Some Comments on Bilocal Holography

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(based on work to appear with Garreth Kemp and Jaco Van Zyl)

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Introduction

The duality between vector models and higher spin gravity is a simple but instructive example of holography. For this example **bilocal holography** gives a construction of the dual gravity theory starting from CFT.

A central ingredient in the construction is a holographic map. The physical input to the map follows by:

1. Reducing gravity to physical and independent degrees of freedom.
2. Reducing CFT to its independent degrees of freedom.
3. Identifying the complete set of degrees of freedom of CFT with those of gravity.

Although we work entirely at large N today, the construction reproduces the complete set of $1/N$ corrections to gauge invariant operators whose dimension is held fixed as $N \to \infty$.

The mapping reproduces general expectations of holography including bulk reconstruction, subregion duality and the holography of information.

Introduction

At a most basic level, as we review below, the holographic map has two ingredients

1. A change to invariant (collective) field variables.
2. A change of spacetime coordinates.

The first ingredient is well understood and there is a systematic approach to writing down the collective fields and to working out the measure - collective field theory.

The second ingredient is guessed.

The goal of today's talk is to

1. Significantly simply the procedure of reducing to physical and independent degrees of freedom. This makes use of gravity in Metsaev's modified de Donder gauge.
2. To outline how to derive the change of spacetime coordinates. The key physical input is bulk locality.

Outline

Review Higher Spin / Vector model duality

Review bilocal holography for the vector model, on the light front.

Modified de Donder gauge in gravity

Equal time bilocal holography

The role of bulk locality

Conclusions

Vector model / Higher spin duality

Duality between higher spin gravity theory in AdS_4 and $O(N)$ vector models (Klebanov, Polyakov, arXiv:hep-th/0210114, Sezgin, Sundell, arXiv:hep-th/0305040).

A simple, nontrivial example of AdS/CFT:

1. Spectrum of operators in CFT not renormalized at infinite N (Giombi, Minwalla, Prakash, Trivedi, Wadia, et al. arXiv:1110.4386), and
2. Spectrum of fields in the bulk theory is simple. (Vasiliev, arXiv:hep-th/0106149 [hep-th])
3. Correlation functions in higher spin gravity and boundary $O(N)$ model CFT agree. Giombi, Yin, arXiv:0912.3462 [hep-th] showed match for free and critical vector models.

Singlet sector of free $O(N)$ $2+1$ vector model: $S=\int d^3x\sum_{a=1}^{N}\left(\frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a\right)$

Single trace primaries: $O_{\Delta=1}(t,\vec{x})=\sum_{a=1}^{N}\phi^a(t,\vec{x})\phi^a(t,\vec{x})$

$$J_{\mu_1\mu_2\cdots\mu_{2s}}(t,\vec{x})\alpha^{\mu_1}\alpha^{\mu_2}\cdots\alpha^{\mu_{2s}}=\sum_{a=1}^{N}\sum_{k=0}^{2s}\frac{(-1)^k\ :(\alpha\cdot\partial)^{2s-k}\phi^a\,(\alpha\cdot\partial)^k\phi^a:}{k!(2s-k)!\Gamma(k+\frac{1}{2})\Gamma(2s-k+\frac{1}{2})}$$

Higher spin gravity in AdS_4 has a bulk scalar and a gauge field $A_{\mu_1\cdots\mu_{2s}}$ for each s.

Large N Higher spin equations of motion

Work in lightcone gauge $A^{+\mu_2\cdots\mu_{2s}} = 0$

in Poincare patch of AdS_4 (Metsaev, arXiv:hep-th/9906217)

$$X^{\pm} \equiv X^2 \pm X^0, \; X \equiv X^1, \; Z \qquad ds^2 = \frac{dX^+ dX^- + dX^2 + dZ^2}{Z^2}$$

Components $A^{-\mu_2\cdots\mu_{2s}}$ determined by constraints. Dynamical fields are X, Z polarizations: $A^{XZXZ\cdots ZZ}$. Gauge field is symmetric and traceless

$\Rightarrow$ **two independent physical degrees of freedom at each spin.**

Free equation of motion is (obtained after gauge fixing and solving the constraint)

$$\left(\frac{\partial}{\partial X^+}\frac{\partial}{\partial X^-} + \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Z^2}\right)\frac{A^{XXZ\cdots ZX}}{Z} = 0$$

Conformal symmetry

To work out the so(2,3) AdS isometry (= conformal) generators after reducing to physical degrees of freedom we need to:

• Fix a gauge and solve the associated gauge constraint. Isometries are generated using the Killing vectors as usual.

• Since conformal transformations move out of lightcone gauge, each conformal transformation must be supplemented with a compensating gauge transformation, that restores the gauge.

• Reduce to independent degrees of freedom by solving the symmetric and traceless constraints.

Result is a set of transformation defined on $A^{XX\cdots X}$ and $A^{ZX\cdots X}$ fields.

Repackaging Higher Spin Gravity

Infinite number of spinning fields in AdS_4 $\equiv$ single field on $\mathrm{AdS}_4 \times \mathrm{S}^1$

Co-ordinates: $X^\pm \equiv X^2 \pm X^0,\ X \equiv X^1,\ Z$ Metric: $ds^2 = \frac{dX^+ dX^- + dX^2 + dZ^2}{Z^2}$

Fields: $A^{XX\cdots X}(X^+, X^-, X, Z)$, $A^{ZX\cdots X}(X^+, X^-, X, Z)$, $\Phi(X^+, X^-, X, Z)$

Co-ordinates: Add θ Metric: As above.

