Quantum Symmetries in $\mathcal{N} = 2$ SCFT's

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Based on arXiv: 2106.08449 (with E. Pomoni and R. Rabe) and ongoing work

Outline

- Explain what is meant by "quantum" symmetry
- Review $\mathcal{N} = 2$ superconformal theories, arising as orbifolds of the $\mathcal{N} = 4$ theory
- Briefly discuss the types of spin chains that arise in the planar limit of these theories
- Construct the quantum symmetry of the $\mathbb{Z}_2$ orbifold theory

Lie groups and Lie algebras

- Lie groups $\leftrightarrow$ Symmetry in physics

$$G = e^{i\sum_k \alpha^k T^k}$$

- T^k are Lie algebra generators
- $\mathrm{SU}(2) \to$ theory of spin in Quantum Mechanics

$$\vec{S} = \hat{x} S_x + \hat{y} S_y + \hat{z} S_z$$

where

$$S_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ S_y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ S_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Product is non-commutative, e.g. $S_x S_y \neq S_y S_x$

The coproduct

- Algebra elements act naturally on a representation space $\{|\psi\rangle\}$ (Hilbert space in QM)

$$S_z\,|\uparrow\rangle = \frac{\hbar}{2}\,|\uparrow\rangle\,, \quad S_z\,|\downarrow\rangle = -\frac{\hbar}{2}\,|\downarrow\rangle$$

- How does $\vec{S}$ act on $|\psi_1\rangle \otimes |\psi_2\rangle$?

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$$\Delta(\vec{S})(|\psi_1\rangle \otimes |\psi_2\rangle) = (\vec{S}\,|\psi_1\rangle) \otimes |\psi_2\rangle + |\psi_1\rangle \otimes (\vec{S}\,|\psi_2\rangle)$$

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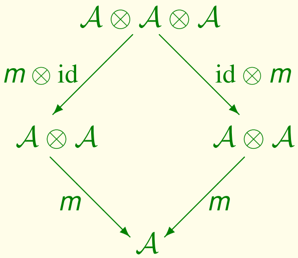
- We have just used a coproduct:

$$\Delta(X) = X \otimes 1 + 1 \otimes X$$

- Δ is co-commutative: $\tau_{12} \circ \Delta(X) = \Delta(X)$
- Hopf algebra: Allow Δ to be non-co-commutative

Hopf Algebras

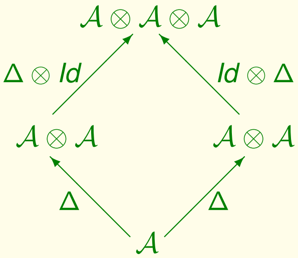
- An *algebra* $\mathcal{A}$ (over a field k) is a vector space together with a product $m : \mathcal{A} \otimes \mathcal{A} \to \mathcal{A}$ and a unit map $\eta : k \to \mathcal{A}$



(+ more diagrams)

Hopf Algebras

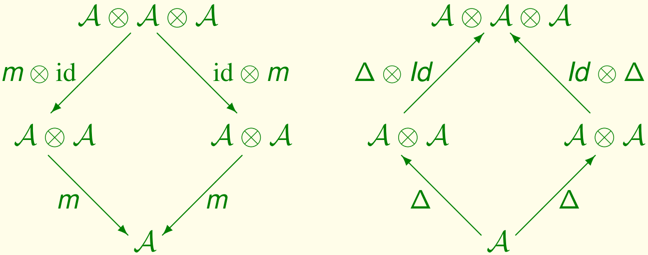
- A *coalgebra* $\mathcal{A}$ is instead equipped with a coproduct $\Delta : \mathcal{A} \to \mathcal{A} \otimes \mathcal{A}$ and a counit $\epsilon : \mathcal{A} \to k$



(+ more diagrams)

Hopf Algebras

- A *bialgebra* is both an algebra and a coalgebra in a compatible way



(+ more diagrams)

