Black holes in AdS/CFT 1

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Black holes & quantum gravity

Black holes universally exhibit thermal behaviors:

- Black hole thermodynamics from event horizon $S = \frac{A}{4G}$, $T = \frac{(\text{surface gravity})}{2\pi}$, ……
- This is so precise (e.g. Hawking radiation) that we believe it has a statistical origin.

As the history of physics proves, thermodynamics is a window to new physics.

- Black body to quantum physics
- Black holes to quantum gravity

Microscopically understanding the black hole thermodynamics is

- not just an application of string theory (or more broadly quantum gravity), but
- also a window to explore the quantum gravity physics in extreme conditions.

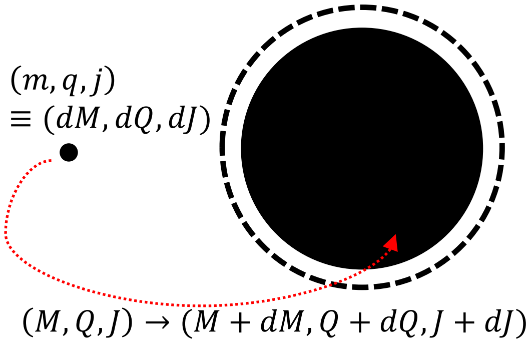
Today's topic: interplay of the two themes quantum gravity & black holes

- Quantum gravity (say string theory) explains black hole physics.
- Black holes reflect/constrain/predict novel quantum gravity physics.

No hair, missing information

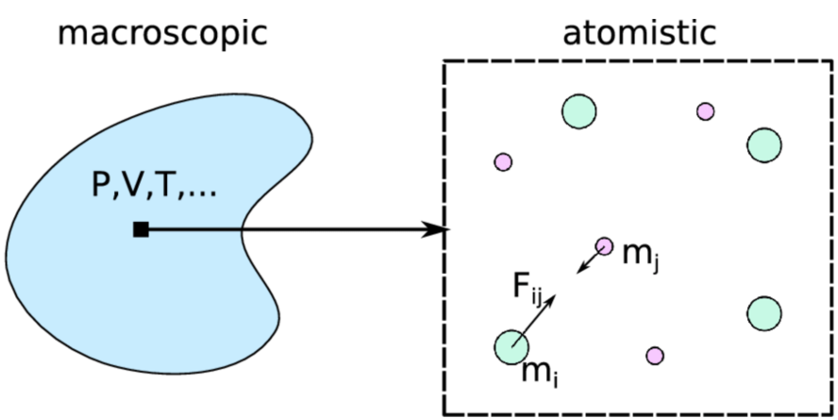
"No hair theorem"

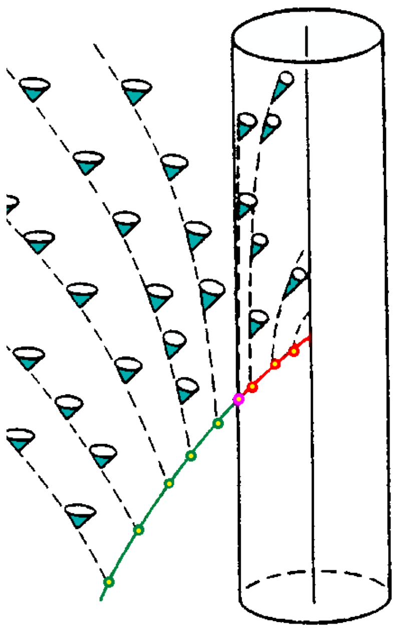
- Black holes (often) settle down to simple solutions labeled by conserved charges
- Mass (~energy), charge, angular momentum
- No information on matters which collapse to form black holes.



The "no hair" behavior, or missing information, is related to the existence of "event horizon"

A bit like macroscopic systems: Information is "hard" to access.

