Quantum Thermodynamics from Holography

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Work in Progress

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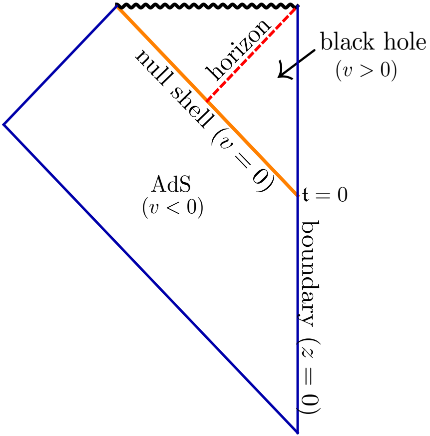

Figure taken from 1311.1200.

Outline

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Gravitational Energy Conditions: Classical and Quantum

- Expect energy density to be positive in General Relativity.
- Null energy condition is $T_{ab}k^a k^b \geq 0$.
- All energy conditions of GR violated by quantum effects.
- Averaged null energy condition, $\int d\lambda \langle T_{ab} \rangle k^a k^b \geq 0$, proven.
  1605.08072, 1610.05308

Quantum Null Energy Condition

- For CFT_2, QNEC is

$$\mathcal{Q}_\pm = 2\pi \langle T_{\pm\pm} \rangle - \left(S'' + \frac{6}{c} S'^2 \right) \geq 0,$$

where S is the entanglement entropy (EE) for any interval, and prime denotes variations of EE w.r.t. null deformations of one of the end-points of the interval. 1506.02669

- Can be rephrased in terms of relative entropy,

$$S(\sigma|\rho) = -\operatorname{Tr} \rho(\log \sigma - \log \rho),$$

as

$$\lim_{y' \to y} \frac{\delta^2 S(\sigma|\rho)}{\delta V(y) \delta V(y')} \geq 0,$$

Outline

Quenches in CFT_2

- Dynamics of isolated quantum systems subject to intense research.
- Quenches in $1+1$-dimensions an exciting arena to study thermalisation, theoretically, numerically, and experimentally. 1603.02889, 1008.3477, 1603.04409
- Evolution of EE after quenches is of particular interest. 1305.7244
- CFT_2 amenable to analytic calculations.
  Holographic dual AdS_3 gravity similarly tractable.

Quantum Thermodynamics

- Generalization of thermodynamics to interacting finite-dimensional quantum systems using measures of accessible quantum information. 1806.06107
- Widely applicable, from quantum engines to chemical reactions.
- Generalized Clausius inequalities: 1005.4495

$$\Delta S_{\text{irr}} = S(\rho_\tau | \rho_\tau^{\text{eq}}) \geq 2 \frac{d^2(\rho_\tau, \rho_\tau^{\text{eq}})}{d^2(e^{1,1}, e^{2,2})} \ .$$

- $\Delta S_{\text{irr}}/\tau$ is also bounded from above. Related to Bekenstein bound.

Outline

Banados Geometries

- States with $\mathcal{Q}_{\pm} = 0$ termed quantum equilibrium states. 1901.04499
- Dual AdS_3 described by Banados geometries: with $L_{\pm} \equiv L_{\pm}(x^{\pm})$,

$$ds^2 = 2dr\,dt - \left(r^2 - 2(L_+ + L_-)\right)dt^2 + 2(L_+ - L_-)dt\,dx + r^2 dx^2.$$

- Boundary energy-momentum tensor has components

$$\langle t_{\pm\pm} \rangle = \frac{c}{12\pi} L_{\pm}(x^{\pm}).$$

- $L_{\pm} = \mu_{\pm}^2$ are BTZ geometries, with

$$T_H = \frac{2}{\pi} \frac{\mu_+ \mu_-}{\mu_+ + \mu_-}, \qquad s = \frac{c}{6}(\mu_+ + \mu_-).$$

Generalizing Banados Geometries

- Consider the metric

$$ds^2 = 2dr\,dt - \left(r^2 - 2m(t,x)\right)dt^2 + 2j(t,x)dt\,dx + r^2dx^2,$$

supported by the bulk energy-momentum tensor

$$T_{tt} = \frac{q}{r} + \frac{\partial_x p}{r^2} + \frac{p\,j}{r^3}, \qquad T_{tx} = \frac{p}{r}.$$

- Einstein equations satisfied if

$$\partial_t m - \partial_x j = 8\pi G q, \qquad \partial_t j - \partial_x m = 8\pi G p.$$