- A Hopf Algebra is a bialgebra equipped with an antipode $S : \mathcal{A} \to \mathcal{A}$

$$m\,(S \otimes \mathrm{id}) \circ \Delta = m\,(\mathrm{id} \otimes S) \circ \Delta = \eta \circ \epsilon\ .$$

Quasitriangular Hopf algebras

- Would like both m and Δ to be non-commutative
- However, there can still be a relation between Δ and $\tau_{12}\Delta$.

$$\tau_{12} \circ \Delta(a) = R(\Delta(a))R^{-1}$$

- $R : \mathcal{A} \otimes \mathcal{A} \to \mathcal{A} \otimes \mathcal{A}$ is called an *R-matrix.*
- Quantum Yang–Baxter Equation (QYBE)

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12} \qquad \left(R^{i\ \,j}_{\ s\ \,r}R^{s\ \,k}_{\ l\ \,p}R^{r\ \,p}_{\ m\ \,n} = R^{j\ \,k}_{\ s\ \,p}R^{i\ \,p}_{\ r\ \,n}R^{r\ \,s}_{\ l\ \,m}\right)$$

- A Hopf algebra with R satisfying the YBE is called quasitriangular $\to$ “Quantum Group”
- The YBE guarantees that the algebra is not too trivial

Drinfeld twists

- Given an initial Hopf algebra, we can twist it to produce (in general) a quasi-Hopf algebra
- Drinfeld twist $F : \mathcal{A} \otimes \mathcal{A} \longrightarrow \mathcal{A} \otimes \mathcal{A}$

$$\Delta_F(a) = F(\Delta(a))F^{-1} \,, \qquad\qquad R_F = F_{21} \cdot R \cdot F^{-1}$$

- To get a Hopf algebra, F should satisfy the cocycle condition

$$(1 \otimes F)(\mathrm{id} \otimes \Delta)F = (F \otimes 1)(\Delta \otimes \mathrm{id})F$$

- Then, R_F will satisfy the QYBE
- If not $\Rightarrow$ quasi-Hopf algebra (non-associative)

The $\mathcal{N} = 4$ SYM theory

- Maximal supersymmetry in $d = 4$
- Contains a gauge field A_μ, 4 spinors ψ^A_α, and 3 complex scalars ϕ^i.
- All in the adjoint of $\mathrm{SU}(N)$ gauge group $\to N \times N$ matrices
- Convenient to use $\mathcal{N} = 1$ superspace notation

$$\mathcal{L} = \int d^4\theta \mathrm{Tr} e^{gV} \overline{\Phi}_i e^{-gV} \Phi^i + \left(\int d^2\theta \mathcal{W} + \int d^2\bar{\theta} \overline{\mathcal{W}} \right) + \cdots$$

- Chiral Superfields $\Phi^i = \phi^i + \theta^\alpha \psi^i_\alpha + \theta^2 F^i, \ i = 1, 2, 3$
- $\mathcal{N} = 4$ superpotential:

$$\mathcal{W} = g \ \mathrm{Tr} \ \Phi^1 [\Phi^2, \Phi^3] = \frac{g}{3} \epsilon_{ijk} \ \mathrm{Tr} \ \Phi^i \Phi^j \Phi^k$$

- The actual potential of the QFT is derived as

$$V = \frac{\partial \bar{\mathcal{W}}}{\partial \bar{\phi}_i} \frac{\partial \mathcal{W}}{\partial \phi^i}$$

Reducing supersymmetry

- In previous work, looked at the $\mathcal{N}=1$ marginal deformations of $\mathcal{N}=4$ SYM [Dlamini, KZ '19]

$$\mathcal{W}_{LS} = \kappa \mathrm{Tr}\left(\Phi^1[\Phi^2,\Phi^3]_q + \frac{h}{3}\left((\Phi^1)^3 + (\Phi^2)^3 + (\Phi^3)^3\right)\right)$$

- q–commutator $[X,Y]_q = XY - qYX$
- Identified a Drinfeld twist which takes us from $(q,h)=(1,0)$ to the general case $\Rightarrow \mathrm{SU}(3)_{q,h}$
- Does not satisfy the cocycle condition
- Today we will look at $\mathcal{N}=2$ theories, which are not purely superpotential deformations.

