Deep Learning Bulk Spacetime from Boundary Optical Conductivity

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Overview

AdS/CFT (QCD,CMT,QI) : Bottom-up approach

Fields
Particles
Interactions

Action
Equation of motion

Solution of response
Physical quantity
: Transport coefficient

Linear Axion Model

EOM of perturbation

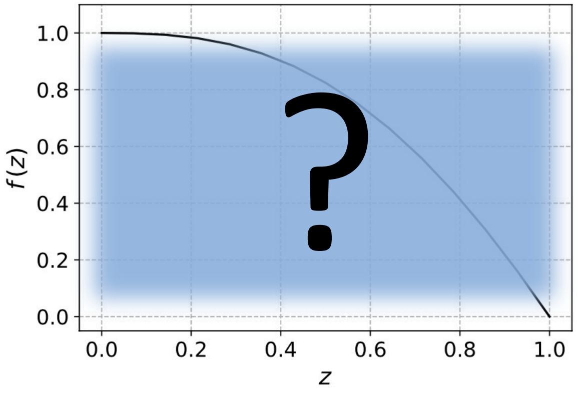
Optical conductivity

AdS/Deep Learning Correspondence : Inverse problem

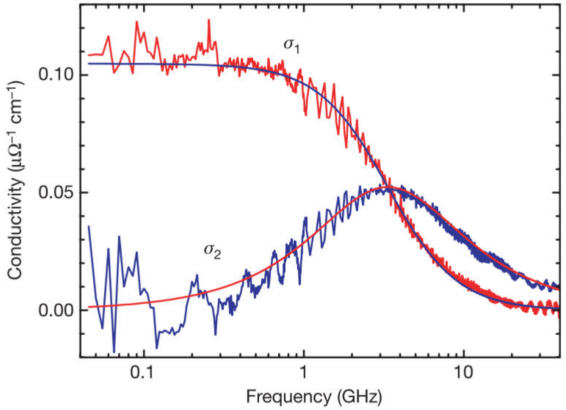
Bulk
Background Metric

Equation of motion
with unknown functions

Input data
from the boundary



Experimental data
of UPd2Al3



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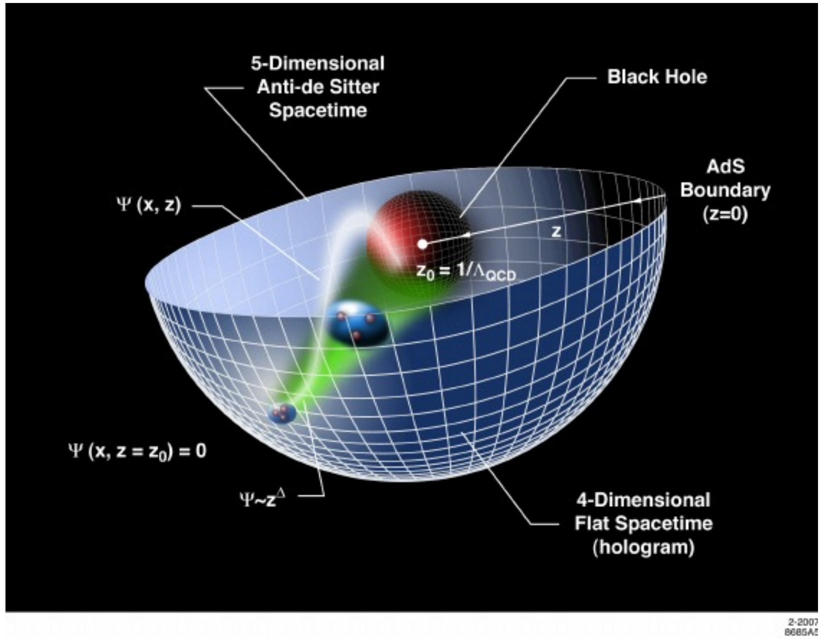
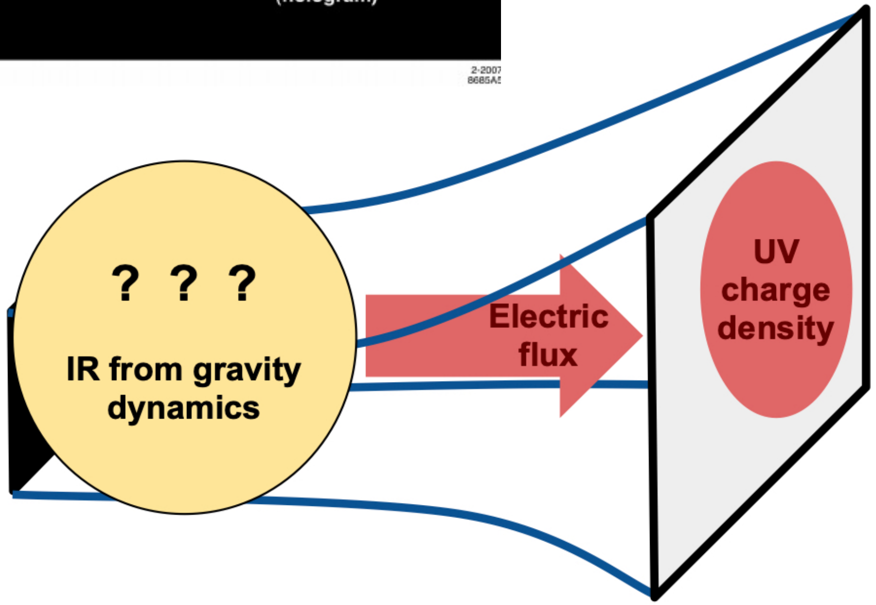
AdS/CMT and Holographic Conductivity

Part 1

AdS/CFT Correspondence





Action

↓

Metric, Field, Perturbation

↓

Equation of motion

↓ Holography

Asymptotic behavior of solution

↓

Physical Quantity

Brodsky, Teramond, 0802.0514, Hartnoll, 1106.4324

AdS/CMT – Linear Axion Model

GENERAL ACTION & FIELDS

$$\mathcal{S} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1,2}(\partial X_I)^2\right)$$