Banados-Vaidya Geometries

- Choose

$$q = \frac{\delta(t)}{8\pi G}\left(L_+^f - L_+^i + L_-^f - L_-^i\right),$$
$$p = \frac{\delta(t)}{8\pi G}\left(L_+^f - L_+^i - L_-^f + L_-^i\right),$$

to get

$$m = \theta(-t)\left(L_+^i + L_-^i\right) + \theta(t)\left(L_+^f + L_-^f\right),$$
$$j = \theta(-t)\left(L_+^i - L_-^i\right) + \theta(t)\left(L_+^f - L_-^f\right).$$

- Boundary energy-momentum is now

$$\langle t_{\pm\pm}\rangle = \frac{c}{24\pi}(m \pm j).$$

- Dual to CFT_2 states with homogeneous or inhomogeneous quench.

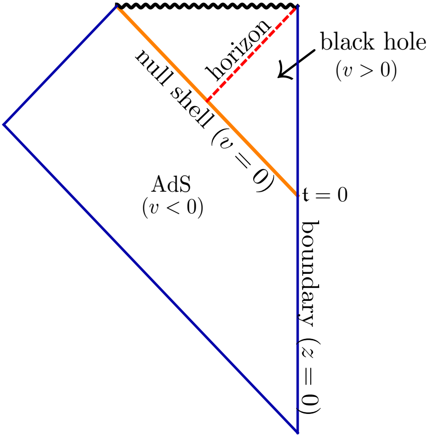


Figure taken from 1311.1200.

Uniformizing Banados Geometries

- All Banados geometries are locally AdS_3: $R = -6$.
- Can be uniformized to the Poincare patch metric, $L_\pm = 0$,

$$R = \frac{r - \frac{X^{+\prime\prime}}{2X^{+\prime}} - \frac{X^{-\prime\prime}}{2X^{-\prime}}}{\sqrt{X^{+\prime}X^{-\prime}}}, \qquad X = \frac{1}{2}\left(X^+ - X^- + \frac{X^{+\prime} - X^{-\prime}}{r - \frac{X^{+\prime\prime}}{2X^{+\prime}} - \frac{X^{-\prime\prime}}{2X^{-\prime}}}\right),$$

$$T = \frac{1}{2}\left(X^+ + X^- + \frac{X^{+\prime} + X^{-\prime} - 2\sqrt{X^{+\prime}X^{-\prime}}}{r - \frac{X^{+\prime\prime}}{2X^{+\prime}} - \frac{X^{-\prime\prime}}{2X^{-\prime}}}\right).$$

where $\mathrm{Sch}(X^\pm(x^\pm), x^\pm) = -2L_\pm$.

Outline

Calculating EE in CFT_2

- Equivalent to computation of geodesic lengths in the AdS_3 bulk.
- All AdS_3 geometries with a dual CFT_2 can be uniformized to a Poincare patch.
- Geodesic lengths in Poincare patch of AdS are given by

$$\ell = \log(\xi + \sqrt{\xi^2 - 1})$$

where

$$\xi = \frac{Z_1^2 + Z_2^2 + \eta_{\mu\nu} X^\mu X^\nu}{2 Z_1 Z_2} \ .$$

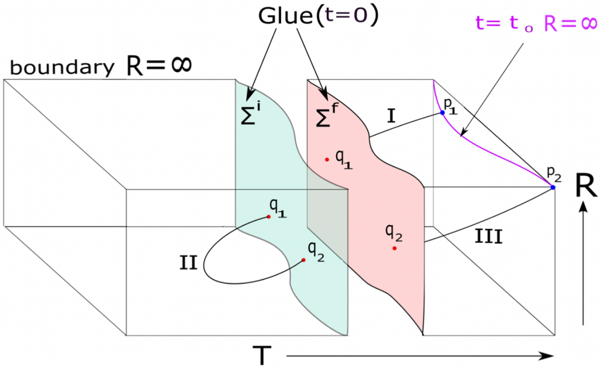
Geodesics in Poincare AdS_3

- In coordinates (z, t, x) Poincare AdS_3 has geodesics

$$z(\sigma) = \frac{\operatorname{sech}\sigma}{\sqrt{\lambda^2 - \epsilon^2}}\ , \qquad \text{for } \lambda > \epsilon,$$
$$z(\sigma) = -\frac{\operatorname{csch}\sigma}{\sqrt{\epsilon^2 - \lambda^2}}\ , \qquad \text{for } \epsilon > \lambda.$$

- Here, $\epsilon = p_a \partial_t^a, \lambda = p_a \partial_y^a$, and p^a is the tangent vector.
- Can solve for the other components, and express in terms of the two endpoints.