$\mathbb{Z}_2$ orbifold of $\mathcal{N} = 4$ SYM

- Start with $\mathcal{N} = 4$ SYM with $\mathrm{SU}(2N)$ gauge group
- 6 real (3 complex) scalar fields: $\mathrm{SO}(6) \sim \mathrm{SU}(4)$ R-symmetry group
- Project $(V, X, Y, Z) \to (V, -X, -Y, Z)$ in R-symmetry space
- Project by $[\cdots] \to \gamma[\cdots]\gamma$ in colour space, where

$$\gamma = \begin{pmatrix} I_{N\times N} & 0 \\ 0 & -I_{N\times N} \end{pmatrix}$$

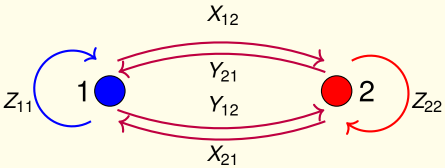
- End up with $\mathcal{N} = 2$ SYM with $\mathrm{SU}(N)_1 \times \mathrm{SU}(N)_2$ gauge group

$$Z = \begin{pmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{pmatrix}, \; X = \begin{pmatrix} 0 & X_{12} \\ X_{21} & 0 \end{pmatrix}, \; Y = \begin{pmatrix} 0 & Y_{12} \\ Y_{21} & 0 \end{pmatrix}$$

- Z's adjoints, X, Y bifundamentals

$\mathbb{Z}_2$ orbifold of $\mathcal{N} = 4$ SYM

- Represent using a quiver diagram:



- Superpotential: $\mathcal{W}_{N=4} = ig\mathrm{Tr}(X[Y, Z]) \rightarrow$

$$\mathcal{W}_{N=2} = ig\left(\mathrm{Tr}_2(Y_{21}Z_{11}X_{12} - X_{21}Z_{11}Y_{12}) - \mathrm{Tr}_1(X_{12}Z_{22}Y_{21} - Y_{12}Z_{22}X_{21})\right)$$

- The $\mathrm{SU}(4)_R$ symmetry is (naively) broken to $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)$.

Marginally deformed orbifold

- Move away from the orbifold point: $g_1 \neq g_2$

$$\mathcal{W} = ig_1 \,\mathrm{Tr}_2(Y_{21} Z_{11} X_{12} - X_{21} Z_{11} Y_{12}) - ig_2 \,\mathrm{Tr}_1(X_{12} Z_{22} Y_{21} - Y_{12} Z_{22} X_{21})$$

- Still preserves $\mathcal{N} = 2$ supersymmetry
- Studied in detail in [Gadde,Pomoni,Rastelli '10].
- Leads to interesting spin chains in the planar limit
- Focus on holomorphic $\mathrm{SU}(3)$ sector
  - Unbroken $\mathrm{SU}(2)$ subsector made up of X, Y fields
  - "$\mathrm{SU}(2)$-like" subsector made up of X, Z fields
- First recall how spin chains arise in $\mathcal{N} = 4$ SYM

Spin chains from $\mathcal{N} = 4$ SYM

[Minahan-Zarembo '02]

- Take planar limit $N \to \infty$
- Observables are gauge invariant operators
- E.g. $SU(2)$ scalar sector: X, Y

$$O(x) = Tr\left[\cdots XXXYXXXYXX \cdots\right]$$

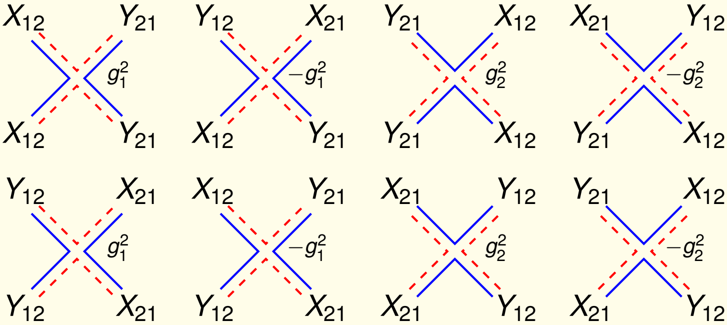
- We want to diagonalise the dilatation operator of the theory