Field: $\Phi(X^+, X^-, X, Z, \theta) = \sum_{s=0}^{\infty} \left(\cos(2s\theta) \frac{A^{XX\cdots XX}}{Z} + \sin(2s\theta) \frac{A^{XX\cdots XZ}}{Z} \right)$

Summary: Higher Spin Gravity

Equation of motion for physical d.o.f.:

$$\left(\frac{\partial}{\partial X^+}\frac{\partial}{\partial X^-}+\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Z^2}\right)\frac{A^{XXZ\cdots ZX}}{Z}=0$$

Repackaged the complete set of physical and independent fields into a single field, which is a function of 5 co-ordinates:

$$\Phi(X^+,X^-,X,Z,\theta)=\sum_{s=0}^{\infty}\left(\cos(2s\theta)\frac{A^{XX\cdots XX}}{Z}+\sin(2s\theta)\frac{A^{XX\cdots XZ}}{Z}\right)$$

Action of conformal symmetry on $\frac{A^{XX\cdots X}}{Z}$, $\frac{A^{ZX\cdots X}}{Z}$ is known: for $L^A\in so(2,3)$

$$L^A_{\oplus}\Phi(X^+,X^-,X,Z,\theta)=\sum_{s=0}^{\infty}\left(\cos(2s\theta)L^A_{2s}\frac{A^{XX\cdots XX}}{Z}+\sin(2s\theta)L^A_{2s}\frac{A^{XX\cdots XZ}}{Z}\right)$$

CFT

Spinning currents ($d = 2+1$) are symmetric, traceless, conserved rank $2s$ tensors $J_{\mu_1 \cdots \mu_{2s}}$. There are 2 independent symmetric, traceless, conserved rank $2s$ tensors.

The number of independent spinning primaries match the number of physical and independent gauge field components.

Reduced theory "solves" current conservation equation. Represent current as

$$|J_s(t, \vec{x}, a^\mu)\rangle \quad = \quad J_s^{\mu_1 \mu_2 \cdots \mu_s}(x^\nu) a_{\mu_1} \cdots a_{\mu_s} |0\rangle$$

where $\quad [\bar{a}^\mu, a^\nu] = \eta^{\mu\nu} \qquad \mu, \nu = 0, 1, 2 \qquad \bar{a}^\mu |0\rangle = 0$

Conservation and traceless conditions: $\quad \bar{a}^\nu \partial_\nu |J_s(t, \vec{x}, a^\mu)\rangle \; = \; 0 \; = \; \bar{a}^\nu \bar{a}_\nu |J_s(t, \vec{x}, a^\mu)\rangle$

Reduction:

$$|J_{(s)}\rangle = \exp\left(-a^+ \left[\frac{\bar{a}^+ \partial^- + \bar{a}^b \partial^b}{\partial^+}\right]\right) |i_{(s)}\rangle \equiv \mathcal{P} |i_{(s)}\rangle$$

$$|i_{(s)}\rangle = j^{(s)}_{i_1 i_2 \cdots i_s} a^{i_1} a^{i_2} \cdots a^{i_s} |0\rangle \qquad i_k = -, b$$

Operators O acting on the original currents $|J_{(s)}\rangle$ become $\tilde{O} = \mathcal{P}^{-1} O \mathcal{P}$.

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Bilocal Holography

Basic claim: Holography is accomplished by a change to gauge invariant (bilocal) field variables in the CFT. A change of spacetime coordinates is needed to give the bulk interpretation of the bilocal collective field theory.

Das and Jevicki, hep-th/0304093

Starting from the CFT and carrying out these two steps, one obtains the higher dimensional gravitational theory. In this way, bilocal holography is a constructive approach towards demonstrating AdS/CFT.

There is a clear motivation for both steps.

Bilocal Holography

Basic claim: Holography is accomplished by a change to gauge invariant (bilocal) field variables in the CFT. A change of spacetime coordinates is needed to give the bulk interpretation of the bilocal collective field theory.

Das and Jevicki, Phys. Rev. D **68** (2003), 044011

The loop expansion parameter of the original CFT is $\hbar$. After changing to invariant (bilocal) variables the loop expansion parameter is $1/N$ matching the loop expansion parameter of the dual gravity. Example of **collective field theory**.

Jevicki and Sakita, Nucl. Phys. B **165** (1980), 511

The bilocal transforms in a tensor product. The complete collection of higher spin fields transform in a direct sum. The natural change of basis

$$V_{[\frac{1}{2},0]} \otimes V_{[\frac{1}{2},0]} \longrightarrow V_{[1,0]} \oplus \bigoplus_{s=2,4,\cdots} V_{[s+1,s]}$$

determines a map between CFT and bulk coordinates.

(Think of addition of angular momentum: $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$.)

Change to Bilocal Field Variables

To solve QFT "all" we need to do is evaluate a complicated integral.

$$\int d\phi^a e^{-\frac{1}{\hbar}S(\phi^a)} \qquad a = 1, \cdots, N$$

Hard to do when $N \to \infty$, but things simplify when the theory has an $O(N)$ symmetry, so the action is an $O(N)$ invariant.

Suppose the ϕ^a are in vector rep of $O(N)$. Then S is a function of $\sigma = \phi^a\phi^a$, the unique invariant. One integration variable and not N - much simpler!

$$\int d\sigma e^{-N\tilde{S}(\sigma)}$$

N appears because we had a total of N variables. Saddle point approximation produces an expansion with $1/N$ the loop counting parameter.

