The long & short of strings on deformed $AdS_4 \times \mathbb{CP}^3$

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Overview

I. The gauge/gravity correspondence

II. Integrable deformations of AdS/CFT

III. $AdS_4 \times \mathbb{CP}^3 \longrightarrow AdS_4 \times \mathbb{CP}^3_\gamma$

IV. Long strings: Giant magnons & spikes

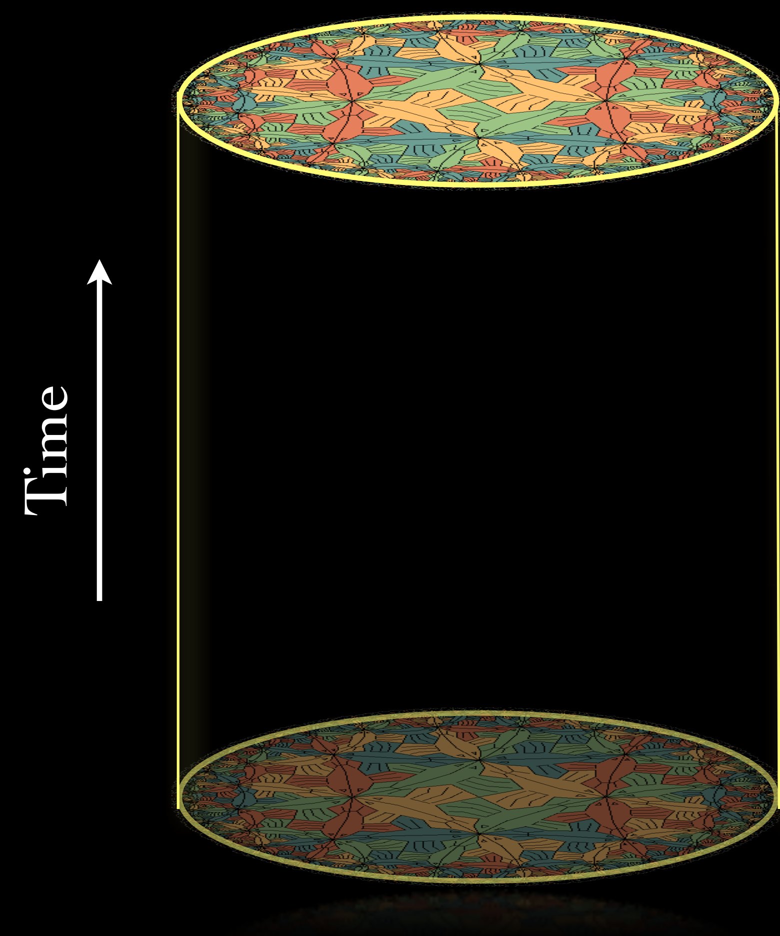
V. Short strings: The pp-wave

The gauge/gravity connection

* Gravity and gauge theory appear to be different manifestations of the same physics ! (t'Hooft-1974)

* AdS/CFT is the first concrete example of a gauge/gravity duality (Maldacena -1998)

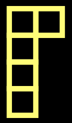
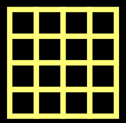
Type IIA string theory on $AdS_5 \times S^5$	'='	$\mathcal{N} = 4$, $SU(N)$, SYM on $\partial(AdS_5)$



* What is the big picture?
  * Can we build a strong coupling description of QCD out of gravity?
  * Could we use gauge theory to figure out the correct degrees of freedom of quantum gravity?

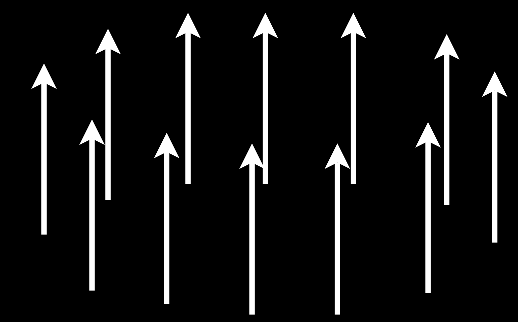
* So what is the problem?
  * AdS/CFT is a strong/weak duality!

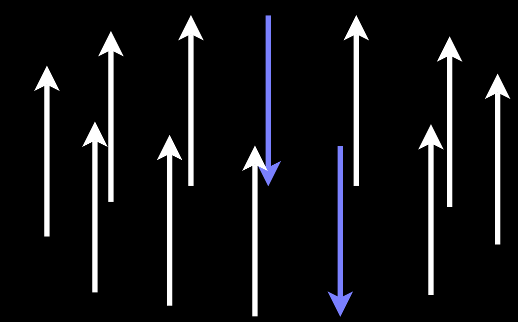
The AdS/CFT dictionary

String Theory	Gauge Theory	
Effective curvature: $\frac{R^4}{\alpha'^2}$	t'Hooft coupling: $\lambda = g_{YM}^2 N$	
String coupling: $4\pi g_s$	Yang-Mills coupling: g_{YM}	
Graviton	$\underbrace{\mathrm{Tr}\,(Z\cdots Z)}_{O(1)}$	(Witten-98)
String	$\underbrace{\mathrm{Tr}\,(Z\cdots Z)}_{O(\sqrt{N})}$	(BMN-01)
D-brane	χ  (Z)	(Balasubramanian et. al. 01 Corley,Jevicki,Ramgoolam-01)
New Geometry	χ  (Z)	(de Mello Koch, 08)

The spin-chain link

* Single-trace operators can be identified with a spin chain where: (Minahan & Zarembo-2003)

