Chaos in General Holographic Space-times

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WITS

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Outline

1. Motivation & Introduction

2. Chaos, Butterfly-effect & Holography

3. Chaos in various systems

4. Summary & Future Directions
Chaos refers to sensitive dependence on initial conditions, i.e. initially very similar states can evolve to be quite different.

Chaos in context of thermalization.

In Quantum Information Theory and Black holes this is also known as scrambling.

Black Holes are fastest scramblers in nature: \( t_s \sim \beta \log S \).

Hayden & Preskill '07, Sekino & Susskind '08

Largest Lyapunov exponent is bounded by Black hole result: \( \lambda_L \leq \frac{2\pi k_B T}{\hbar} \). Theories where the bound is saturated should have a gravity dual.

Maldacena, Shenker, & Stanford '15
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Diagnostic of Chaos: $C(t) = -\langle[W(t), V(0)]^2\rangle_\beta \sim \mathcal{O}(1)$

- In semi classical limit, $V = p$ and $W = q(t)$:
  $$C(t) = \hbar^2 \left(\frac{\partial q(t)}{\partial q(0)}\right)^2 \sim \hbar^2 e^{2\lambda L t}.$$ 

- $C(t) \sim \mathcal{O}(1)$ at $t_* \sim \frac{1}{\lambda L} \log \frac{1}{\hbar}$

- Dissipation time $t_d$: $\langle V(0)V(t)\rangle \sim e^{-\frac{t}{t_d}}$.

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**Bound on Chaos-2**

\[ F(t) = \text{Tr} [yVyW(t)yVyW(t)] \text{ where } y^4 = \frac{1}{Z} e^{-\beta H}. \]

Conjecture \((t \gg t_d, t \ll t_*)\):

\[
\frac{d}{dt} (F_d - F(t)) \leq \frac{2\pi}{\beta} (F_d - F(t))
\]

Chaotic system: \( (F_d - F(t)) \sim \epsilon e^{\lambda_L t} \)

\[ \rightarrow \lambda_L \leq \frac{2\pi}{\beta} \]

Holographically \((2 + 1 \text{ dim bulk})\):

\[ F(t) = f_0 - \frac{f_1}{N^2} e^{\frac{2\pi}{\beta} t} \text{ Shenker & Stanford '13} \]
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For such chaotic/thermal systems, a small perturbation can change the pattern of correlation drastically: Disruption of entanglement/Butterfly effect.

The diagnostic used in this case is Thermo-Mutual Information Morrison & Roberts '12.

It is recently studied in context of AdS/CFT by various people including Shenker, Stanford, Susskind, Roberts, Leichenauer, ...

We have extended their work in context of:
- Black $Dp$-branes
- Lifshitz Black branes
- Higher Derivative Black brane
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“AdS/CFT Correspondence” or Holography

- Duality between $\mathcal{N} = 4 \ SU(N)$ Super-Yang Mills’ theory in $(3 + 1)$-dim and type IIB Super-strings in $AdS_5 \times S^5$. Maldacena '97; Gubser, Klebanov, Polyakov '98 ; Witten '98

- Simplifies in the limit of large 't Hooft coupling ($\lambda = g_{YM}^2 N \gg 1$) and large $N \gg 1$ to a duality between classical type IIB super-gravity and full quantum Super-Yang Mills’ theory at leading order in $\lambda$ and $N$.

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- Presently refers to more general class of dualities.
Consider an entangling surface $\Sigma$ which divides the space into two separate sub-systems.

Integrate out the degrees of freedom living “outside” (region $B$).

The reduced system is now described by a density matrix $\rho_A$.