FIELDS ANSATZ

$$\mathrm{d}s^2 = \frac{1}{z^2}\left[-f(z)\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + \mathrm{d}x^2 + \mathrm{d}y^2\right]$$

$$A = \mu\,(1-z)\,\mathrm{d}t\,, \qquad X_1 = \alpha\, x\,, \quad X_2 = \alpha\, y\,,$$

Chemical potential : Gauge field

Momentum relaxation : Massless scalar field

BLACK HOLE SOLUTION

$$f(z) = 1 - \frac{\alpha^2}{2}z^2 - \left(1 - \frac{\alpha^2}{2} + \frac{\mu^2}{4}\right)z^3 + \frac{\mu^2}{4}z^4$$

Horizon : z = 1, Boundary : z = 0

$$T_H := -\frac{f'(1)}{4\pi} = \frac{12 - 2\alpha^2 - \mu^2}{16\pi}$$

Temperature : Black hole geometry

AdS/CMT - Holographic Conductivity

EQUATION OF MOTION : Perturbation + Ingoing BC

htx : metric perturbation, ax : gauge perturbation, ψx : axion perturbation

$$\delta g_{tx} = e^{-i\omega t}\frac{h_{tx}(z)}{z^2}\,, \qquad \delta A_x = e^{-i\omega t} a_x(z)\,, \qquad \delta X_1 = e^{-i\omega t}\frac{\psi_x(z)}{\alpha}\,,$$

$$a_x''(z) + \frac{f'(z)}{f(z)}a_x'(z) + \left(\frac{\omega^2}{f(z)^2} - \frac{\mu^2 z^2}{f(z)}\right)a_x(z) - \frac{i\mu z}{f(z)}\phi(z) = 0\,,$$

$$\phi''(z) + \frac{f'(z)}{f(z)}\phi'(z) + \left(\frac{\omega^2}{f(z)^2} - \frac{\alpha^2}{f(z)} - \frac{f'(z)}{zf(z)}\right)\phi(z) + \frac{i\alpha^2\mu z}{f(z)}a_x(z) = 0\,,$$

ϕ : field redefinition including axion perturbation

$$\phi(z) := -\frac{f(z)\psi_x'(z)}{\omega\, z}$$

HOLOGRAPHIC CONDUCTIVITY

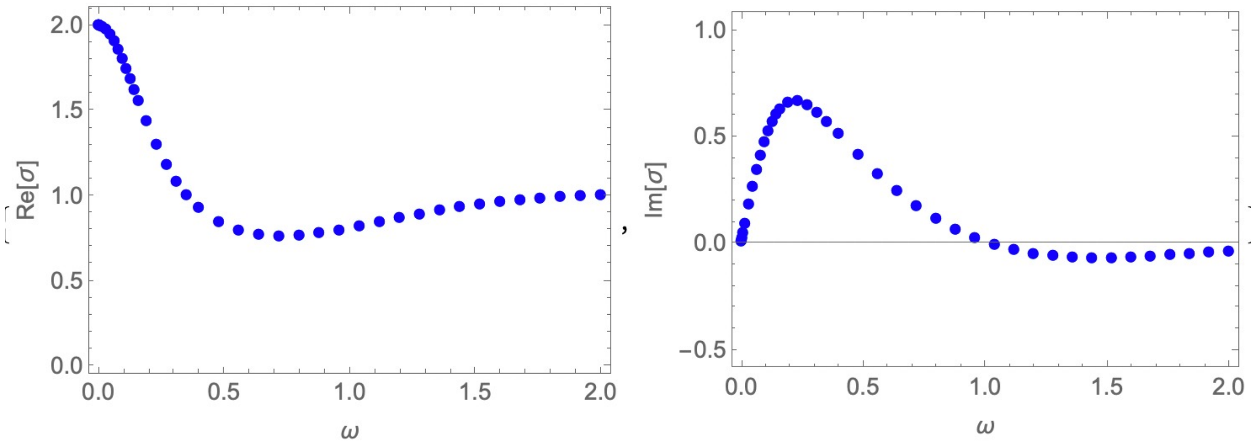
$$a_x = a_x^{(S)} + a_x^{(R)} z + \cdots\,,$$

$$\phi = \phi^{(S)} + \phi^{(R)} z + \cdots\,,$$

$$\sigma(\omega) = \frac{1}{i\omega}G^R_{j^x j^x}(\omega) = \frac{1}{i\omega}\frac{a_x^{(R)}}{a_x^{(S)}}\,,$$