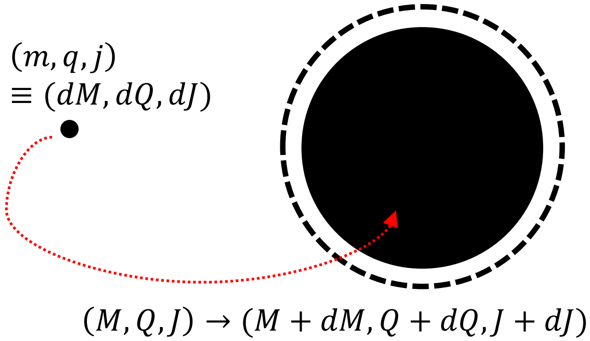




Black hole thermodynamics

Surprisingly, the missing information of black holes induces "thermal behaviors"

- 1st law: Slowly perturb BHs: behaves as if it absorbs "heat"



$$dM = \frac{\kappa}{8\pi G} dA + \Omega dJ + \Phi dQ$$

κ: surface gravity at the event horizon
A: area of the even horizon

- 2nd law: obeys "area law" [Hawking] (1971)

$$\frac{dA}{dt} \geq 0$$

Hawking radiations strongly implies that this should be real thermodynamics.

$$\frac{c^2 \kappa}{8\pi G} dA = T dS \quad \rightarrow \quad S_{BH} = \frac{A\, c^3}{4G\, \hbar}$$ Bekenstein-Hawking entropy, $$T = \frac{\kappa\, \hbar}{2\pi\, c}$$ Hawking temperature

Further studies over the past ~50 years:

- Establishing that this thermodynamics is based on statistical mechanics.
- Exploring the implications of the novel thermodynamics of black holes.

Entropy

Entropy is a very subtle notion in many ways.

- It is a quantum property: #(states) at given macroscopic data

$$S(E, V, \ldots) = \log(\#\, of\ states\ at\ fixed\ E, V, \ldots)$$

- Measures "possibilities" rather than a definite phenomenon.
- Measures "ignorance": Information unknown
- Good master function for thermodynamic observables, but otherwise an abstract quantity.

$$\frac{1}{T} = \frac{\partial S}{\partial E} \quad , \quad \text{heat} = \Delta Q = T\Delta S \quad , \quad \ldots\ldots$$

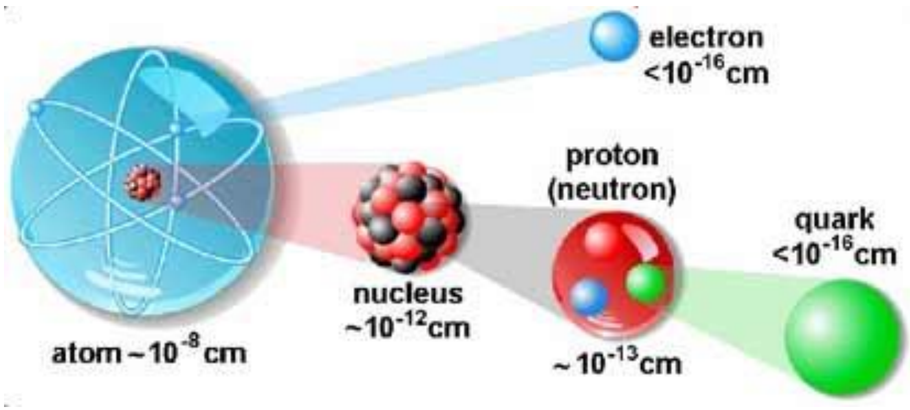
But it has been a key "theoretical observable" of studying quantum black holes.

- Statistical account from a quantum system which makes the black hole. [Strominger, Vafa]

Bekenstein bound of entropy

Sounds strange that the information on the matter (forming BH) is definitely given.

- We rarely see short-distance information by looking at big bodies, especially in the particle physics/quantum field theory which usually have UV cutoffs.