“Entanglement entropy” or von Neuman entropy:

$$S_{EE} = -\text{Tr}(\rho_A \log \rho_A).$$
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Holographic Entanglement Entropy

- \( S_{EE} = \min_{\partial \gamma_A = \Sigma} \left( \frac{\text{Area}(\gamma_A)}{4G_N} \right) \)  
  - Ryu & Takayanagi '06

- Generalized to time dependent situations (Covariant prescription): \( \min \rightarrow \text{extremum} \)  
  - Hubeny, Rangamani, & Takayanagi '07

- In presence of dilaton: \( \text{Area} \rightarrow \text{Area in Einstein frame} \)  
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Mutual Information

- $I(A; B) = S(A) + S(B) - S(A \cup B)$
- Finite quantity, UV divergence in EE cancels.
- Strong sub-additivity: $I(A; B) \geq 0$
- Measures total classical and quantum correlation between two regions.

$I(A; B) \geq \frac{1}{2} \left( \frac{\langle O_A O_B \rangle - \langle O_A \rangle \langle O_B \rangle}{\|O_A\| \|O_B\|} \right)^2$  \[\text{Wolf et al., '08 i.e.}\]

$I(A; B) = 0 \implies \langle O_A O_B \rangle = \langle O_A \rangle \langle O_B \rangle$
 Mutual Information

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\[
I(A; B) = 0 \implies \langle O_A O_B \rangle = \langle O_A \rangle \langle O_B \rangle
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Holographic Mutual Information

\[ \min(\text{Area}(\gamma_{A \cup B})) = \text{Area}(\gamma_{A \cup B}) \]

if \( \text{Area}(\gamma_{A \cup B}) < \text{Area}(\gamma_A) + \text{Area}(\gamma_B) \)

\[ = \text{Area}(\gamma_A) + \text{Area}(\gamma_B) \] otherwise

For later case,

\[ I(A; B) = S(A) + S(B) - S(A \cup B) = 0 \]
Black Holes and Finite Temperature QFT

- QFT at Temperature $T \equiv$ Black Holes with Hawking temperature $T$.

- Black Hole metric:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + d\Sigma^2$$

$$f(r_H) = 0; \quad T = \frac{1}{\beta} = \frac{|f'(r_H)|}{4\pi}$$

- For asymptotically AdS black holes: $f(r) \to \frac{r^2}{L^2}$ as $r \to \infty$
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For asymptotically AdS black holes: $f(r) \rightarrow \frac{r^2}{L^2}$ as $r \rightarrow \infty$
Black Holes: Kruskal-Szekeres Coordinates

- Define Kruskal-Szekeres Coordinates \((u, v)\):

\[
\begin{align*}
u v &= -e^{4\pi T r_*(r)}, \quad u/v = e^{-4\pi T t} \\
\text{ds}^2 &= -\frac{4f(r)}{16\pi^2 T^2} e^{-4\pi T r_*(r)} du dv + d\Sigma^2_{\perp} \\
\text{dr}_* &= \frac{dr}{f(r)}.
\end{align*}
\]

- We can further re-define coordinates ("Penrose Diagram") region by: \(U = \tan^{-1}(u), \ V = \tan^{-1}(v)\).

- AdS black holes: AdS boundary at \(uv = -1\) and Singularity at \(uv = 1\).
Define Kruskal-Szekeres Coordinates \((u, v)\):

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Eternal $AdS$ Black Holes

Kruskal-Szekeres Diagram

Penrose Diagram

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Consider two QFTs with isomorphic Hilbert Spaces $\mathcal{H}_L$ and $\mathcal{H}_R$. Thermofield double is a particular entangled state in $\mathcal{H}_L \otimes \mathcal{H}_R$:

$$|	ext{TFD}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n} |n\rangle_L |n\rangle_R$$

$$Z(\beta) = \sum_n e^{-\beta E_n}$$

$$\rho_L = \text{Tr}_R |\text{TFD}\rangle \langle \text{TFD}| = \frac{1}{Z(\beta)} \sum_n e^{-\beta E_n} |n\rangle_L \langle n|_L$$

$\leftarrow$ Thermal Density Matrix
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← Thermal Density Matrix
Chaos in General Holographic Space-times

Nilanjan Sircar

Motivation & Introduction

Chaos, Butterfly-effect & Holography

Chaos in various systems

Summary & Future Directions

TFD \equiv \text{Maximally extended Black Holes}

- TFD is dual to the maximal extension of the eternal black hole. The pair of QFTs living on the two boundaries correspond to the two QFTs in the definition of TFD.  
  *Israel '76, Maldacena '01*