$$DO(x) = (L + \gamma)O(x)$$ γ: anomalous dimension

- Difficult problem because of operator mixing
- At one-loop, D acts exactly like a Heisenberg Hamiltonian
- Integrability $\Rightarrow$ Solution of the $\mathcal{N} = 4$ spectral problem

XY sector: Diagrams

- F-term contributions to the Hamiltonian



- Will rescale by $g_1 g_2$ and define $\kappa = g_2/g_1$.

XY sector: Hamiltonian

- $\mathcal{N} = 2$ picture

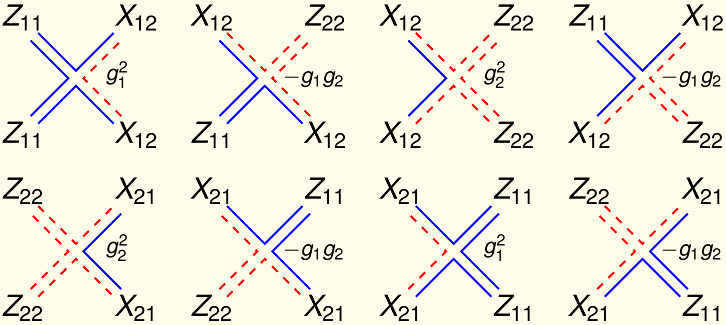
$$\mathcal{H}_{\ell,\ell+1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa^{-1} & -\kappa^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\kappa^{-1} & \kappa^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa & -\kappa & 0 \\ 0 & 0 & 0 & 0 & 0 & -\kappa & \kappa & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ on: } \begin{pmatrix} X_{12}X_{21} \\ X_{12}Y_{21} \\ Y_{12}X_{21} \\ Y_{12}Y_{21} \\ X_{21}X_{12} \\ X_{21}Y_{12} \\ Y_{21}X_{12} \\ Y_{21}Y_{12} \end{pmatrix}$$

- "Dynamical $\mathcal{N} = 4$" picture

$$\mathcal{H}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa^{-1} & -\kappa^{-1} & 0 \\ 0 & -\kappa^{-1} & \kappa^{-1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \ \mathcal{H}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa & -\kappa & 0 \\ 0 & -\kappa & \kappa & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ on: } \begin{pmatrix} XX \\ XY \\ YX \\ YY \end{pmatrix}_i$$

XZ sector: Diagrams

- F-term contributions to the Hamiltonian



- Will again rescale by $g_1 g_2$ and define $\kappa = g_2/g_1$.

XZ sector: Hamiltonian

- $\mathcal{N} = 2$ picture

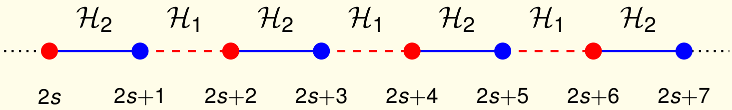
$$\mathcal{H}_{i,i+1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & \kappa^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa^{-1} & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & \kappa & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ on: } \begin{pmatrix} X_{12}X_{21} \\ X_{12}Z_{22} \\ Z_{11}X_{12} \\ Z_{11}Z_{11} \\ X_{21}X_{12} \\ X_{21}Z_{11} \\ Z_{22}X_{21} \\ Z_{22}Z_{22} \end{pmatrix} .$$