2+1 Minkowski in light cone coordinates $x^+ \equiv x^0 + x^2$, $x^- \equiv x^2 - x^0$, $x \equiv x^1$. Invariant variables are equal x^+ bilocals

$$\sigma(x^+, x_1^-, x_1, x_2^-, x_2) = \sum_{a=1}^{N} \phi^a(x^+, x_1^-, x_1)\phi^a(x^+, x_2^-, x_2)$$

Comment on the change to Bilocal Field Variables

$$\int D\phi^a(x) e^{i\frac{1}{\hbar}S[\phi^a]} \qquad a = 1, \cdots, N$$

$\phi^a(t, \vec{x})$ is not a gauge invariant variable - not a physical field - because it is not gauge invariant.

$$\int D\sigma(x_1, x_2) e^{iN\tilde{S}[\sigma]}$$

$\sigma(x_1, x_2)$ is gauge invariant.

$$\sigma(x_1, x_2) = \frac{1}{N}\sum_{a=1}^{N} \phi^a(x_1)\phi^a(x_2)$$

$=$ mean of N identically distributed independent random variables $\Rightarrow$ by the central limit theorem $\sigma(x_1, x_2)$ approaches a definite classical field at large N.

Change to Bilocal Field Variables

2+1 Minkowski in light cone coordinates $x^+ \equiv x^0 + x^2$, $x^- \equiv x^2 - x^0$, $x \equiv x^1$. Invariant variables are equal x^+ bilocals

$$\sigma(x^+, x_1^-, x_1, x_2^-, x_2) = \sum_{a=1}^{N} \phi^a(x^+, x_1^-, x_1)\phi^a(x^+, x_2^-, x_2)$$

The field $\sigma(x^+, x_1^-, x_1, x_2^-, x_2)$ develops a large N expectation value. It is the fluctuation $\eta(x^+, x_1^-, x_1, x_2^-, x_2)$ about this large N background that maps to bulk AdS fields.

$$\sigma(x^+, x_1^-, x_1, x_2^-, x_2) = \sigma_0(x^+, x_1^-, x_1, x_2^-, x_2) + \frac{1}{\sqrt{N}}\eta(x^+, x_1^-, x_1, x_2^-, x_2)$$

$\sigma_0(x^+, x_1^-, x_1, x_2^-, x_2)$ is the large N two point function.

RdMK, Jevicki, Jin and Rodrigues, Phys. Rev. D **83** (2011) 025006.

Change of Spacetime Co-ordinates

The bilocal transforms in $V_{\frac{1}{2},0} \otimes V_{\frac{1}{2},0}$ $\qquad (L^A \in so(2,3))$

$$L^A_\otimes \sigma \;=\; \Big(L^A \phi^a(x^+, x_1^-, x_1)\phi^a(x^+, x_2^-, x_2) + \phi^a(x^+, x_1^-, x_1) L^A \phi^a(x^+, x_2^-, x_2) \Big)$$

$$V_{\frac{1}{2},0} \otimes V_{\frac{1}{2},0} = V_{1,0} \oplus \bigoplus_{s=2,4,6,\cdots} V_{s+1,s}$$

The complete collection of higher spin fields fill out the reducible representation $V_{1,0} \oplus \bigoplus_{s=2,4,6,\cdots} V_{s+1,s}$

$$L^A_\oplus \Phi(X^+, X^-, X, Z, \theta) = \sum_{s=0}^{\infty} \left(\cos(2s\theta) L^A_{2s} \frac{A^{XX\cdots XX}}{Z} + \sin(2s\theta) L^A_{2s} \frac{A^{XX\cdots XZ}}{Z} \right)$$

We want to change from the natural representation ($L^A_\otimes$) of the CFT to the representation that is natural for the bulk gravity ($L^A_\oplus$).

Change of Spacetime Coordinates

Bilocal field $\eta(x^+, x_1^-, x_1, x_2^-, x_2)$. 5 coordinates in CFT: $x^+, x_1^-, x_1, x_2^-, x_2$.

Higher spin gravity field $\Phi(X^+, X^-, X, Z, \theta)$. 5 coordinates in gravity: X^+, X^-, X, Z, θ.

Symmetry: $X^- \to X^- + a$ in gravity and $x^- \to x^- + b$ in CFT motivates the Fourier transform:

$$\eta(x^+, p_1^+, x_1, p_2^+, x_2) \;=\; \int \frac{dx_1^-}{2\pi} \int \frac{dx_2^-}{2\pi}\, \eta(x^+, x_1^-, x_1, x_2^-, x_2)\, e^{-ip_1^+ x_1^- - ip_2^+ x_2^-}$$

5 coordinates in CFT: $x^+, p_1^+, x_1, p_2^+, x_2$.

$$\Phi(X^+, P^+, X, Z, \theta) \;=\; \int \frac{dX^-}{2\pi}\, \Phi(X^+, X^-, X, Z, \theta)\, e^{-iP^+ X^-}$$

5 coordinates in gravity: X^+, P^+, X, Z, θ.

Change of Spacetime Coordinates

Bilocal field $\eta(x^+, p_1^+, x_1, p_2^+, x_2)$. 5 coordinates in CFT: $x^+, p_1^+, x_1, p_2^+, x_2$.