- Entanglement between the two QFTs is given by the thermal entropy of the black hole.
TFD $\equiv$ Maximally extended Black Holes

- TFD is dual to the maximal extension of the eternal black hole. The pair of QFTs living on the two boundaries correspond to the two QFTs in the definition of TFD. \textit{Israel '76, Maldacena '01}

- Entanglement between the two QFTs is given by the thermal entropy of the black hole.
Consider the mutual information for strips of size $L$:

$$I_{AB} = S_A + S_B - S_{A \cup B}$$

For BTZ Black Hole (2 + 1-dimension):

$$I_{AB} = \max\left(\frac{L_{AdS}}{G_N} \log \sinh(\pi L T), 0\right)$$

Shenker & Stanford '13
There exists a critical strip size $L = L_c$ beyond which the Mutual Information is non zero.

Shenker & Stanford ’13, Leichenauer ’14, Sonnenschein-NS-Tangarife ’16
Nilanjan Sircar

Outline

1. Motivation & Introduction

2. Chaos, Butterfly-effect & Holography

3. Chaos in various systems

4. Summary & Future Directions
Consider the Kruskal coordinates \((\tilde{u}, \tilde{v}), (u, v)\) to the left and right of the perturbation respectively. In the limit of small perturbation \(\frac{\delta M}{M} \ll 1\) and large \(t_w \gg 1\): \(\tilde{v} = v + \alpha, \tilde{u} = u\) with,

\[
\alpha = \frac{c_2 \delta M \beta}{c_1 S_{BH}} e^{-\frac{2\pi}{\beta} (r_*(\infty) - t_w)}
\]

Heuristic Calculation of Scrambling time

$t_W$ should correspond to scrambling time $t_*$ when the effect of the perturbation is order one.

\[ \alpha \sim 1 \]

which gives for perturbation $\delta M \sim T$,

\[ t_* = \frac{\beta}{2\pi} \log S_{BH} + \frac{\beta}{2\pi} \log \left( \frac{c_1}{c_2} e^{r_*(\infty)} \right) \]

\[ S_{BH} \sim N^2 \gg 1. \text{Shenker & Stanford '13} \]
We are interested in looking for signature of disruption of entanglement/correlation due to a small perturbation.

So in the Shockwave geometry corresponding to a small perturbation of the Thermofield double we can calculate two sided mutual information (thermo-mutual information (TMI)).

As seen before the TMI is non zero only for beyond some critical strip size $L > L_c$.

Now with $L > L_c$ we can calculate the TMI in the Shockwave geometry.
Diagnostics for Chaos -I

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As seen before the TMI is non zero only for beyond some critical strip size $L > L_c$.

Now with $L > L_c$ we can calculate the TMI in the Shockwave geometry.
In this geometry TMI is function of
\[ I_{AB}(L, \alpha(t_W)) = S_A(L) + S_B(L) - S_{A\cup B}(L, \alpha(t_W)). \]

We define scrambling time \( t_* \) as when
\[ I_{AB}(L, \alpha(t_W = t_*)) = 0 \]
for a given \( L > L_c \).

\[ t_* = \frac{\beta}{2\pi} \log S_{BH} + \mathcal{O}(N^0), \]
same conclusion as \( \alpha \sim 1 \) analysis.
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In this geometry TMI is function of 
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\[ t_* = \frac{\beta}{2\pi} \log S_{BH} + \mathcal{O}(N^0) \], same conclusion as \(\alpha \sim 1\) analysis.
General behavior of TMI in Shockwave geometries except for some parameter region of higher derivative gravity.