Entropy has un upper bound: $S \leq \frac{2\pi k_B R E}{\hbar c}$. (E: energy, R: "size" of body) [Bekenstein] (1973,1981)

- The black hole entropy S_{BH} saturates this bound.
- May thus be regarded as the most fundamental, fine-grained, information of Nature.

S_{BH} is an exceptional channel to see the fundamental information of Nature.

- What are the matters made of, at the most elementary level?

Exotic entropy of black holes

Have negative specific heat $c < 0$: Lose energy and become hotter.

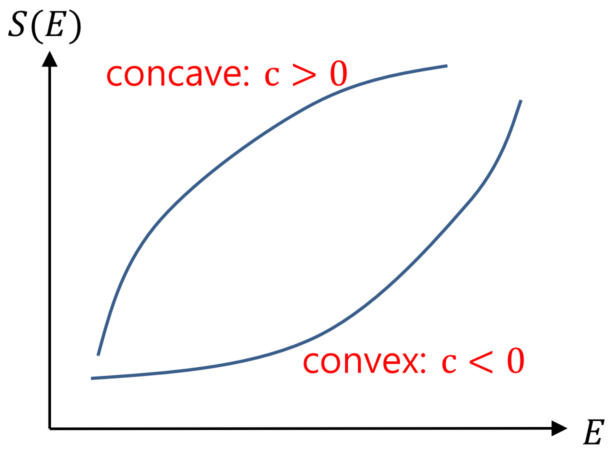
- E.g., for Schwarzschild black holess,

$D = 3 + 1$: $$T_{\rm H} = \frac{\hbar c^3}{8\pi G M k_{\rm B}} \approx 6.169 \times 10^{-8}\ {\rm K} \times \frac{M_\odot}{M}$$

$D \geq 3 + 1$: $GM \sim r^{D-3}$, $S \sim A/G \sim r^{D-2}/G$

$$\to S(M) \sim G^{\frac{1}{D-3}} M^{\frac{D-2}{D-3}} \to \frac{dS(E)}{dE} = \frac{1}{T} \sim M^{1/(D-3)}$$

- Why? Entropy grows too fast: $\frac{dS}{dE} > 0$ and $\frac{d^2S}{dE^2} > 0$.



Of course, ∃systems w/ $c < 0$: due to interactions in special situations: E.g.

- Collapsing stars: release E but get hotter by virial theorem ($K \propto |V|$)
- Plasma-balls near phase transition [Aharony, Minwalla, Wiseman] (2005)

If S_{BH} reflects fundamental asymptotic d.o.f., perhaps there are simpler explanations:

- Demands unusual structures of the Hilbert space.
- Very fast growing entropy in the high energy regime (~ black hole mass).

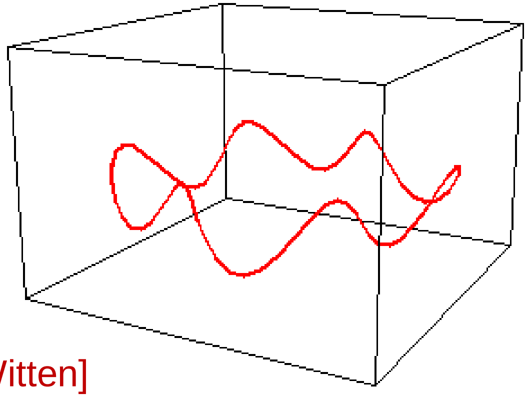
Exotic entropy of quantum gravity

Fast-growing entropy at high E is unfamiliar in standard particle physics.