Higher spin gravity field $\Phi(X^+, P^+, X, Z, \theta)$. 5 coordinates in gravity: X^+, P^+, X, Z, θ.

$$x_1 = X + Z\tan\left(\frac{\theta}{2}\right) \qquad x_2 = X - Z\cot\left(\frac{\theta}{2}\right) \qquad x^+ = X^+$$

$$p_1^+ = P^+\cos^2\left(\frac{\theta}{2}\right) \qquad p_2^+ = P^+\sin^2\left(\frac{\theta}{2}\right)$$

$$X = \frac{p_1^+ x_1 + p_2^+ x_2}{p_1^+ + p_2^+} \qquad Z = \frac{\sqrt{p_1^+ p_2^+}(x_1 - x_2)}{p_1^+ + p_2^+}$$

$$P^+ = p_1^+ + p_2^+ \qquad \theta = 2\tan^{-1}\left(\sqrt{\frac{p_2^+}{p_1^+}}\right)$$

$$L_\oplus^A \Phi = 2\pi P^+ \sin\theta \; L_\otimes^A \eta$$

RdMK, Jevicki, Jin and Rodrigues, Phys. Rev. D **83** (2011) 025006.

Summary: Bilocal Holography

$$
\begin{aligned}
\sigma(x^+, x_1^-, x_1, x_2^-, x_2) &= \sum_{a=1}^{N} \phi^a(x^+, x_1^-, x_1)\phi^a(x^+, x_2^-, x_2) \\
&= \sigma_0(x^+, x_1^-, x_1, x_2^-, x_2) + \frac{1}{\sqrt{N}}\eta(x^+, x_1^-, x_1, x_2^-, x_2)
\end{aligned}
$$

$$
X = \frac{p_1^+ x_1 + p_2^+ x_2}{p_1^+ + p_2^+} \qquad Z = \frac{\sqrt{p_1^+ p_2^+}(x_1 - x_2)}{p_1^+ + p_2^+} \qquad X^+ = x^+
$$

$$
P^+ = p_1^+ + p_2^+ \qquad \theta = 2\tan^{-1}\left(\sqrt{\frac{p_2^+}{p_1^+}}\right)
$$

$$
\begin{aligned}
\Phi &= \sum_{s=0}^{\infty}\left(\cos(2s\theta)\frac{A^{XX\cdots XX}(X^+, P^+, X, Z)}{Z} + \sin(2s\theta)\frac{A^{XX\cdots XZ}(X^+, P^+, X, Z)}{Z}\right) \\
&= 2\pi P^+ \sin\theta\ \eta(X^+, P^+\cos^2\frac{\theta}{2}, X + Z\tan\frac{\theta}{2}, P^+\sin^2\frac{\theta}{2}, X - Z\cot\frac{\theta}{2})
\end{aligned}
$$

We are studying the free O(N) vector model. Its a UV fixed point.

By perturbing and flowing we can reach an IR fixed point. The bilocal holography for this fixed point has also been worked out. Mulokwe and Rodrigues, JHEP 11 (2018) **047** and Johnson, Mulokwe and Rodrigues, Phys. Lett. B **829** (2022) 137056.

Bulk Reconstruction

Does the proposed bulk field $\Phi(X^+, P^+, X, Z, \theta)$ obey the correct bulk equation of motion with the correct boundary condition? CFT equation of motion:

$$\left(\frac{\partial}{\partial x^+}\frac{\partial}{\partial x^-} + \frac{\partial^2}{\partial x^2}\right)\phi^a(x^+, x^-, x) = 0$$

implies

$$\left(\frac{\partial}{\partial X^+}\frac{\partial}{\partial X^-} + \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Z^2}\right)\Phi\left(X^+, X^-, X, Z, \theta\right) = 0$$

Boundary condition: The GKPW dictionary is formulated in de Donder gauge: $D^A A_{AA_2 \cdots A_{2s}} = 0$. After transforming to light cone gauge the GKPW dictionary exactly reproduces the boundary behaviour of the bilocal. Mintun and Polchinski, arXiv:1411.3151

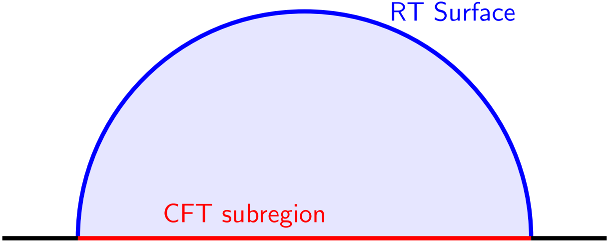
$\Rightarrow$ bilocal holography gives a correct bulk reconstruction.

All single trace primaries must appear together and the complete set of bulk fields are reconstructed.

Subregion Duality

Which subregion of the CFT (if any) is dual to a given subregion of the bulk?

Entanglement wedge reconstruction claims that everyting from the boundary up to the RT surface can be reconstructed.



Subregion Duality

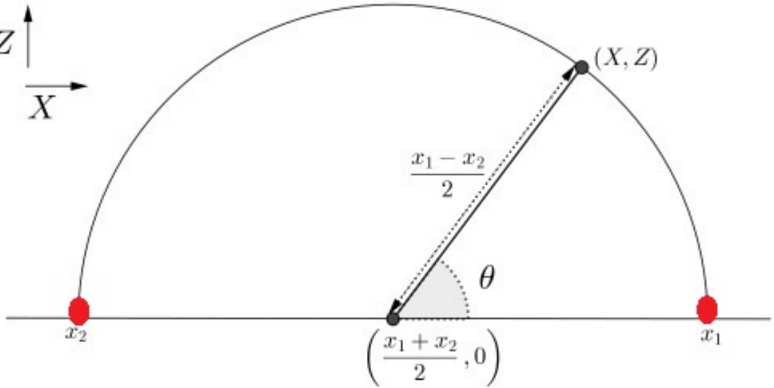


Figure: The bilocal describing excitations localized at (x_1, p_1^+) and (x_2, p_2^+) corresponds to a bulk excitation localized at (X, Z) as shown. This figure lives on a constant $x^+ = X^+$ slice. The angle θ is $\theta \ = \ 2\tan^{-1}\left(\sqrt{\frac{p_2^+}{p_1^+}}\right)$.

