Exact Quantum Entropy of Black Holes in String Theory

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- ***A.D. Joao Gomes, Monica Guica, Murthy, Ashoke Sen***

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- ***A.D., Gomes, Murthy*** arXiv:0802.0761

- ***A. D., Davide Gaiotto, Suresh Nampuri***

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References

- **Dijkgraaf, Verlinde, Verlinde;**
- **Cardoso, de Wit, Kappelli , Mohaupt**
- **Kawai;**
- **Gaiotto, Strominger, Xi, Yin**
- **David, Jatkar, Sen; Banerjee,Srivastava**
- **Cheng and Verlinde**

Heterotic on $\mathbf{T^6}$

- Total rank of the four dimensional theory is **16** ($\mathrm{E_8} \times \mathrm{E_8}$) + **12** ($\mathrm{g_{\mu m}}, \mathrm{B_{\mu m}}$) = **28**

- **N=4** supersymmetry in **D=4**

- Duality group

$$\mathbf{SO(22,6;Z) \times SL(2;Z)}$$

Charges and T-duality Invariants

- A charge vector is specified as

$$\Gamma = \begin{pmatrix} Q \\ P \end{pmatrix}$$

- Transforms as a vector of the T-duality group and a doublet of S-duality group.
- If the vectors Q and P are parallel, one can preserve eight susys (half-BPS) otherwise only four susys (quarter-BPS)

Spectrum of Half-BPS States

- Degeneracies given by Fourier coefficients of a genus-one modular form

$$d(Q^2) \sim \oint_{\mathcal{C}} d\tau \, \frac{e^{-i\pi I Q^2 \tau}}{\psi_{12}(\tau)}$$

- Here $\psi_{\mathbf{12}}(\tau) = \eta^{\mathbf{24}}(\tau)$ is a well-known modular form of weight 12 of the group SL(2, Z) = Sp(1, Z). Genus one partition function of left-moving heterotic string.

Spectrum of quarter-BPS dyons

- Now the spectrum is expected to have a sensitive moduli dependence.
- There are many inequivalent duality orbits so we first need to classify them.
- Surprisingly, both these problems have been solved in the recent years and one now has a counting formula for ***all*** dyons at ***all*** points in the moduli space.

Duality Orbits

- Define an *arithmetic* duality invariant

$$\mathbf{I} = \gcd(\mathbf{Q} \wedge \mathbf{P})$$

- All inequivalent duality orbits are labeled essentially by this single integer.

A. D., Gaiotto, Nampuri; Banerjee,Sen

Chemical potentials

- Define a matrix of T-duality invariants

$$\mathbf{\Lambda} = \begin{pmatrix} Q\cdot Q & Q\cdot P \\ Q\cdot P & P\cdot P \end{pmatrix} = \begin{pmatrix} 2n & l \\ l & 2m \end{pmatrix}$$

- Define the matrix of chemical potentials

$$\mathbf{\Omega} = \begin{pmatrix} \tau & z \\ z & \rho \end{pmatrix}$$

Spectrum of quarter-BPS dyons

- For I=1, degeneracies are given by the Fourier coefficients

$$d(\Lambda;\mu) \sim \oint_{\mathcal{C}} d\Omega \, \frac{e^{-i\pi(\Lambda,\Omega)}}{\Phi_{10}(\Omega)}$$

- Here $\mathbf{\Phi_{10}}$ is a well-known Siegel modular form of weight 10 of group Sp(2, Z) and is a genus two partition function of the left-moving heterotic string.

Moduli dependence

- The contour depends on moduli in a precise way. *All* dependence on the moduli μ is captured by dependence of contours on the moduli.
- Changing moduli deforms the contour. Degeneracy remains constant for smooth contour deformation but jumps if one encounter a pole of the Fourier integral.

Walls and Poles

- Moduli space divided into regions separated by walls of marginal stability where a quarter-BPS state decays into two half-BPS states.
- Walls correspond to poles the Fourier integral at the zeros of the Siegel form.
- Jumps in degeneracy upon crossing a wall precisely equals the residue of the Fourier integral at the poles. Nontrivial check.

General duality orbits

- Dyons with nontrivial values of the arithmetic duality invariant ***I*** can be mapped to charge vectors of the form $\Gamma = \begin{pmatrix} IQ_0 \\ P \end{pmatrix}$
- Define for ***s*** which divides ***I***

$$\mathbf{\Lambda_s} = \begin{pmatrix} Q\cdot Q/s^2 & Q\cdot P/s \\ Q\cdot P/s & P\cdot P \end{pmatrix}$$

Degeneracy of all dyons

$$\mathbf{d_I}(\mathbf{\Lambda}, \mu) = \sum_{\mathbf{s}|\mathbf{I}} \mathbf{d}(\mathbf{\Lambda_s}, \mu)$$

- Passes many nontrivial checks for small and large values of charges.

Banerjee,Sen,Srivastava;

A.D,Gomez,Murthy

Comparison with Entropy

- Comparison of ***S= log (d)*** is impressive for this and many other compactifications with N=4 supersymmetry—CHL orbifolds.
- Both macroscopic and microscopic entropy can be obtained by the minimum value of the *same* function F of two variables **a** and σ

Entropy function

- The entropy function is given by

$$F = \frac{\pi}{2}\left[\frac{a^2+\sigma^2}{\sigma}P^2 + \frac{1}{\sigma}Q^2 - 2\frac{a}{\sigma}Q\cdot P + 128\pi\phi(a,\sigma) + \ldots\right]$$

$$\phi(a,\sigma) = -\frac{1}{64\pi^2}\{(n+2)\log\sigma + \log|f^{(n)}(a+i\sigma)|^2\}$$

$$f^{(n)}(\tau) \equiv \eta(\tau)^{n+2}\,\eta(N\tau)^{n+2}$$

- For our case N=1 and n=10.

Conclusions

- We have seen that for many models one can compute exactly the *quantum microscopic degeneracies of black holes*.
- Sub-leading corrections in the asymptotic expansion for large charges match beautifully with Wald entropy to that order.
- Such exact information can help deepen our understanding of nonpeturbative quantum structure of gravity.

Work in progress

- It seems possible to define a full *quantum macroscopic partition function* given our knowledge on the microscopic side. One can view this as an instance of precision holography of AdS2/CFT1.
- This can shed light on a number of subtle questions about the nonperturbative string partition function in AdS backgrounds.