- Relativistic particles in D space dimension at high E (← almost massless particle):
- On dimensional grounds ($c = 1$, $\hbar = 1$), one finds

$$S \propto VT^D \quad , \quad E \propto VT^{D+1} \quad \rightarrow \quad S(E,V) \propto V^{\frac{1}{D+1}} E^{\frac{D}{D+1}}$$

- $S \propto E^{\alpha}$ with $\alpha = \frac{D}{D+1} < 1$ is not too fast: $d^2S/dE^2 \sim \alpha(\alpha - 1)E^{\alpha-2} < 0$



However, familiar in quantum gravity ~ string theory

- Elementary strings: $S(E) \sim E/T_H$ at high E
- Due to ∞ tower of string oscillation modes:
- We call this "Hagedorn growth" [Hagedorn] (1965) [Sundborg] [Atick, Witten]

  (Originally found in the context of hadron physics)

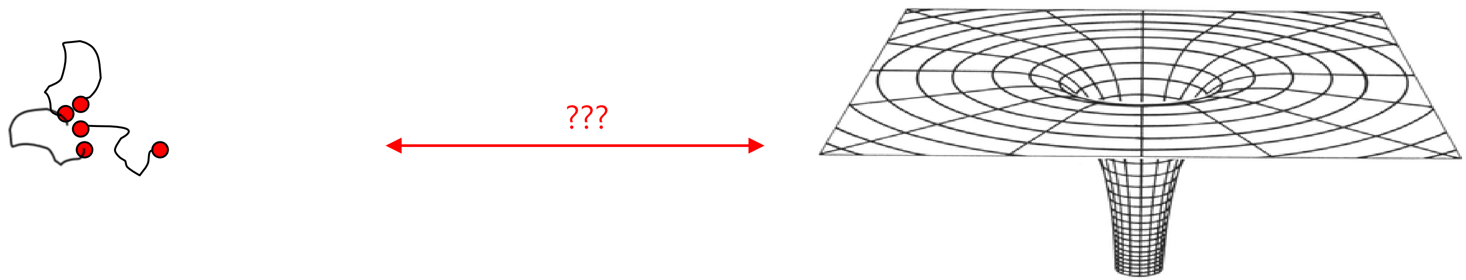
However, this perturbative entropy <u>doesn't grow fast enough</u>: $S_{BH} \propto M^{\frac{D-2}{D-3}} \gg M$.

- Non-perturbative degrees of freedom should play roles.

Statistical entropy of extremal black holes

To form such black holes, non-perturbative objects like “D-branes” are needed.

- New bound states of D-branes [Polchinski] (1995) & open strings → faster growth.



Statistical mechanics of D-brane QM or QFT

Accounts for the gravitational entropy?

Statistical interpretation of S_{BH} justified w/ extremal “BPS” black holes:

- Carry electric charge & saturate the bound $M \geq \# \, Q$.
- $D = 5$: $S = 2\pi Q^{3/2}$ [Strominger, Vafa] (1996) …
- $D = 4$: $S = 2\pi Q^2$ [Maldacena,Strominger,Witten] [Vafa] (1997) …

Since Q is roughly M, qualitatively the same growth as Schwarzschild BH:

- $S_{BH} \propto M^{(D-2)/(D-3)}$

Black hole entropy & quantum gravity

The fast growth of S_{BH} is related to the fundamental structure of the Hilbert space of quantum gravity, as least in string theory models.

- Historically, introducing so many extra d.o.f. to Einstein's relativity as strings was to avoid very technical inconsistencies of quantum gravity (UV divergences).
- Now we may also understand it naturally as a constraint from black hole thermodynamics.

It will be interesting to review how other quantum gravity models explain black hole thermodynamics, related to the number of fundamental d.o.f. they have.

- And also, if the picture/scenario I presented has interesting caveats.

For instance,

- Higher spin gravity, loop quantum gravity, asymptotically free quantum gravity [Weinberg] …

Currently I don't have good enough insights to answer these questions.