- "Dynamical $\mathcal{N} = 4$" picture

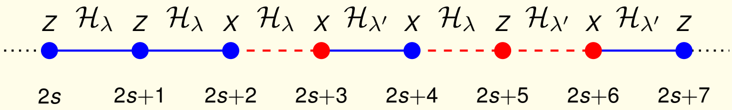
$$\mathcal{H}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa & -1 & 0 \\ 0 & -1 & \kappa^{-1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \mathcal{H}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa^{-1} & -1 & 0 \\ 0 & -1 & \kappa & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ on: } \begin{pmatrix} XX \\ XZ \\ ZX \\ ZZ \end{pmatrix}_i$$

Dynamical spin chains

- The XY chain is strictly alternating:



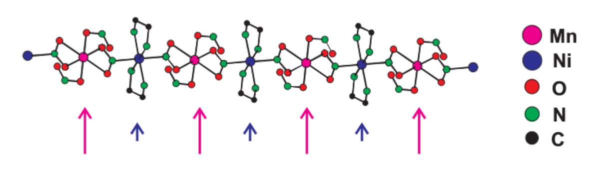
- The XZ chain is “dynamical”: The Hamiltonian depends on the number of X’s crossed.



- Introduced a “dynamical” parameter taking two values λ, λ' (more later)
- $\lambda \leftrightarrow \lambda'$ when crossing X, Y, unchanged when crossing Z

Alternating chains

- There is extensive condensed-matter literature on alternating chains, though mostly for the antiferromagnetic case
- E.g. the bimetallic chain $\mathrm{MnNi(NO_2)_4(en)_2(en = ethylenediamide)}$



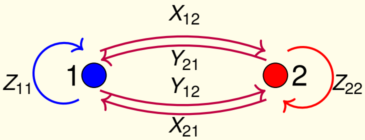
[Feyerherm, Mathonière, Kahn, J. Phys. Condens. Matter 13, 2639 (2001)]

- Have been studied with various techniques such as the recursion method [Viswanath,Müller '94], also long-wavelength approximations [Huang et al. '91]
- It is not known whether such chains are integrable (solvable by some type of Bethe Ansatz)

Quantum Symmetry

[work with E. Andriolo, H. Bertle, E. Pomoni and X. Zhang]

- First, understand the symmetries better



- Naively, $\mathrm{SU}(4)_R \to \mathrm{SU}(2)_L^{i=1,2} \times \mathrm{SU}(2)_R^{i=3,4} \times \mathrm{U}(1)$
- Eight broken generators: $R^1{}_3\ ,\ R^1{}_4\ ,\ R^2{}_3\ ,\ R^2{}_4$ + conjugates
- Relate fields which now belong to different $\mathrm{SU}(N) \times \mathrm{SU}(N)$ representations
- Claim: Can upgrade them to true generators in a *quantum* version of $\mathrm{SU}(4)_R$
- E.g. want to write: $\boxed{R^3{}_2 X^a{}_{\hat{a}} = Z^a{}_a\ ,\quad R^2{}_3 Z^a{}_a = X^a{}_{\hat{a}}}$

Quantum Symmetry

- Gauge indices of all fields to the right need to be flipped

$$\cdots Z_{11} X_{12} Z_{22} Y_{21} X_{12} \cdots \xrightarrow{\Delta(\sigma_{-}^{XZ})} \cdots Z_{11} Z_{11} Z_{11} Y_{12} X_{21} \cdots$$

- Can achieve this with a suitable coproduct $\Rightarrow$ Quantum algebra
- Structure is that of a quantum groupoid [Lu '96, Xu '99]
- Path groupoid: Like a group, but not all compositions of elements are allowed. The allowed paths are those given by the quiver.
- Unbroken generators have the Lie algebraic coproduct $\Delta_o(a) = I \otimes a + a \otimes I$
- For the broken generators we define:

$$\Delta_o(a) = I \otimes a + a \otimes \gamma \quad , \quad \text{where} \quad \gamma(X_i) = X_{i+1}$$

- To complete the algebra we also need $\Delta(\gamma) = \gamma \otimes \gamma$

Twist

- Can move away from the orbifold point by a Drinfeld twist

$$\Delta(a) = F\Delta_o(a)F^{-1}$$

- (For β-deformation see [Garus '17], also [Dlamini, KZ '16,'19] for LS)