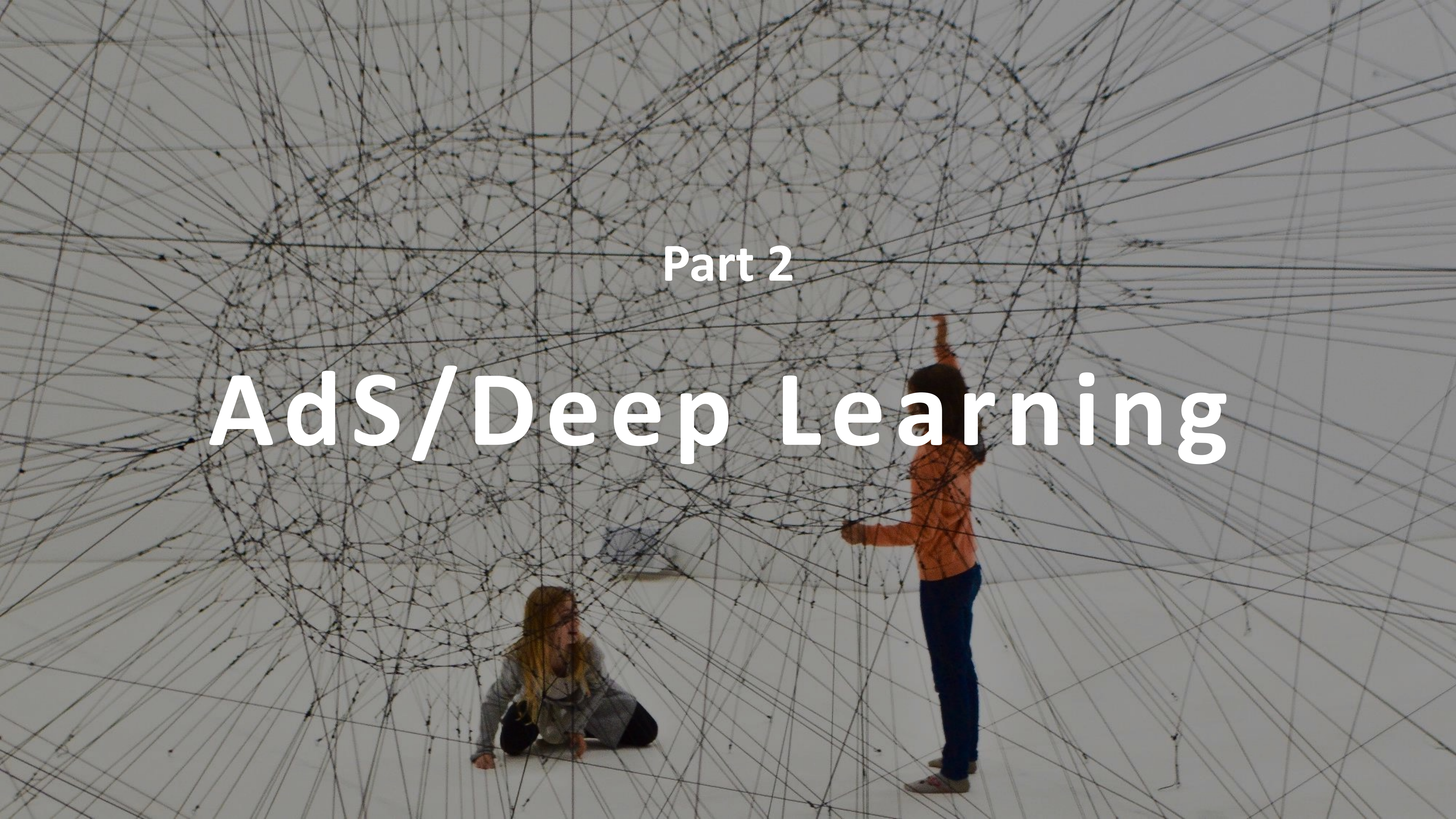
Sourceless condition : $\phi^{(S)} = 0$

AC Conductivity (μ=1, α=1)



