}\bar{w} = \bar{\eta}\bar{\mathsf{D}}\bar{\eta}$$

Superconformal transformation

$$z \to w(z,\theta), \quad \theta \to \eta(z,\theta), \quad \bar{z} \to \bar{w}(\bar{z},\bar{\theta}), \quad \bar{\theta} \to \bar{\eta}(\bar{z},\bar{\theta})$$

$$\mathrm{D}w = \eta \mathrm{D}\eta \ , \qquad \bar{\mathrm{D}}\bar{w} = \bar{\eta}\bar{\mathrm{D}}\bar{\eta}$$

$$w(z,\theta) = F\left(z + \theta\Psi(z)\right) = F(z) + \theta\Psi(z)\partial_z F(z)$$

$$\eta(z,\theta) = \sqrt{\partial_z F(z)}\left(\theta + \Psi(z) + \frac{1}{2}\theta\Psi(z)\partial_z\Psi(z)\right)$$

Asymptotic solution of F, Ψ in the order of $(1-r)$ can be found as

$$F = -ie^{i\tau(u)}\left(1 - (1-r)\tau'(u) + \frac{(1-r)^2}{2}\left(\tau'(u)^2 - \tau'(u) - i\tau''(u)\right) + \mathcal{O}\left((1-r)^3\right)\right)$$

$$\Psi = e^{\frac{i\tau(u)}{2}}\left(\psi(u) + (1-r)\left(i\psi'(u) - \frac{\psi(u)}{2}\right) - \frac{(1-r)^2}{2}\left(\psi''(u) + \frac{\psi(u)}{4}\right) + \mathcal{O}\left((1-r)^3\right)\right)$$

$$\mathcal{K} - \mathcal{K}_0 = 4\epsilon^2\left(\mathrm{sSch}[\tau,\xi;u,\vartheta] + \frac{1}{2}\xi(\mathcal{D}\xi)^3 - \frac{1}{2}\vartheta\right) + \mathcal{O}(\epsilon^3)$$

One more step to get the super-Schwarzian

$$I_{sJT} \quad \longrightarrow \quad 4\int du d\vartheta (\mathcal{D}\xi)^{-2}\left(\text{sSch}[\tau,\xi;u,\vartheta] + \frac{1}{2}\xi(\mathcal{D}\xi)^3 - \frac{1}{2}\vartheta\right) + \mathcal{O}(\epsilon)$$

Inversion

$$(u,\vartheta) \quad \longleftrightarrow \quad (\tau,\xi)$$

$$\text{Sch}[\tau,\xi;u,\vartheta] = -(D\xi)^3\,\text{Sch}[u,\vartheta;\tau,\xi], \quad du d\vartheta = d\tau d\xi (D\xi)^{-1}$$

super-Schwarzian action at finite temperature

$$I_{sJT} \quad \longrightarrow \quad -4\int d\tau d\xi \left(\text{sSch}[u,\vartheta;\tau,\xi] + \frac{1}{2}\vartheta(D_\xi\vartheta)^3 - \frac{1}{2}\xi\right) + \mathcal{O}(\epsilon)$$

Summary

- Two descriptions to derive the gravitational edge mode dynamics
- Confirmed the derivation of the $\mathcal{N} = 1$ **super-Schwarzian** from super-JT gravity
- We directly derived the **finite temperature** super-Schwarzian action without relying on the transformation from the zero temperature one.
- We also worked out the derivation of the $\mathcal{N} = 1$ super-Schwarzian action from the **BF theory** with supergroup $\mathrm{OSp}(2,1)$.

Discussion

Gravitational edge modes…

on the manifold with nontrivial topology?

on non-AdS geometry?

at finite boundary?

in higher-dimensional gravity?

Thank

YU!