- We require that Δ preserves the F-term relations:

$$\Delta(\sigma_{\pm}^{XZ}) \triangleright \left(X_{12}Z_{22} - \frac{1}{\kappa}Z_{11}X_{12}\right) = 0$$

- A suitable twist is:

$$F = I \otimes \kappa^{-\frac{s}{2}} \quad \text{where} \quad s = \begin{cases} 1 & \text{if the gauge index is 1} \\ -1 & \text{if the gauge index is 2} \end{cases}$$

- Recall that γ flips the gauge index $\Rightarrow s \circ \gamma = -\gamma \circ s$

Twisted coproduct

- Twisting the unbroken generators has no effect:

$$\Delta(\sigma_3) = (I \otimes \kappa^{-\frac{s}{2}})(I \otimes \sigma_3 + \sigma_3 \otimes I)(I \otimes \kappa^{\frac{s}{2}}) = (I \otimes \sigma_3 + \sigma_3 \otimes I)$$

- But on the broken generators we find:

$$\Delta(\sigma_\pm) = (I \otimes \kappa^{-\frac{s}{2}})(I \otimes \sigma_\pm + \sigma_\pm \otimes \gamma)(I \otimes \kappa^{\frac{s}{2}}) = (I \otimes \sigma_\pm + \sigma_\pm \otimes \gamma\kappa^s)$$

- Defining $K = \gamma\kappa^s$, and also $\Delta_o(s) = s \otimes I$, our final coproducts are:

$$\boxed{\Delta(\sigma_\pm) = I \otimes \sigma_\pm \ + \ \sigma_\pm \otimes K \ \ , \ \ \Delta(K) = K \otimes K}$$

- $K^2 = 1 \quad \Rightarrow$ Compatibility of the coproduct with the algebra product

$$\Delta([\sigma_+, \sigma_-]) = [\Delta(\sigma_+), \Delta(\sigma_-)]$$

- The $\mathrm{SU}(2)$ commutation relations are not deformed, unlike in $U_q(\mathrm{sl}(2))$

Iterated coproduct

- The twist satisfies the cocycle condition

$$F_{12} \circ (\Delta_o \otimes \mathrm{id})(F) = F_{23} \circ (\mathrm{id} \otimes \Delta_o)(F) =: F_{(3)}$$

  giving

$$\Delta^{(3)}(a) = F_{(3)} \Delta_o^{(3)}(a) F_{(3)}^{-1} = I \otimes I \otimes a + I \otimes a \otimes K + a \otimes K \otimes K$$

- Similarly we find the L-site coproduct for the broken/revived generators:

$$\boxed{\Delta^{(L)}(a) = \sum_i \cdots I \otimes I \otimes a_i \otimes K \otimes K \cdots}$$

- By construction, the coproduct preserves the quantum plane relations
- The superpotential is now invariant under all $\mathrm{SU}(3)$ generators

$$\Delta^{(3)}(\sigma^{XY}_{\pm,3}) \triangleright \mathcal{W} = \Delta^{(3)}(\sigma^{XZ}_{\pm,3}) \triangleright \mathcal{W} = \Delta^{(3)}(\sigma^{YZ}_{\pm,3}) \triangleright \mathcal{W} = 0$$

Is this useful?

- The Hamiltonian does not commute with $\Delta(a)$ (for the broken a's).
- So we do not expect κ-deformed multiplets to map 1-1 to eigenstates of the Hamiltonian
- Algebraic Bethe Ansatz: Assume there exists an R-matrix $R(u)$, depending on a spectral parameter u
- Our twist is in the quantum plane limit ($u \to \infty$ for rational integrable models)
- The full twist will also be u-dependent, such that

$$R(u,\kappa) = F(u)_{21} R(u,\kappa\!=\!1) F(u)_{12}^{-1}$$

- So we expect a different twist/coproduct for each u (i.e. each eigenvalue of $\mathcal{H}$)
- For BPS states, it turns out that $\Delta^{BPS}(a,\kappa) = \Delta(a,1/\kappa)$.
- Agrees with the direct diagonalisation in [Gadde,Pomoni,Rastelli '10]