Shenker & Stanford '13, Leichenauer '14, Sonnenschein-NS-Tangarife '16
Analytic Results in BTZ black hole

- In $2 + 1$-dimensional BTZ black hole calculation can be performed analytically. \textit{Shenker & Stanford '13}

- For $L > L_c$: $I(A, B) = \frac{L_{\text{AdS}}}{G_N} \left( \log \sinh \frac{\pi L}{\beta} - \log \left(1 + \frac{\alpha}{2}\right) \right)$

- $\alpha = \frac{E \beta}{2S_{BH}} e^{2\pi t_w/\beta}$

- For high temperature, $LT \gg 1$: $t_* = \frac{L}{2} + \frac{\beta}{2\pi} \log \frac{2S_{BH}}{\beta E}$. 
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- For high temperature, $LT \gg 1$: $t_\star = \frac{L}{2} + \frac{\beta}{2\pi} \log \frac{2S_{BH}}{\beta E}$. 
\( F(t) \) in BTZ Black Hole

In geodesic approximation \(^{(\text{Cornalba et al '06, Shenker & Stanford '13,})}\),

\[
F(t) \sim \left( \frac{1}{1 + \frac{\alpha}{2}} \right)^{2ml}
\]

where \( \alpha = \frac{E \beta}{2S_{BH}} e^{2\pi t_w/\beta} \).

\( F(t) \) is initially order 1 but starts decaying exponentially at time \( t \sim t_\ast \).

Note in BTZ case, the calculation of \( F(t) \) and Mutual Information are essentially same and given by geodesics.
Outline

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2. Chaos, Butterfly-effect & Holography
3. Chaos in various systems
4. Summary & Future Directions
Black $Dp$ Branes correspond to $p + 1$-dimensional Super Yang Mills’ theories with 16 supercharges. $p = 3$ corresponds to usual AdS/CFT. Itzhaki, Maldacena, Sonnenschein, & Yankielowicz

- $p \neq 3$ corresponds to non-conformal field theories. The Yang-Mills’ coupling is dimension full for $p \neq 3$.

- $p \neq 3$ has a non-trivial dilaton which couples to metric. So we need modified Ryu-Takayanagi principle to calculate entanglement entropy. Ryu & Takayanagi ’06, Klebanov et al. ’07.

- Validity of super-gravity solution requires

$$1 \ll g_{\text{eff}}^2 \ll N^{\frac{4}{7-p}}$$

where, $g_{\text{eff}}^2 = g_{YM}^2 N \frac{r^{p-3}}{l_s^{2(p-3)}}$. 

$$\text{Nilanjan Sircar}$$

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So we need to put an UV-cutoff $\Lambda$ and the validity at the other end can be achieved by choosing an appropriate temperature.

Temperature and Entropy is given as,

$$s_{BH} = c(p)(g_{YM}^2 N)^{\frac{p-3}{5-p}} N^2 T^{\frac{9-p}{5-p}}$$

Holography is not well defined for $p \geq 5$. 
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Black $Dp$ Branes: Results

\[ t_* = r_*(\Lambda) + \frac{\beta}{2\pi} \left( \log s_{BH}(\beta, p, \lambda, N) + \log \frac{c_1}{c_2}(p) + \log \alpha_*(\frac{L}{\beta}, p) \right) \]

\[ r_*(\Lambda) = \frac{2\ell}{n(p-5)} \left( \frac{\Lambda}{\ell} \right)^{\frac{n}{2}(p-5)}, \quad p \neq 5 \]

\[ = \ell \log \left( \frac{\Lambda}{r_h} \right), \quad p = 5 \]

\[ n = \frac{8}{8+(7-p)(3-p)} > 0 \]
We are here interested in non-relativistic scale invariant theories with:

\[ t \rightarrow \lambda^z t \quad ; \quad x \rightarrow \lambda t \quad \text{for} \quad z \neq 1. \]

The holographic dual to such field theories at finite temperature is generically called Lifshitz Black Holes. Kachru, Liu, & Mulligan '08

\[ S_{BH} = \frac{1}{4G_N^{(4)}} \left( \frac{2\pi \ell}{\lambda} \right)^{2/z} T^{2/z}. \]

General lore is that the Ryu-Takayanagi principle is not modified for this case.
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Lifshitz Black Branes: Results

\[ t_\ast = \frac{\beta}{2\pi} \log s_{BH} - \frac{l_\ast^2}{z^2\Lambda} + \frac{\beta}{2\pi} \log \left( \frac{1}{2} e^{-\psi(z/2)-\gamma} \right) + \frac{\beta}{2\pi} \log \alpha_\ast (LT^{1/z}) \]
Higher derivative Black Branes

- We consider $AdS$ black-brane solutions in case of Einstein gravity corrected with higher curvature terms.