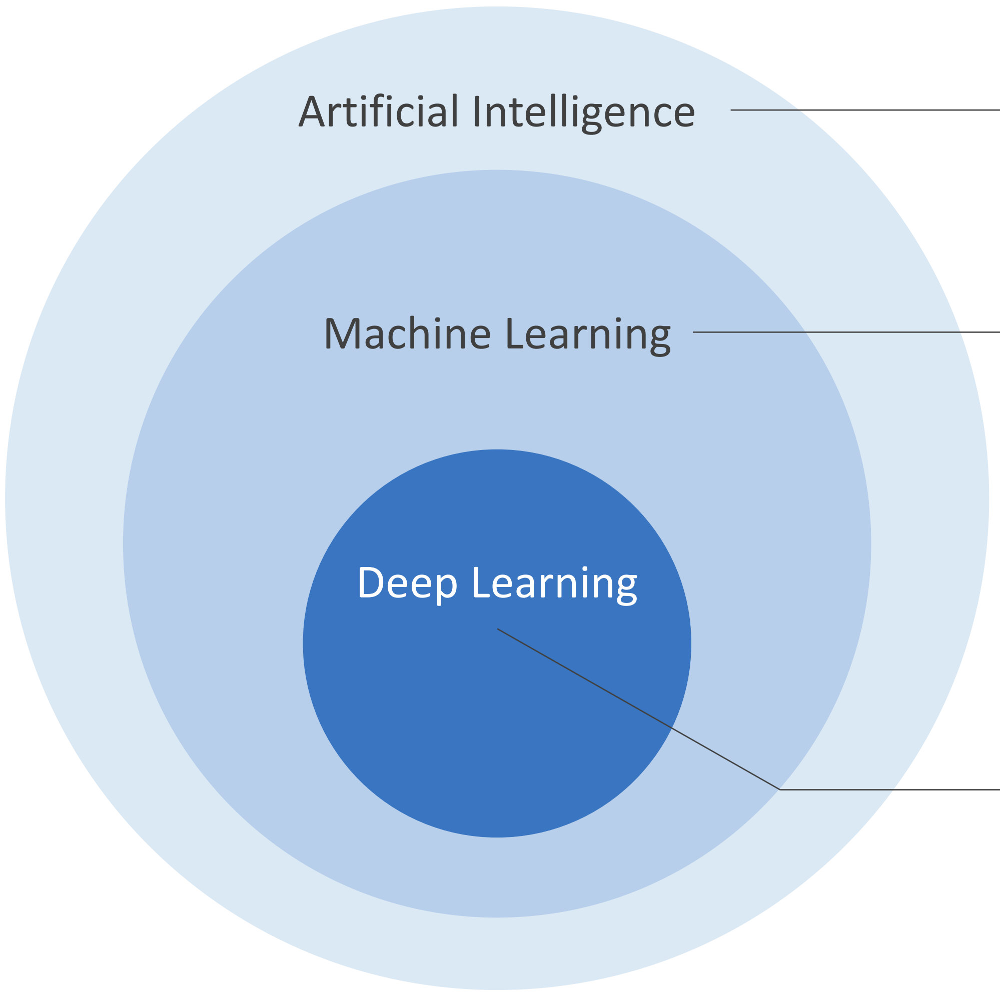
Part 2

AdS/Deep Learning

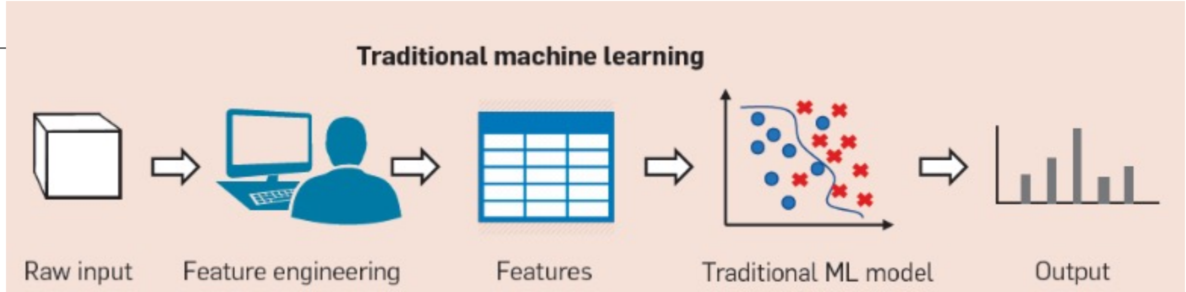


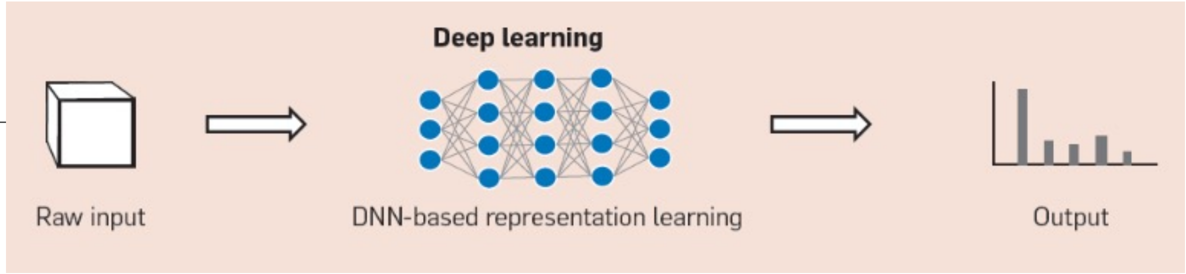
Part 2

Machine Learning/Deep Learning



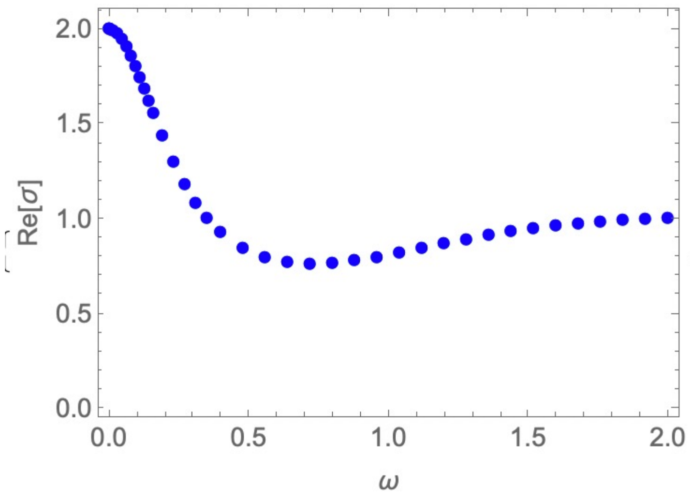
Artificial intelligence (AI) makes it possible for machines to learn from experience, adjust to new inputs and perform human-like tasks.





AdS/DL – Construction & Neural ODE

INPUT DATA : AC conductivity obtained from hologrpahy



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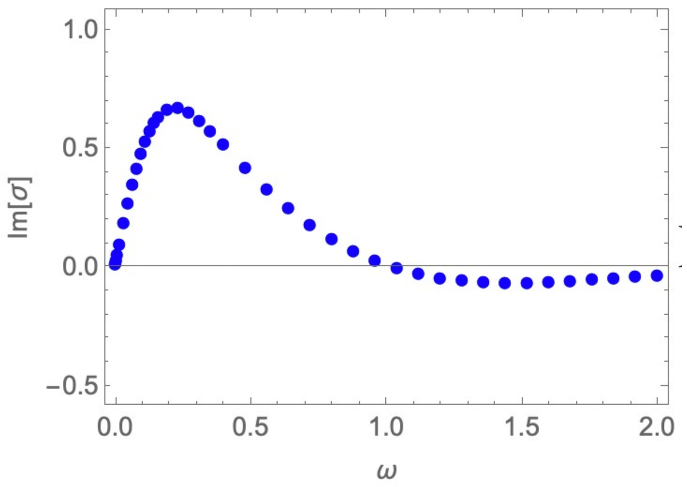


SOLVER : Neural ordinary differential equation (Neural ODE)

DATA SET : Frequency ω

TRAING VARIABLE : Metric function

Polynomial & Continuous

$$f(z) = \sum_{i=0}^{4} b_i \, z^i \, ,$$

Enhance accuracy
Natural interpretation as a metric

PHYSICAL CONSTRAINTS : AdS black hole geometry

Boundary : z = 0, Horizon : z = 1, cutoff = 0.0001, layers = 10,000

Asymptotically AdS : f(z) → 1 $\quad b_0 = 1 \, ,$

Blackening factor : f(1) = 0 $\quad b_4 = -(1 + b_1 + b_2 + b_3) \, .$

Coordinate transformation $\quad b_1 = 0$

AdS/DL - Construction

EQUATION OF MOTION : Perturbation

$$A_x(z) := (1-z)^{-\frac{i\omega}{f'(1)}} a_x(z)\,, \qquad \Phi(z) := (1-z)^{-\frac{i\omega}{f'(1)}} \phi(z)\,,$$

$$\partial_z^2 A_x = \zeta\,\partial_z A_x + \left(\frac{z^2\mu^2}{f} - \xi\right) A_x + \frac{iz\mu}{f}\Phi\,, \qquad \zeta := \frac{2i\omega}{(1-z)f'(1)} - \frac{f'(z)}{f(z)}\,,$$

$$\partial_z^2 \Phi = \zeta\,\partial_z \Phi + \left(\frac{\alpha^2}{f} + \frac{f'}{zf} - \xi\right)\Phi - \frac{iz\alpha^2\mu}{f} A_x\,, \quad \xi := \frac{\omega^2}{f(z)^2} + \frac{i\omega}{(1-z)f'(1)}\left(\frac{i\omega}{(1-z)f'(1)} - \frac{1}{1-z} - \frac{f'(z)}{f(z)}\right)$$