Bulk Reconstruction

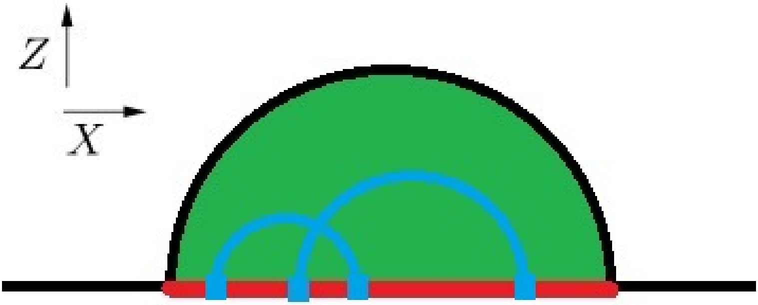


Figure: Using bilocals restricted to the red region of the CFT we are able to reconstruct bulk field living in the area shaded in green. This figure is for fixed time $x^+ = X^+$.

What is the interpretation of the boundary of the green region? The boundary is a geodesic so that the green region is reminiscent of the entanglement wedge.

The Principle of the Holography of Information

In a theory of quantum gravity, a copy of all the information available on a Cauchy slice is also available near the boundary of the Cauchy slice. This redundancy in description is already visible in the low-energy theory.

Changing the values of fields within R changes field values in the green band. The split property fails.

Raju, arXiv:2110.05470.

Location of single trace primaries

Where do single trace primaries map to in the AdS_4 bulk?

The complete set of single trace primary operators, after mapping to the dual gravity, are supported in an arbitrarily small neighbourhood of the boundary.

Which CFT bilocals map to operators localized deep in the bulk of AdS_4?

$$Z = \frac{\sqrt{p_1^+ p_2^+}(x_1 - x_2)}{p_1^+ + p_2^+}$$

p_1^+ and p_2^+ are both positive $\Rightarrow$ the ratio $0 < \frac{\sqrt{p_1^+ p_2^+}}{p_1^+ + p_2^+} < 1$.

To obtain a large value for Z we must consider a large separation between the two fields in the bilocal, i.e. $x_1 - x_2$ must be large.

Holography of information and OPE

HOI predicts bulk operators can be expressed as elements of the boundary algebra.

Single trace primaries are supported in arbitrarily small neighbourhood of the boundary.

By separating x_1 and x_2 to be arbitrarily distant, the bilocal $\eta(x^+, x_1^-, x_1, x_2^-, x_2)$ is located arbitrarily deep in the bulk ($Z = \frac{\sqrt{p_1^+ p_2^+}(x_1 - x_2)}{p_1^+ + p_2^+}$).

HOI is true if we can replace the bilocal field $\eta(x^+, x_1^-, x_1, x_2^-, x_2)$ by a sum of single trace primaries $\mathcal{O}^{\mu_1 \cdots \mu_{2s}}(x^\mu)$. This is exactly what the OPE does!

$$\begin{aligned} \eta(x^+, x_1^-, x_1, x_2^-, x_2) &= :\phi^a(x^+, x_1^-, x_1)\phi^a(x^+, x_2^-, x_2): \\ &= \sum_s C(x_1^- - x_2^-, x_1 - x_2)_{\mu_1 \cdots \mu_{2s}} J^{\mu_1 \cdots \mu_{2s}}(x^+, x_1^-, x_1) \end{aligned}$$

Burden of the proof is to show the OPE converges.

(Pappadopulo, Rychkov, Espin, Rattazzi, arXiv:1208.6449, Qiao, arXiv:2005.09105)

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Modified de Donder gauge in gravity

Spin-s Fronsdal field $A_{\mu_1\mu_2\cdots\mu_s}$ is symmetric and double tracelessness $A_\nu{}^\nu{}_\rho{}^{\rho\mu_5\cdots\mu_s}=0$. The higher spin gauge symmetry is

$$A'^{\mu_1\cdots\mu_s}=A^{\mu_1\cdots\mu_s}+\nabla^{(\mu_1}\Lambda^{\mu_2\cdots\mu_s)}$$

$\Lambda^{\mu_1\cdots\mu_{s-1}}$ is symmetric and traceless. ∇_μ is the AdS covariant derivative. e^A_μ converts spacetime to frame indices $A^{\mu\cdots}\to\Phi^{A\cdots}$. In the Poincaré patch

$$e^A_\mu=\frac{1}{z}\delta^A_\mu$$

Again employ an auxiliary Fock space description

$$\Phi=\sum_{s=0}^{\infty}\Phi_{A_1\cdots A_s}\alpha^{A_1}\cdots\alpha^{A_s}|0\rangle \qquad [\bar\alpha^A,\alpha^B]=\eta^{AB}$$

The AdS covariant derivative in frame field indices is

$$D_A\equiv\hat\partial_A+\frac{1}{2}\omega_A{}^{BC}\eta_{BD}\eta_{CE}M^{DE} \qquad M^{BC}=\alpha^B\bar\alpha^C-\alpha^C\bar\alpha^B$$

with $\hat\partial_A\equiv e^\mu_A\partial_\mu$ and $\omega_A{}^{BC}$ is the frame field spin connection for Poincaré AdS.