Cannot be true…

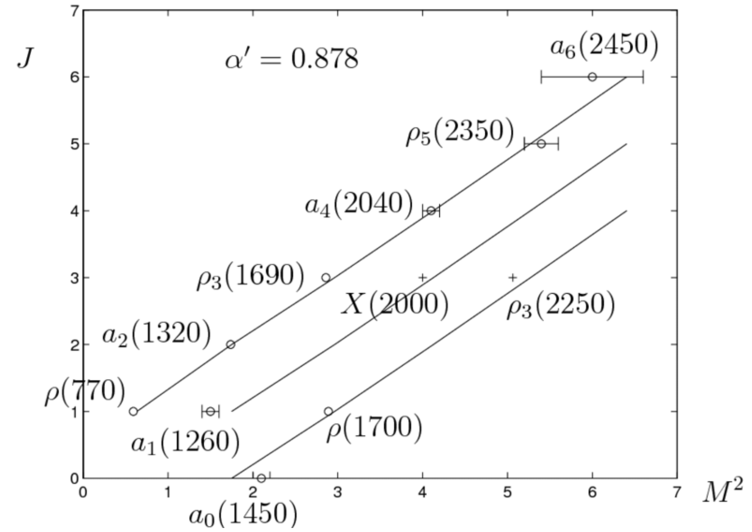
Systems with $c < 0$ are thermodynamically unstable in canonical ensemble.

- This might be an exotic feature of quantum gravity, but sounds somewhat weird.
- It is also possible that quantum gravity at high T/E is fundamentally reformulated.

Analogue in QCD: tower of hadrons vs. quark-gluon

→ QCD at high T is in “quark-gluon plasma phase”

- Tempting to speculate that QG states that we just saw from black holes originate from more fundamental (& fewer) d.o.f. visible at even higher E.



One more general subtlety of gravity at fixed T.

- Needs “finite volume” or IR regulators. (Extensive quantities ∝ “volume”)
- Should put gravity in a box consistently, subject to the equivalence principle.

The questions raised on this page can also be answered by studying black holes.

- I’ll present a model of gravity in a “box” and better phrase these questions.

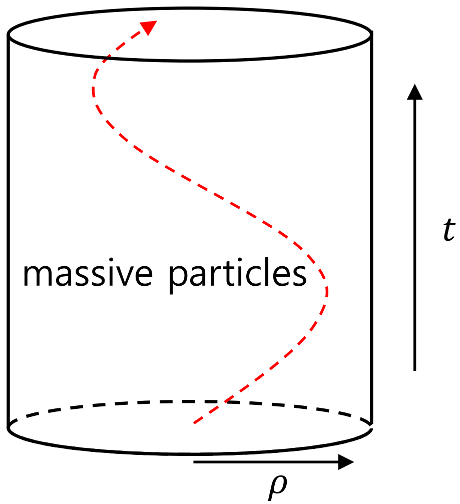
Anti de Sitter (AdS)

A consistent setup for the “box” or “trap”

- Put gravity in AdS: [Hawking, Page] (1983)

$$ds^2_{D+1} = d\rho^2 - \cosh^2\frac{\rho}{\ell}\,dt^2 + \ell^2\sinh^2\frac{\rho}{\ell}\,ds^2(S^{D-1})$$

- Technically, the above metric is for the “global AdS” spacetime.



- Confines to $\rho = 0$: $\Phi \approx -g_{tt}(\rho)/2$. Massive particles (also black holes) cannot escape.
- Renders many thermal questions of QG well defined.