- It corresponds to finite $T$ Conformal Theories with central charges $a \neq c$.

- Most general higher curvature gravity theory with second order equation of motion is known as Lovelock theory.

- We will consider 4 + 1-dimensional Lovelock theory, which corresponds to just addition of Gauss-Bonnet term along with usual Einstein-Hilbert term in the action.

- Higher curvature corrections in bulk $\leftrightarrow \lambda'_{tHooft}$ corrections in dual field theory.
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Higher curvature corrections in bulk $\Longleftrightarrow \lambda_t^{tHooft}$ corrections in dual field theory.
Gauss Bonnet Action

\[ S_{\text{grav}} = \frac{1}{16\pi G_N} \int d^5x \sqrt{-g} \left( R + \frac{12}{L^2} + \frac{\lambda_{\text{GB}} L^2}{2} \chi_4 \right) \]

\[ \chi_4 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \]

The entropy density of Gauss-Bonnet black brane:\text{Cai '02}

\[ s_{\text{BH}} = \frac{L^3}{4G_N} s(\lambda_{\text{GB}}) T^3 \]

The value of \( \lambda_{\text{GB}} \) is bounded by causality constraints:\text{Brigante et al. '08, Buchel et al. '09}

\[-7/36 \leq \lambda_{\text{GB}} \leq 9/100\]

It is recently pointed out that Gauss-Bonnet as an exact theory violates causality for any \( \lambda_{\text{GB}} \).\text{Camanho et al. '14}
Holographic Entanglement Entropy in Higher Derivative theories

\[ S_{EE} = \frac{1}{4G_N} \int d^3\sigma \sqrt{\gamma} \left(1 + \lambda_{GB} L^2 R_{\gamma}\right) + \lambda_{GB} L^2 \frac{1}{2G_N} \int_{\partial\gamma} d^2\sigma \sqrt{h K} \]

de Boer et al. ’11, Hung et al. ’11
For positive $\lambda_{GB}$ the result is qualitatively similar to $\lambda_{GB} = 0$ case.

$$t_* = \frac{\beta}{2\pi} \log s_{BH} + \frac{\beta}{2\pi} \left( \frac{\pi}{2} - 4\lambda_{GB} \right)$$

Similar jumps was noticed in time evolution of Entanglement entropy with higher derivative correction Caceres et. al. '15.
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We have shown that the dual thermal field theory scrambles information at time scales given by $\frac{\beta}{2\pi} \log S_{BH}$ for various geometries.

We have used vanishing of thermo-Mutual Information as a signature for scrambling.

The results are qualitatively similar in conformal ($a = c$), non-conformal and non-relativistic cases.

For conformal theories with ($a \neq c$), dual to Gauss-Bonnet Black hole the behavior can be very different depending on the sign of coupling.
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Future directions-1

- **Thermo-Mutual Information**, used as a probe for scrambling involves regions in two different copies of the field theory. It would be useful to express this as a probe in a single field theory.

- **Exponential fall of out of time 4 point correlation function** is also used as a probe of Scrambling. Connection to Chaos is more transparent in this definition, also definition of Lyapunov exponent is natural. In case of BTZ black hole, the two definitions can be shown to be equivalent, at least for some heavy operators. Precise connection in general is missing.
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Future directions-2

- Whether the higher derivative correction beyond GB term can smooth out the jumps? If not, significance of such jumps?

- Extension to non-commutative theories, General Higher Derivative gravity confining theories... Reynolds et al 1604.04099, Huang et al 1609.08841, Alishahiha et al 1610.02890
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Thank You