Example: BPS spectrum

$$X_{12}X_{21}X_{12}X_{21}$$

$$\Big\downarrow \Delta^{BPS}(\sigma_-^{XZ})$$

$$X_{12}X_{21}X_{12}Z_{22} + \kappa X_{12}X_{21}Z_{11}X_{12} + X_{12}Z_{22}X_{21}X_{12} + \kappa Z_{11}X_{12}X_{21}X_{12}$$

$$\Big\downarrow \Delta^{BPS}(\sigma_-^{XZ})$$

$$\kappa X_{12}X_{21}Z_{11}Z_{11} + X_{12}Z_{22}X_{21}Z_{11} + \tfrac{1}{\kappa}X_{12}Z_{22}Z_{22}X_{21} + \kappa Z_{11}X_{12}X_{21}Z_{11} + Z_{11}X_{12}Z_{22}X_{21} + \kappa Z_{11}Z_{11}X_{12}X_{21}$$

$$\Big\downarrow$$

$$\dots$$

- To get a closed eigenstate, add the state with $\{1 \leftrightarrow 2, \kappa \leftrightarrow \kappa^{-1}\}$ and impose cyclicity. We find the following BPS state:

$$\kappa \mathrm{Tr}_1(X_{12}X_{21}Z_{11}Z_{11}) + \mathrm{Tr}_1(X_{12}Z_{22}X_{21}Z_{11}) + \frac{1}{\kappa}\mathrm{Tr}_1(X_{12}Z_{22}Z_{22}X_{21})$$

- This state is not protected by $\mathcal{N} = 2$ supersymmetry. The fact that it still has $E = 0$ is a consequence of the quantum symmetry

Twisted $\mathrm{SU}(4)$ groupoid

- We have extended this to multiplets in the full deformed $\mathrm{SU}(4)$ sector
  [Andriolo, Bertle, Pomoni, Zhang, KZ, to appear]
- Mainly focused on $L = 2$ $(\mathbf{20'}, \mathbf{15})$ and $L = 3$ $(\mathbf{50}, \mathbf{10})$ etc.
- The non-BPS multiplets of the closed Hamiltonian at $\kappa = 1$ break up into several multiplets as $\kappa \neq 1$
- Main idea: Can partially untwist the Hamiltonian to make the open multiplets agree with those at the orbifold point, while leaving the closed spectrum unchanged. Schematically:

$$R'(u,\kappa) = G(u)_{21} R(u,\kappa) G(u)_{12}^{-1} \quad \Rightarrow \quad H'_{\text{open}} = H_{\text{open}} + \delta H_{\text{open}} \quad (\text{but } \; H'_c = H_c)$$

- In this basis the splitting is only due to the closed boundary conditions
- First step towards constructing $F(u)$

Not discussed

- Coordinate Bethe Ansatz for the 2-magnon problem
- 3-magnon scattering in special cases (D. Bozkurt, E. Pomoni)
- Dynamical Yang-Baxter equation (Felder)
- Interpretation as dilute RSOS model (with A. Roux)
- Hints of integrability in the XY sector (with M. de Leeuw, E. Pomoni, A. Retore)
- More general $\mathcal{N} = 2$ orbifolds, e.g. $\mathbb{Z}_k, D_k$ (with J. Bath)

Summary

- Spin chains for $\mathcal{N} = 2$ orbifold theories are dynamical
- The naively broken $\mathrm{SU}(4)_R$ generators are not lost but can be upgraded to generators of a quantum groupoid
- Found a simple twist that takes us away from the orbifold point
- The twist leads to a quantum groupoid coproduct
- Studied short chains with the goal of better understanding the twist and the implications of this quantum symmetry

Thanks for your attention!