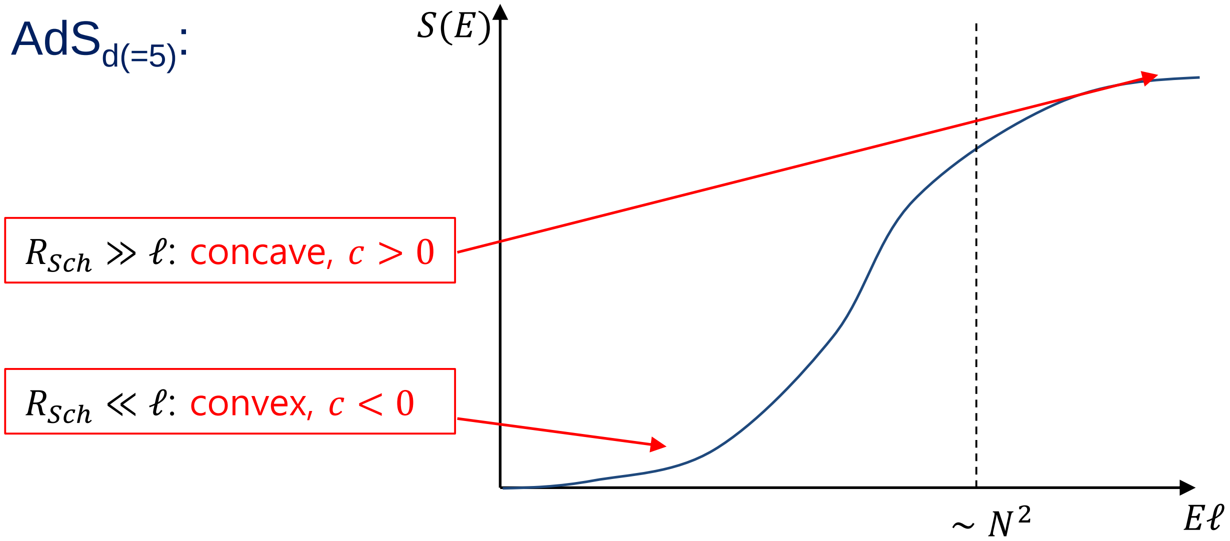
Incidentally, we know a microscopic description of such quantum gravity.

- AdS/CFT correspondence [Maldacena] (1997) ……
- QFT at the boundary $S^{D-1} \times R$ “holographically” describes the QG inside.
- In simple examples, SU(N) gauge theories at $N \gg 1$: large number of “gluons”

Black holes in AdS

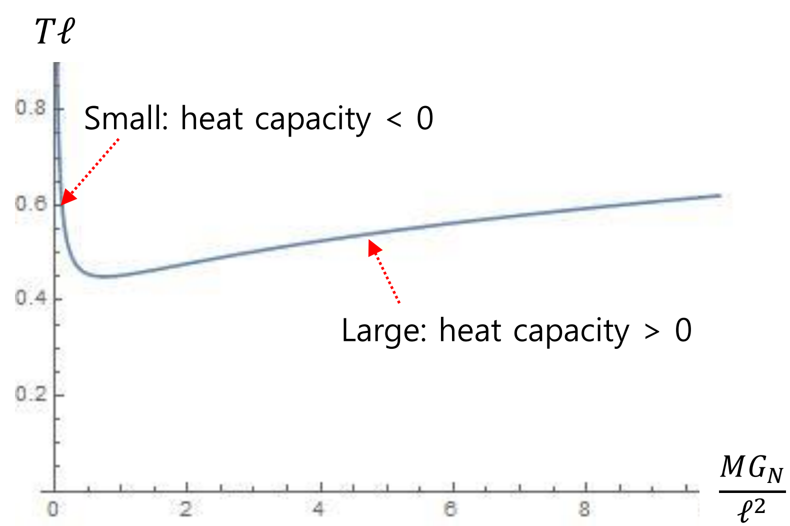
Schwarzschild black holes in AdS$_{d(=5)}$:

[Hawking, Page] (1983)



T-E curve: $T = \frac{r_+}{\pi \ell^2} + \frac{1}{2\pi r_+}$

$$r_+^2 = -\frac{\ell^2}{2} + \ell\sqrt{\frac{\ell^2}{4} + \omega M} \qquad \omega \equiv \frac{16\pi G_N}{3\mathrm{vol}(S^3)}$$



Features:

- Small BH: Like familiar BH's in the sky.
  But exotic thermodynamics.
- Large BH: Novel black holes filling the box.
  But ordinary thermodynamics.
  Determines the phase of the system.