$$\left(D^AD_A+\omega_A{}^{AB}D_B-s^2+2s+2-\alpha D\bar\alpha D+\frac{1}{2}(\alpha D)^2\bar\alpha^2-\alpha^2\bar\alpha^2\right)\Phi=0$$

Modified de Donder gauge in gravity

Terms involving $\bar{\alpha}$ in the equation motion

$$\left(D^A D_A + {\omega_A}^{AB} D_B - s^2 + 2s + 2 - \alpha D\bar{\alpha}D + \frac{1}{2}(\alpha D)^2\bar{\alpha}^2 - \alpha^2\bar{\alpha}^2\right)\Phi = 0$$

imply solving constraints is highly non-trivial - the constraint solution must be inserted into the equations of motion that remain. The equations are **coupled** equations.
In Metsaev's modified de Donder gauge

$$|\phi^{(s)}\rangle \equiv \sum_{s'=0}^{s} \alpha_Z^{s-s'}|\phi_{s'}\rangle \qquad |\phi_{s'}\rangle \equiv \frac{\alpha_{a_1}\ldots\alpha_{a_{s'}}}{s'!\sqrt{(s-s')!}}\,\phi_{s'}^{a_1\ldots a_{s'}}|0\rangle$$

Fields $\phi_{s'}^{a_1\ldots a_{s'}}$ with $s' > 3$ are double-traceless $(\bar{\alpha}\cdot\bar{\alpha})^2|\phi^{(s)}\rangle \equiv (\eta_{ab}\,\bar{\alpha}^a\bar{\alpha}^b)^2|\phi^{(s)}\rangle = 0$ The modified de Donder gauge condition is $\bar{C}_{AdS}|\phi^{(s)}\rangle = 0$ where

$$\begin{aligned}\bar{C}_{AdS} \quad &\equiv \quad \bar{\alpha}\cdot\partial - \frac{1}{2}\alpha\cdot\partial\bar{\alpha}\cdot\bar{\alpha} - \frac{1}{2}\alpha^Z\widetilde{e}_1\left(\partial_Z + \frac{2s+d-5-2N_Z}{2Z}\right)\bar{\alpha}\cdot\bar{\alpha} \\ &+\left(\partial_Z - \frac{2s+d-5-2N_Z}{2Z}\right)\widetilde{e}_1\bar{\alpha}^Z\Pi\end{aligned}$$

In this gauge we obtain decoupled equations of motion

$$\left(\Box + \partial_Z^2 - \frac{1}{Z^2}\left(\nu^2 - \frac{1}{4}\right)\right)|\phi^{(s)}\rangle = 0 \qquad \nu \equiv s + \frac{d-4}{2} - N_Z$$

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In this gauge we obtain decoupled equations of motion

$$\left(\Box + \partial_Z^2 - \frac{1}{Z^2}\left(\nu^2 - \frac{1}{4}\right)\right)|\phi^{(s)}\rangle = 0 \qquad \nu \equiv s + \frac{d-4}{2} - N_Z$$

We can freely set components of the fields to zero to fix residual gauge invariances or to solve constraints. To fix almost all of the residual gauge invariance we can eliminate all z polarizations. The equation of motion then becomes

$$\left(\Box + \partial_Z^2 - \frac{s}{Z^2}(s-1)\right)|\phi^{(s)}\rangle = 0$$

In modified de Donder gauge we preserve the Poincare invariance of CFT.

$\Rightarrow$ modified de Donder gauge is a natural gauge choice for the higher spin gravity when bilocal holography is developed

Solutions of bulk equations of motion

Operators naturally appear in pairs: the primary operator and it's shadow. These have the same value of quadratic conformal Casimir

$$C_2 = \Delta(d-\Delta) + \vec{L}\cdot\vec{L}$$

If the primary has dimension Δ, the shadow operator has dimension $\Delta_s = d - \Delta$. In the dual gravity these correspond to the normalizable and non-normalizable modes of the bulk wave equation. The bulk wave equation produces these solutions as a power of Z times a Bessel function

$$\sim (qZ)^{\frac{d}{2}} J_{\pm\nu}(qZ) \qquad q^2 = \Box = \partial^\mu\partial_\mu \tag{1}$$

$$\Delta, \Delta_s \quad = \quad \frac{d}{2} \pm (\frac{d}{2} - \Delta) \equiv \frac{d}{2} \pm \nu$$

The form of these wave functions was argued by Migdal in his studies of large N Yang-Mills theory (A. Migdal, "Integral Equation for CFT/String Duality," [arXiv:1107.2370 [hep-th]]).