Glueing Banados Geometries



Determining Intersection Points

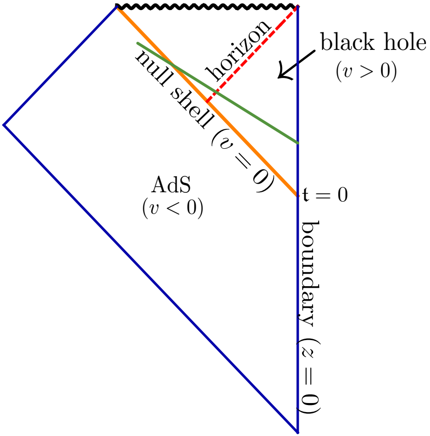
- Pick two spacelike separated points at the boundary.
- Shoot geodesics from the boundary to some points on the shock.
- Form the unique geodesic beyond the shock.
- Enforce appropriate jump conditions at the shock.
- Necessary to use the uniformization map for each of the three segments.

Calculating EE and QNEC in BTZ-Vaidya

- Calculate lengths of the three segments.
- Sum to get EE.
- To calculate QNEC, determine intersection points with deformed points.
- Take derivatives along the deformation.

Outline

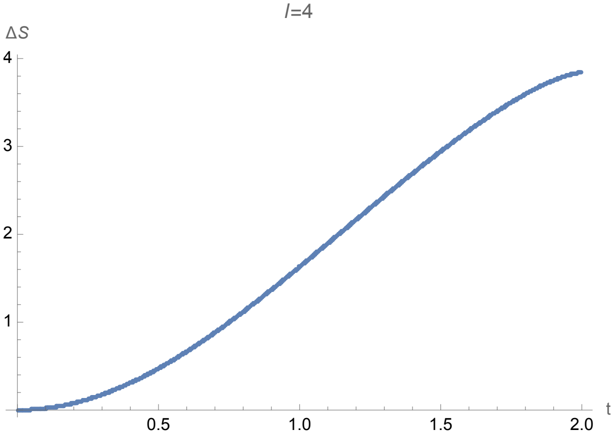
Representative geodesic in AdS_3-Vaidya



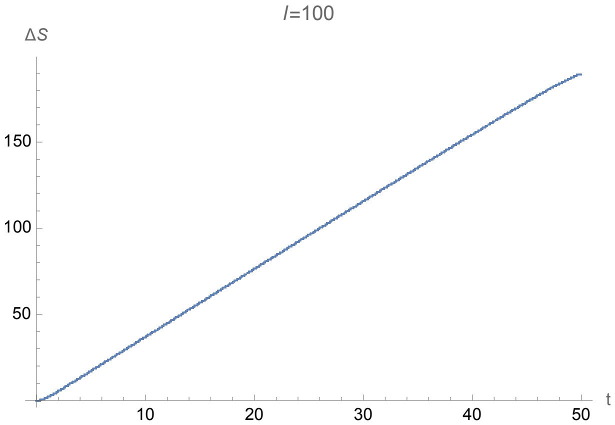
Precis

- Choose the interval with endpoints $(t, 0)$ and (t, l), and look at transitions between arbitrary BTZ geometries.
- For transitions between non-rotating BTZ, can track the complete evolution.
- Can handle all transitions analytically at small times and arbitrary l.
- Need numerics in general to determine the intersection points.

EE for Vacuum to Non-Rotating BTZ



EE for Vacuum to Non-Rotating BTZ



Stages of the Evolution

- Quadratic growth at small times after the quench, $\Delta S \sim Dt^2$, for $0 \leq t \ll 1$. 1302.0853, 1305.7244
- Quasi-linear growth at intermediate times, $\Delta S = v_s t$.
- EE always saturates at $t = l/2$.
- Approach to saturation characterized by an exponent 3/2, i.e.,