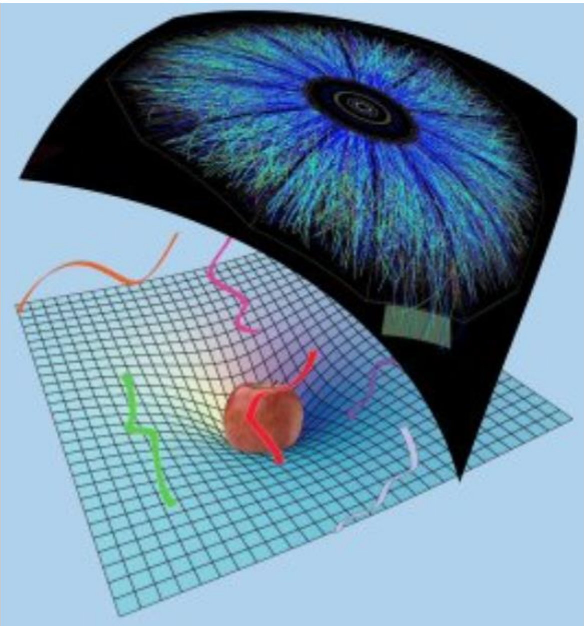
Phase transition & black holes

Quarks/gluons in different phases. (like liquid/gas)

- low T : Confinement. Only see hadrons.

  (like protons ~ uud, mesons ~ $\bar{u}d$, …)

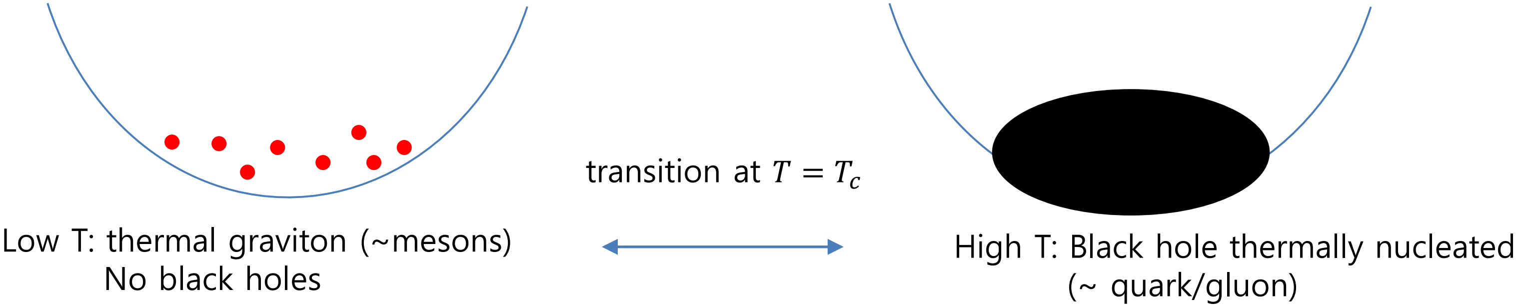
- high T: Deconfinement. Liberated "quark-gluon plasma"



Phase transition at $T_c \sim 1/\ell$. ("radius of curvature")

confinement-deconfinement phase transition

In AdS: Hawking-Page phase transition (1983)



Expect: Deconfined quarks/gluons are boundary images of black holes. [Witten] (1998)

Next lecture

We have learned the followings:

- Asymptotically flat black holes exhibit exotic thermal behaviors which can naturally be explained by quantum gravity w/ large number of high E d.o.f. like string theory
- Putting the quantum gravity in a box (or AdS), we are able to explore a more exotic phase of gravity at high T from black holes.

Next lecture:

- Microscopic studies of the above physics from AdS/CFT
- Both large & small black hole physics
- Will face a universal technical difficulty of having to deal with strongly-coupled QFT.
- Will setup (what I think is) a rather sharp analogue problem in supersymmetric models to enable quantitative calculations and explore these physics.