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Equal Time Bilocal Holography

$$X = \frac{p_1^0 x_1 + p_2^0 x_2}{p_1^0 + p_2^0} \qquad Y = \frac{p_1^0 y_1 + p_2^0 y_2}{p_1^0 + p_2^0}$$

and

$$Z_1 = \frac{\sqrt{p_1^0 p_2^0}}{p_1^0 + p_2^0}(x_1 - x_2) \qquad Z_2 = \frac{\sqrt{p_1^0 p_2^0}}{p_1^0 + p_2^0}(y_1 - y_2)$$

or, equivalently

$$Z = \frac{\sqrt{p_1^0 p_2^0}}{p_1^0 + p_2^0}\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad \tan\psi = \frac{(y_1 - y_2)}{(x_1 - x_2)}$$

$$\left(X - \frac{x_1 + x_2}{2}\right)^2 + \left(Y - \frac{y_1 + y_2}{2}\right)^2 + Z^2 = \frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{4}$$

Equal Time Bilocal Holography

The above map is invertible with the result

$$x_1 = X + \sqrt{\frac{p_2^0}{p_1^0}} Z_1 \qquad x_2 = X - \sqrt{\frac{p_1^0}{p_2^0}} Z_1$$

$$y_1 = Y + \sqrt{\frac{p_2^0}{p_1^0}} Z_2 \qquad y_2 = Y - \sqrt{\frac{p_1^0}{p_2^0}} Z_2$$

The bilocal CFT field maps to the bulk field

$$\begin{aligned} \Phi(t, X, Y, Z_1, Z_2) &= \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} \, \sigma(p_1^\mu, p_2^\mu) \\ &\times \quad e^{-i(p_1^0 + p_2^0)t + i(p_1^x(x_1) + p_1^y(y_1) + p_2^x(x_2) + p_2^y(y_2))} \end{aligned}$$

The CFT equations of motion imply

$$\left(\Box + \frac{\partial^2}{\partial Z_1^2} + \frac{\partial^2}{\partial Z_2^2} \right) \Phi(t, X, Y, Z_1, Z_2) = 0$$

Equal Time Bilocal Holography

The above map is invertible with the result

$$x_1 = X + \sqrt{\frac{p_2^0}{p_1^0}} Z_1 \qquad x_2 = X - \sqrt{\frac{p_1^0}{p_2^0}} Z_1$$

$$y_1 = Y + \sqrt{\frac{p_2^0}{p_1^0}} Z_2 \qquad y_2 = Y - \sqrt{\frac{p_1^0}{p_2^0}} Z_2$$

The bilocal CFT field maps to the bulk field

$$\begin{aligned}\Phi(t, X, Y, Z_1, Z_2) &= \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} \sigma(p_1^\mu, p_2^\mu) \\ &\times \quad e^{-i(p_1^0+p_2^0)t+i(p_1^x(X+\sqrt{\frac{p_2^0}{p_1^0}}Z_1)+p_1^y(Y+\sqrt{\frac{p_2^0}{p_1^0}}Z_2)+p_2^x(X-\sqrt{\frac{p_1^0}{p_2^0}}Z_1)+p_2^y(Y-\sqrt{\frac{p_1^0}{p_2^0}}Z_2))}\end{aligned}$$

The CFT equations of motion imply

$$\left(\Box + \frac{\partial^2}{\partial Z_1^2} + \frac{\partial^2}{\partial Z_2^2}\right) \Phi(t, X, Y, Z_1, Z_2) = 0$$

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The role of bulk locality

Consider an operator localized in the bulk of AdS at the point $X^M = (x^\mu, Z)$ with $Z = z_p$, $x^\mu = 0$.

$$P^\mu = \partial^\mu \qquad D = x\cdot\partial + z\partial_z + \frac{d-1}{2} \to z_p\partial_z + \frac{d-1}{2}$$

$$J^{\mu\nu} = x^\mu\partial^\nu - x^\nu\partial^\mu \to 0$$

$$K^\mu = -\frac{1}{2}(x\cdot x + z^2)\partial^\mu + x_\nu J^{\mu\nu} \to -\frac{1}{2}z_p^2\partial^\mu$$

The point is fixed by the isotropy group $\mathcal{G}$ generated by

$$\left\{ J^{\mu\nu}\,, \frac{z_p^2}{2}P^\mu + K^\mu \right\}$$

Isotropy group $\mathcal{G} = SO(1,d)$. The $d+1$ dimensional coset $SO(2,d)/\mathcal{G}$ is AdS_{d+1}.

The role of bulk locality: wave function

$$D|\phi\rangle = \Delta_\phi|\phi\rangle \qquad J^{\mu\nu}|\phi\rangle = 0 \qquad K^\mu|\phi\rangle = 0$$

Solve

$$J^{\mu\nu}|\Phi\rangle = 0 = \left(\frac{z_p^2}{2}P^\mu + K^\mu\right)|\Phi\rangle$$

Ansatz

$$|\Phi\rangle = \sum_{k=0}^{\infty} C_k (z_p^2 P^\mu P_\mu)^k |\phi\rangle$$

Use algebra

$$[D, P^\mu] = -P^\mu \qquad [D, K^\mu] = K^\mu \qquad [D, J^{\mu\nu}] = 0$$

$$[P^\alpha, J^{\beta\gamma}] = \eta^{\alpha\beta}P^\gamma - \eta^{\alpha\gamma}P^\beta \qquad [K^\alpha, J^{\beta\gamma}] = \eta^{\alpha\beta}K^\gamma - \eta^{\alpha\gamma}K^\beta$$

$$[P^\alpha, K^\beta] = \eta^{\alpha\beta}D - J^{\alpha\beta}$$

The role of bulk locality: wave function

$$D|\phi\rangle \;=\; \Delta_\phi|\phi\rangle \qquad J^{\mu\nu}|\phi\rangle \;=\; 0 \qquad K^\mu|\phi\rangle \;=\; 0$$

Solve

$$J^{\mu\nu}|\Phi\rangle \;=\; 0 \;=\; \left(\frac{z_p^2}{2}P^\mu + K^\mu\right)|\Phi\rangle$$