PHYSICAL CONSTRAINTS : Boundary conditions

Black hole horizon : Regularity condition

$$\partial_z A_x(1) = -\left(\frac{i\omega f''(1)}{2f'(1)^2} + \frac{i\mu^2}{2\omega - if'(1)}\right) A_x(1) + \frac{\mu}{2\omega - if'(1)}\Phi(1)\,,$$

$$\partial_z \Phi(1) = -\frac{\alpha^2\mu}{2\omega - if'(1)} A_x(1) - \left(\frac{i\omega f''(1)}{2f'(1)^2} + \frac{i\alpha^2 + if'(1)}{2\omega - if'(1)}\right)\Phi(1)\,,$$

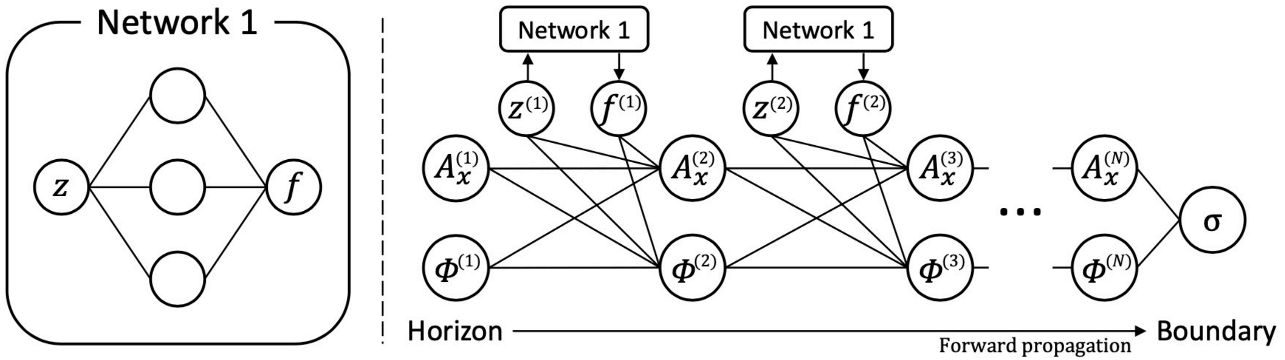
Axion sourceless condition : $\phi(0) \to 0$

LOSS FUNCTION

$$\sigma_m(\omega) = \frac{a_x'(z_{fin})}{i\omega a_x(z_{fin})} = \frac{A_x'(z_{fin})}{i\omega A_x(z_{fin})} - \frac{1}{f'(1)}\,.$$

$$\mathcal{L} = \frac{1}{N}\sum_{\omega}\Big[|\sigma_m(\omega) - \sigma(\omega)| + \beta|\Phi(z_{fin})|\Big]\,.$$