$$\Delta S \propto -\left(\frac{l}{2} - t\right)^{3/2}.$$

Representative Explicit Expressions

- For transition from vacuum to non-rotating BTZ,

$$D = 2\mu^2, \qquad v_s = 4\mu = 2S_{BH}.$$

- For linear growth regime, we obtain for large l,

$$\Delta S = 2S_{BH}t + 2\log\left(1 - \frac{2t}{l}\right) + 4\log 2.$$

- For transitions between arbitrary BTZ geometries, for arbitrary l,

$$D = \mu_{+f}^2 + \mu_{-f}^2 - \mu_{+i}^2 - \mu_{-i}^2.$$

Outline

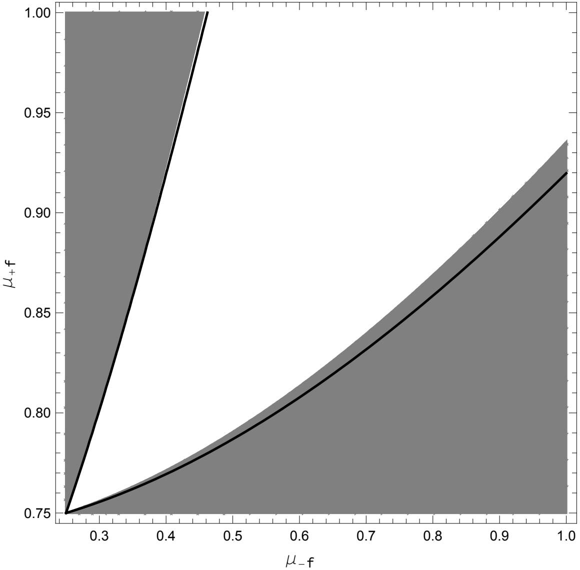
Bounds on Quenches $t = 0$

- For large l, instantaneously after the shock

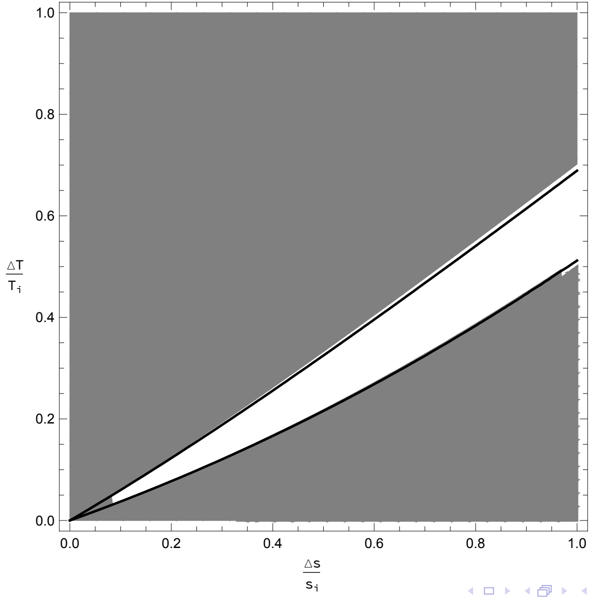
$$\mathcal{Q}_{+} = \frac{1}{4}\left(3\mu_{+f}^2 - \mu_{-f}^2 - 3\mu_{+i}^2 + \mu_{-i}^2\right) \geq 0 \ ,$$
$$\mathcal{Q}_{-} = \frac{1}{4}\left(3\mu_{-f}^2 - \mu_{+f}^2 - 3\mu_{-i}^2 + \mu_{+i}^2\right) \geq 0 \ .$$

- For $\mu_{\pm i} = 0$, these imply $\mu_{+f} \leq \sqrt{3}\mu_{-f} \leq \sqrt{3}\mu_{+f}$.
- For vacuum to non-rotating BTZ at large l, we can analytically show that $\mathcal{Q}_{\pm}(t) = 0$.

Example of (Dis)Allowed Parameter Space



Example of (Dis)Allowed Parameter Space



Key Features

- QNEC non-violation requires that both $\mu_{\pm}$ should increase.
- This is necessary but not sufficient.
- Given initial entropy and the initial and final temperatures, QNEC bounds the allowed final entropy from both above and below.
- Also places bounds on the rates of growth of entanglement.

Outline

Summary

- Investigated QNEC in quenched CFT_2 states.
- Working out the evolution of entanglement entropy in a large class of BTZ-Vaidya geometries.
- QNEC places tight upper and lower bounds on irreversible entropy production, and bounds the rates of growth of entanglement.
- Provide proof of concept that 1+1-dimensional many body systems can be used as quantum memories in 2202.00022.

Remarks on Scope of Validity

- Holography requires the CFT to have large central charge and a sparse spectrum.
- Timescale of the quench should be faster than any other scale involved.
- Temperature scales involved should be smaller than the microscopic energy scale below which the CFT provides a good description.
- Since strongest QNEC bounds arise as $l \to \infty$, we expect the microscopic details to be irrelevant.

Outlook

- Inhomogeneous to inhomogeneous transitions.
- Continuous quenches.
- Higher dimensions.
- Verification in numerical simulations.
- Lattice version of QNEC.
- Non-relativistic version of QNEC?

Thank You