Solution

$$|\Phi\rangle = \sum_{k=0}^{\infty} C_k (z_p^2 P^\mu P_\mu)^k |\phi\rangle \qquad C_k = \prod_{n=1}^{k} \frac{1}{4n\Delta_\phi + 4n^2 - 2nd}$$

$$|\Phi\rangle = (z_p^2 P\cdot P)^{\frac{d}{4}-\frac{\Delta_\phi}{2}} J_{\Delta_\phi - \frac{d}{2}}(z_p\sqrt{P\cdot P})|\phi\rangle$$

By operator-state corespondence

$$\Phi(0,z_p) \;=\; (z_p^2 P\cdot P)^{\frac{d}{4}} J_{\Delta_\phi - \frac{d}{2}}(z_p\sqrt{P\cdot P})\phi(0)$$

The role of bulk locality: radial coordinate

Use input from the bilocal theory: the bulk field is a bilocal in the CFT. Then

$$\frac{z_p^2}{2}P^\mu + K^\mu = \frac{z_p^2}{2}(P_1^\mu + P_2^\mu) + (K_1^\mu + K_2^\mu)$$

Using the generators for the free scalar field

$$-\frac{1}{2}(x_1 \cdot x_1 P_1^\mu + x_2 \cdot x_2 P_2^\mu) + \frac{1}{2}z_p^2(P_1^\mu + P_2^\mu) + x_1 \cdot P_1 x_1^\mu + x_2 \cdot P_2 x_2^\mu = 0$$

We have localized to $X^\mu = 0$ so that in lightfront case

$$x_1 p_1^+ + x_2 p_2^+ = 0 \qquad \Rightarrow \; z_p^2 = \frac{p_1^+}{p_2^+} x_1^2$$

and in equal time case

$$\vec{x}_1 p_1^0 + \vec{x}_2 p_2^0 = 0 \qquad \Rightarrow \; z_p^2 = \frac{p_1^0}{p_2^0} \vec{x}_1 \cdot \vec{x}_1$$

The role of bulk locality: radial coordinate

Consider the condition

$$-\frac{1}{2}(x_1 \cdot x_1 P_1^\mu + x_2 \cdot x_2 P_2^\mu) + \frac{1}{2} z_p^2 (P_1^\mu + P_2^\mu) + x_1 \cdot P_1 x_1^\mu + x_2 \cdot P_2 x_2^\mu \quad = \quad 0$$

The $\mu = +$ equation for lightfront case gives the answer from the map

$$z_p^2 \quad = \quad \frac{p_1^+}{p_2^+} x_1^2$$

The $\mu = 0$ equation for equal time case gives the answer from the map

$$z_p^2 = \frac{p_1^0}{p_2^0} \vec{x}_1 \cdot \vec{x}_1$$

The requirement of bulk locality - that our field is at a specific position in the AdS bulk - correctly determines the z coordinate!

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Following Das and Jevicki, bilocal holography (collective field theory) constructs the higher dimensional dual gravitational theory. The resulting holographic map provides a valid bulk reconstruction and localizes information as expected in a theory of quantum gravity (entanglement wedge reconstruction and holography of information).

We argued that modified de Donder gauge, developed by Metsaev, is convenient to both (i) match to the CFT and (ii) to treat gauge invariance and the associated constraints.

We argued that bulk locality determines bulk wave functions and features of holographic map. Would be attractive to show bulk locality determines the holographic mapping.

For matrix theories, many more gauge invariant variables. Collective field theory applied to single matrix quantum mechanics provides a string field theory for the $c = 1$ string. Das, Jevicki, Mod. Phys. Lett. A **5** (1990) 1639-1650.

For free massless matrix in $2+1$ dimensions, bilocal, trilocal, multi-local operators appear. OPE takes product of separated operators and expresses them in terms of local operators. Non-trivial new examples for collective maps. Bulk locality is equally applicable to determine the holographic coordinate. (Work in progress with Jaco Van Zyl, Pratik Roy)

Conclusions

The holography of information gives a very suggestive idea of the mechanism behind holography (C. Chowdhury, V. Godet, O. Papadoulaki and S. Raju, "Holography from the Wheeler-DeWitt equation," [arXiv:2107.14802 [hep-th]]): solving the Hamiltonian and momentum constraints perturbatively, wave functional in neighborhood of the boundary fix the wave functional completely.

Eliminating the reducndancy of diffeomorphism invariance, the degrees of freedom lie on the boundary.

The collective field is highly redundant: trade $\pi^a(t,\vec{x}),\phi^a(t,\vec{y})$ for $\eta(t,\vec{x}_1,\vec{x}_2),\Pi(t,\vec{y}_1,\vec{y}_2)$

$$[\pi^a(t,\vec{x}),\phi^b(t,\vec{y})]=-i\delta^{ab}\delta(\vec{x}-\vec{y}) \qquad [\Pi(t,\vec{x}_1,\vec{x}_2),\eta(t,\vec{y}_1,\vec{y}_2)]=-i\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2)$$

The redundancy of the collective field is such that the physical degrees of freedom lie at the boundary of spacetime ($Z=\frac{\sqrt{p_1^0p_2^0}}{p_1^0+p_2^0}|\vec{x}_1-\vec{x}_2|$)

$$\langle 0|\cdots\ \eta(t,\vec{x}_1,\vec{x}_2)|0\rangle = \sum_{s=0}^{\infty}\sum_{d=0}^{\infty} c_{sd}\left((x_1-x_2)^\mu\frac{\partial}{\partial x_1^\mu}\right)^{2d}\langle 0|\cdots\ \eta(t,\vec{x}_1,\vec{x}_1)|0\rangle$$

Thanks for your attention!