FORWARD PROPAGATION

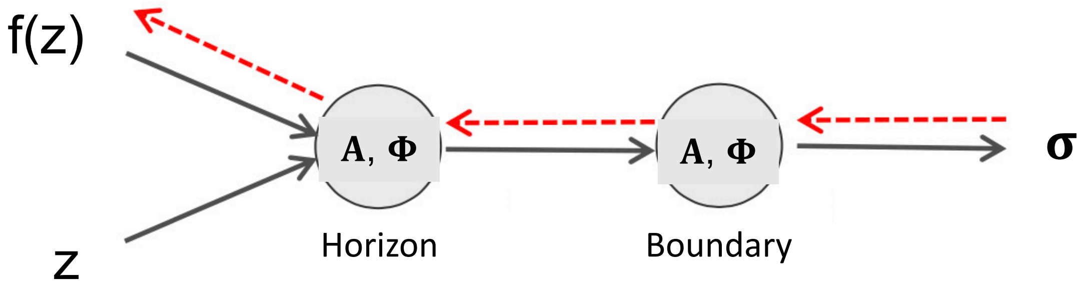


Metric
Unknown function

Horizon
Regularity condition

Boundary
Conductivity

BACKWARD PROPAGATION



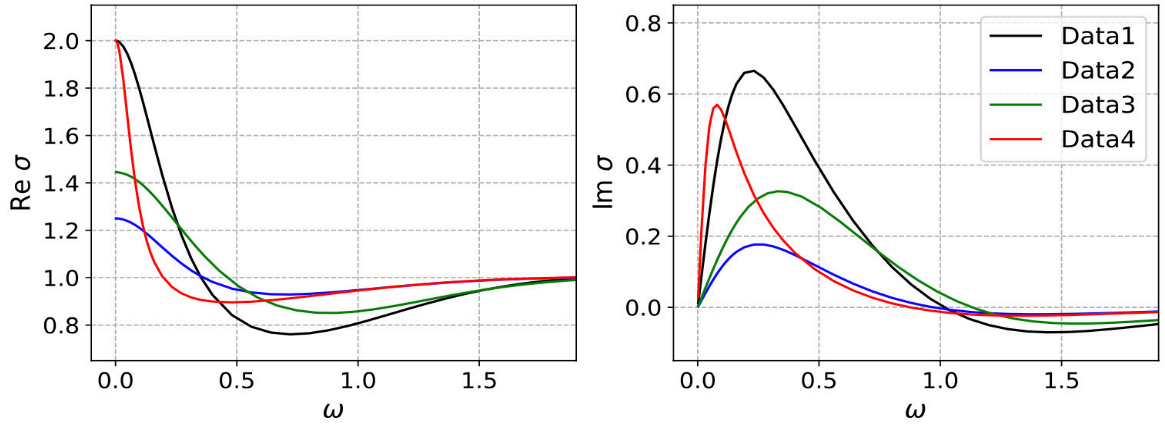
Zhou, Wang, Pang, Shi, 2303.15136

TRAINING f(z) at GIVEN $(\mu, \alpha) \rightarrow (b_2, b_3)$

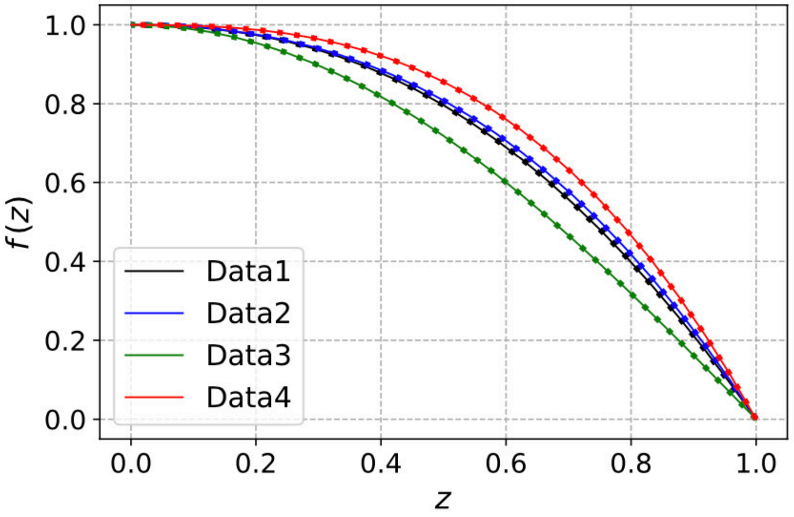
HOLOGRAPHIC CONDUCTIVITY

Data 1 : $\mu = 1.0\,, \quad \alpha = 1.0\,,$ Data 2 : $\mu = 0.5\,, \quad \alpha = 1.0\,,$

Data 3 : $\mu = 1.0\,, \quad \alpha = 1.5\,,$ Data 4 : $\mu = 0.5\,, \quad \alpha = 0.5\,.$



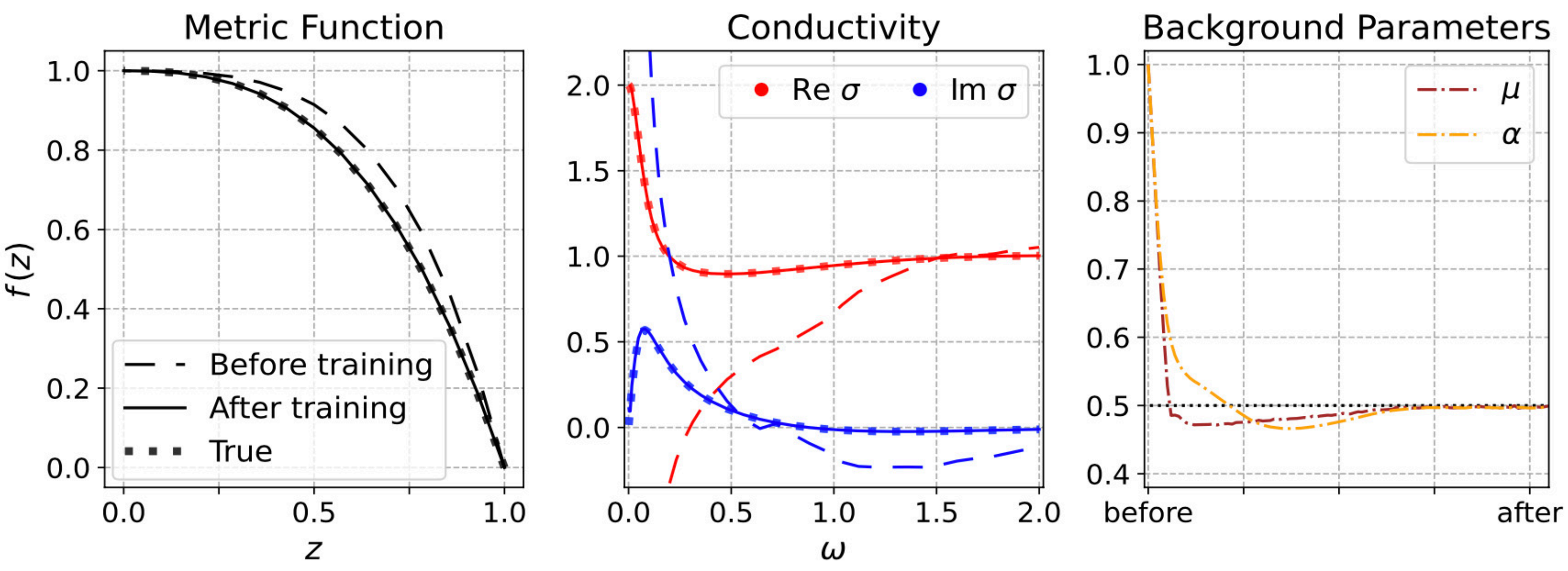
TRAINED METRIC



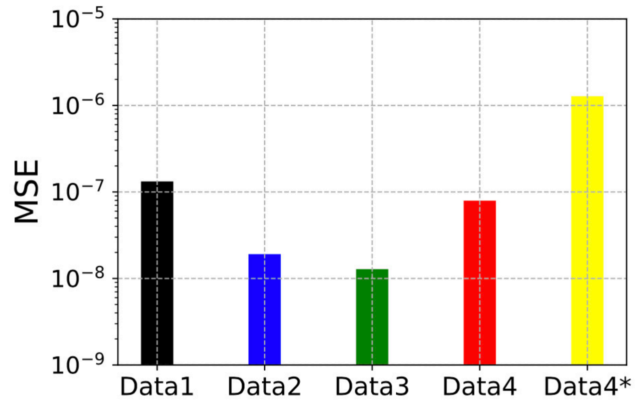
Dotted : Reference data
Solid : Trained data

		Metric function $f(z)$
Data 1	*True*	$f(z;\mu,\alpha) = 1 - 0.5000\,z^2 - 0.7500\,z^3 + 0.2500\,z^4$
	Trained	$f(z;b_2,b_3) = 1 - 0.5048\,z^2 - 0.7435\,z^3 + 0.2483\,z^4$
Data 2	*True*	$f(z;\mu,\alpha) = 1 - 0.5000\,z^2 - 0.5625\,z^3 + 0.0625\,z^4$
	Trained	$f(z;b_2,b_3) = 1 - 0.5056\,z^2 - 0.5481\,z^3 + 0.0537\,z^4$
Data 3	*True*	$f(z;\mu,\alpha) = 1 - 1.1250\,z^2 - 0.1250\,z^3 + 0.2500\,z^4$
	Trained	$f(z;b_2,b_3) = 1 - 1.1294\,z^2 - 0.1146\,z^3 + 0.2440\,z^4$
Data 4	*True*	$f(z;\mu,\alpha) = 1 - 0.1250\,z^2 - 0.9375\,z^3 + 0.0625\,z^4$
	Trained	$f(z;b_2,b_3) = 1 - 0.1274\,z^2 - 0.9271\,z^3 + 0.0544\,z^4$

TRAINING f(z) and $(\mu, \alpha) \to (b_2, b_3, \mu, \alpha)$



MEAN SQUARED ERROR



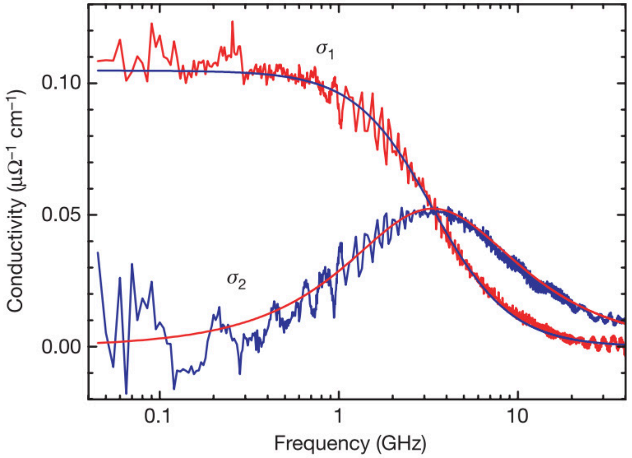
$$\mathrm{MSE} = \frac{1}{N} \sum_z |f(z;\mu,\alpha) - f(z;b_2,b_3)|^2 .$$

COMPARISON BETWEEN FROM TURE DATA 4 AND TRAINED DATA

$$(b_2,\, b_3) = \begin{cases} (-0.1250,\ -0.9375) & (\text{True}) \\ (-0.1487,\ -0.8627) & (\text{Trained}) \end{cases},$$

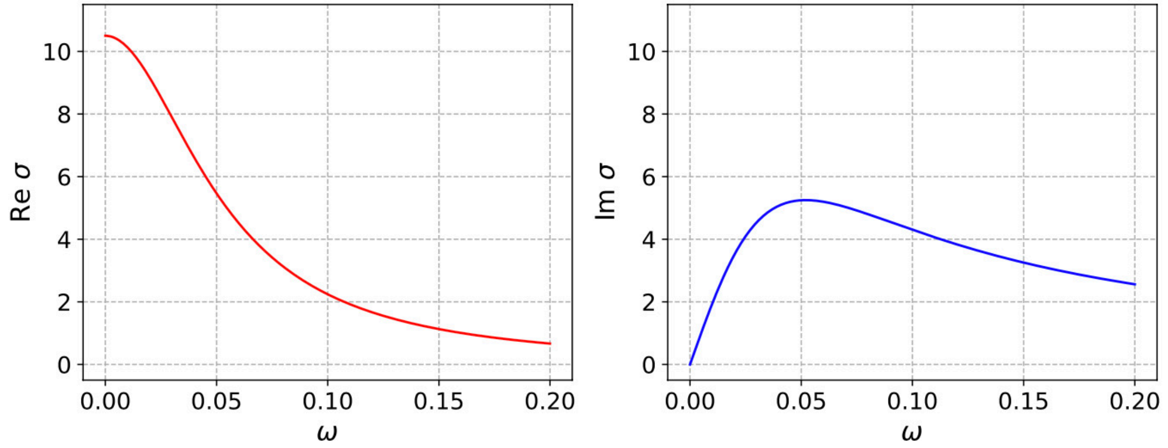
$$(\mu,\, \alpha) = \begin{cases} (0.5000,\ 0.5000) & (\text{True}) \\ (0.5004,\ 0.4997) & (\text{Trained}) \end{cases},$$

EXPERIMENT DATA : UPd2Al3

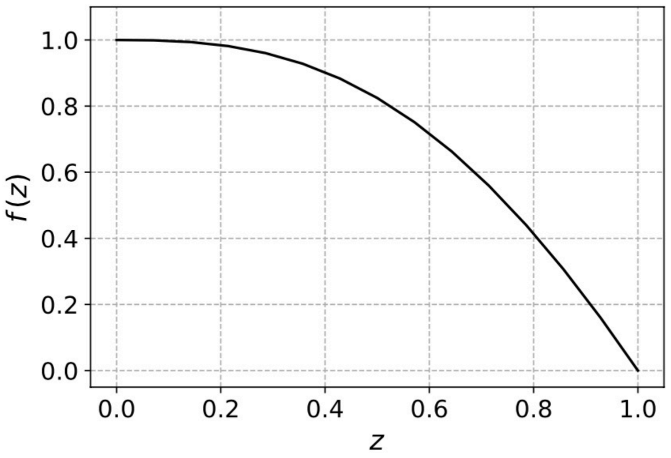


Marc Scheffler, Martin Dressel, Martin Jourdan, Hermann Adrian 2005

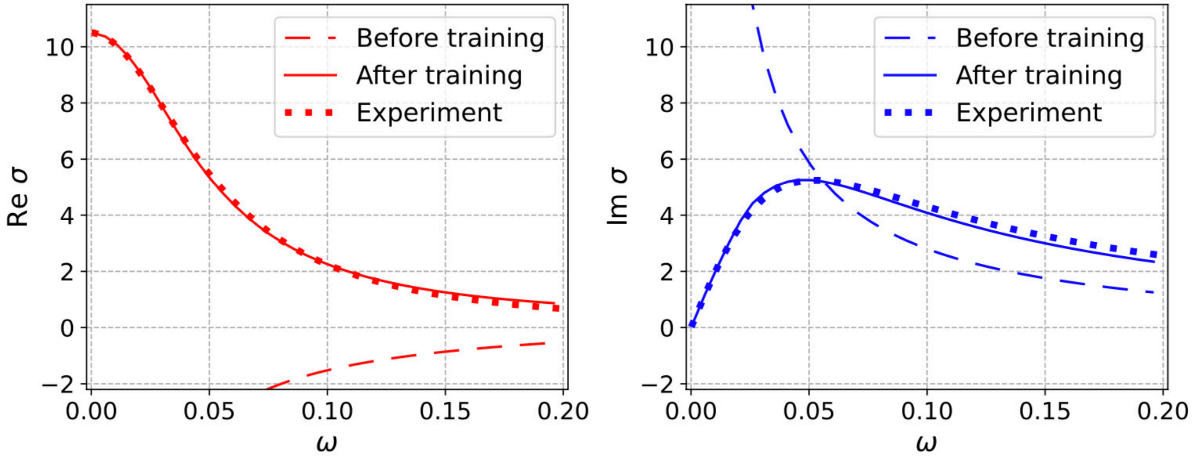
EXTRACT DATA after RESCALING



TRAINED METRIC



CHECK for VALIDITY



Part 3

Conclusion and Further Discussion

Conclusion

- We have studied the bulk geometry of the AdS black hole within the framework of holographic condensed matter theory, with a focus on scenario where translational symmetry is broken.
- Utilizing the neural ordinary differential equation in the AdS/DL correspondence, we derived the bulk metric from the boundary electric conductivity data.
- We explore a scenario involving the use of “experimental” optical electric conductivity data from the real material UPd2Al3, representative of heavy fermion metals in strongly correlated electron systems.

- In our research, there are three improvements:

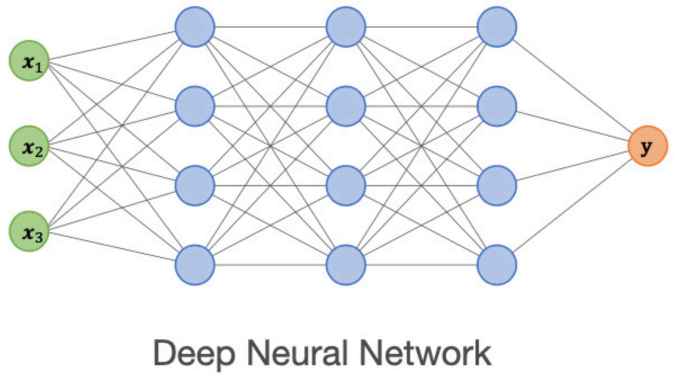
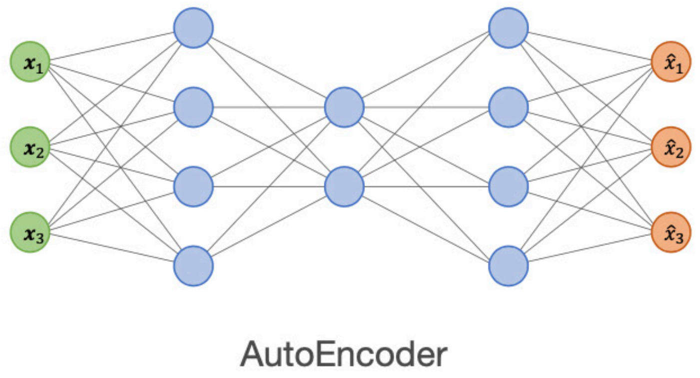
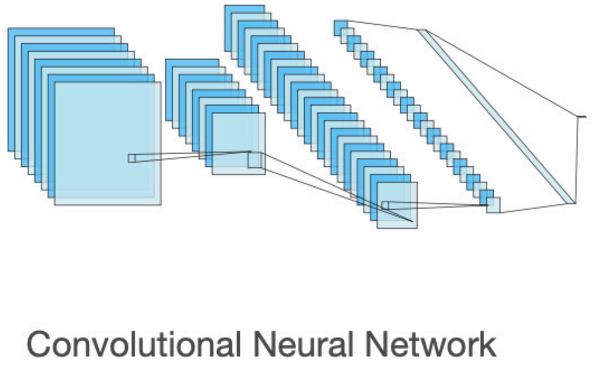
Firstly, we adopted the neural ODE methodology. By choosing a continuous function as the training variable, we not only ensured effective application to coupled bulk equations but also enabled a more natural interpretation as the metric.

Secondly, we utilized optical conductivity data defined strictly at the boundary. By introducing regularity conditions into deep learning, we not only removed constraints in exploring data at various parameter values but also enabled a physically meaningful interpretation.

Lastly, we considered the holographic model with the momentum relaxation.

Part 3

Deep Learning & Physics

Deep Neural Network	Auto Encoder	Convolutional Neural Network
		
	- Reconstruct the input data going through a bottleneck structure. - Bottleneck layer describes the code to more efficiently represent data with latent variables.	- Regularized type of the feed-forward neural network. - It learns feature engineering through filters optimization.
Equations of motion	Effective Degrees of Freedom	Renormalization Group Flow

Zhou, Wang, Pang, Shi, 2303.15136

Thank you for listening!