$|0\rangle$ string vacuum ↔ $\mathrm{tr}\left(Z^J\right)$ ↔ 

$|m\rangle$ excited state ↔ $\mathrm{tr}\left(\Phi Z^m \Phi Z^{J-m}\right)$ ↔ 

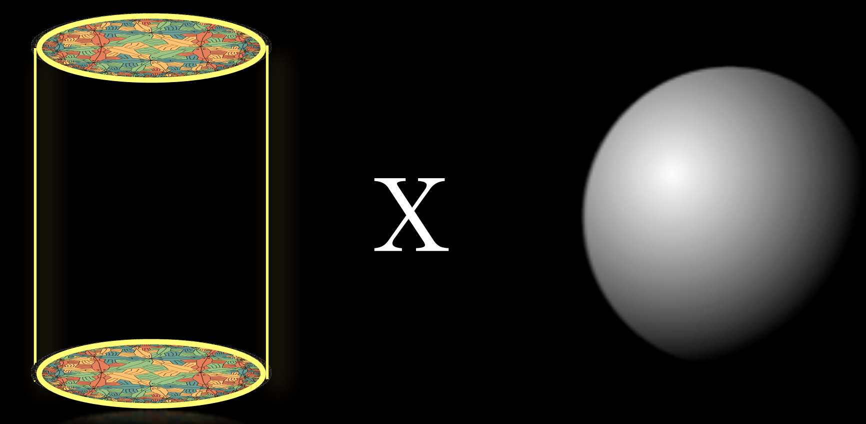
$\mathcal{D}^{(1)}$ one-loop dilatation operator ↔ $$\mathcal{H} = g^2 N \sum_{n=1}^{J} \vec{\sigma}_n \cdot \vec{\sigma}_{n+1}$$

* What do we learn from the spin-chain?
    * The type IIB string is classically integrable - perhaps even quantum also! (Bena,Polchinski,Roiban-04)
    * The thermodynamic limit of the spin-chain reproduces the Polyakov string action. (Kruczenski-04)

Superconformal Deformations

- AdS/CFT good...non-AdS/non-CFT better!

- Lunin-Maldacena deformation (Lunin & Maldacena - 2005)
  - Locate a torus subgroup of the 5-sphere corresponding to a U(1) x U(1) global symmetry
  - Form $\tau \equiv B_{NS} + i\sqrt{g}$
  - Act with an $SL(2,\mathbb{R})$ on τ to get $\tau_\gamma = \dfrac{\tau}{1+\gamma\tau}$



- Properties:
  - Non-singular solutions are mapped to non-singular solutions.
  - The deformation is exactly marginal and background is conformal but less supersymmetric.
  - The deformation is equivalent to a TsT duality. (Frolov - 2005)

BLG, ABJM & M-theory

* M-theory
  * Strong coupling limit of type IIA superstring theory.
  * High energy limit of 11-dimensional supergravity.

* Gauge theory dual?
  * Worldvolume theory of N M2-branes in flat space.
  * Strong coupling limit of worldvolume of N D2 branes.
  * 3-dimensional + conformal + $\mathcal{N} = 8$ supersymmetry

THIS IS A HARD PROBLEM!

* Problems
  * YM term breaks scale invariance + adds unwanted d.o.f.
  * Having no kinetic term removes needed d.o.f
  * CS term breaks parity invariance + introduces new parameter.

BLG, ABJM & M-theory

∗ New idea: Build the theory from 3-algebras (Bagger & Lambert - 2007 Gustavsson - 2007)

$$\mathcal{L} = -\frac{1}{2}\mathcal{D}^\mu X^{Ia}\mathcal{D}_\mu X^I_a + \frac{i}{2}\bar{\psi}^a\Gamma^\mu\mathcal{D}_\mu\psi_a + \frac{i}{4}\bar{\psi}_b\Gamma^{IJ}X^I_c X^J_d\psi_a f^{abcd}$$
$$-V(X) + \frac{1}{2}\epsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}_{\ \ \ g}f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right)$$

∗ Properties:

1. 3-algebra Chern-Simons Matter theory
2. Parity invariant.
3. $\mathcal{N} = 8$ SUSY
4. SO(8) R-charge.

BLG only works for two M2's!

BLG, ABJM & M-theory

* ABJM is a generalisation of BLG where: (Aharony,Bergman, Jafferis & Maldacena - 2008)

$$SU(2)^2_{gauge} \to U(N)^2_{gauge}$$

$$SO(8)_R \to SO(6)_R = SU(4)_R$$

* Properties:

1. 3-dimensional + conformal Chern-Simons-matter theory at level k .
2. $\mathcal{N} = 6$ SUSY
3. Low energy limit of kN M2-branes probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity.
4. A natural ‘t Hooft parameter $\lambda = N/k$
5. At small coupling it reduces to type IIA on $AdS_4 \times \mathbb{CP}^3$

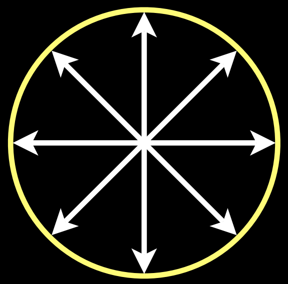
BLG, ABJM & M-theory

- From 11 to 10 and M to D:
  - * Hopf fibre of the 7-sphere:

$$\begin{array}{ccc} S^1 & \longrightarrow & S^7 \\ & & \downarrow \\ & & \mathbb{CP}^3 \end{array}$$

$$ds^2_{S^7} = ds^2_{S^1} \times ds^2_{\mathbb{CP}^3}$$

  - * Orbifold:



$$ds^2_{S^7/\mathbb{Z}_k} = \frac{1}{k} ds^2_{S^1} \times ds^2_{\mathbb{CP}^3}$$

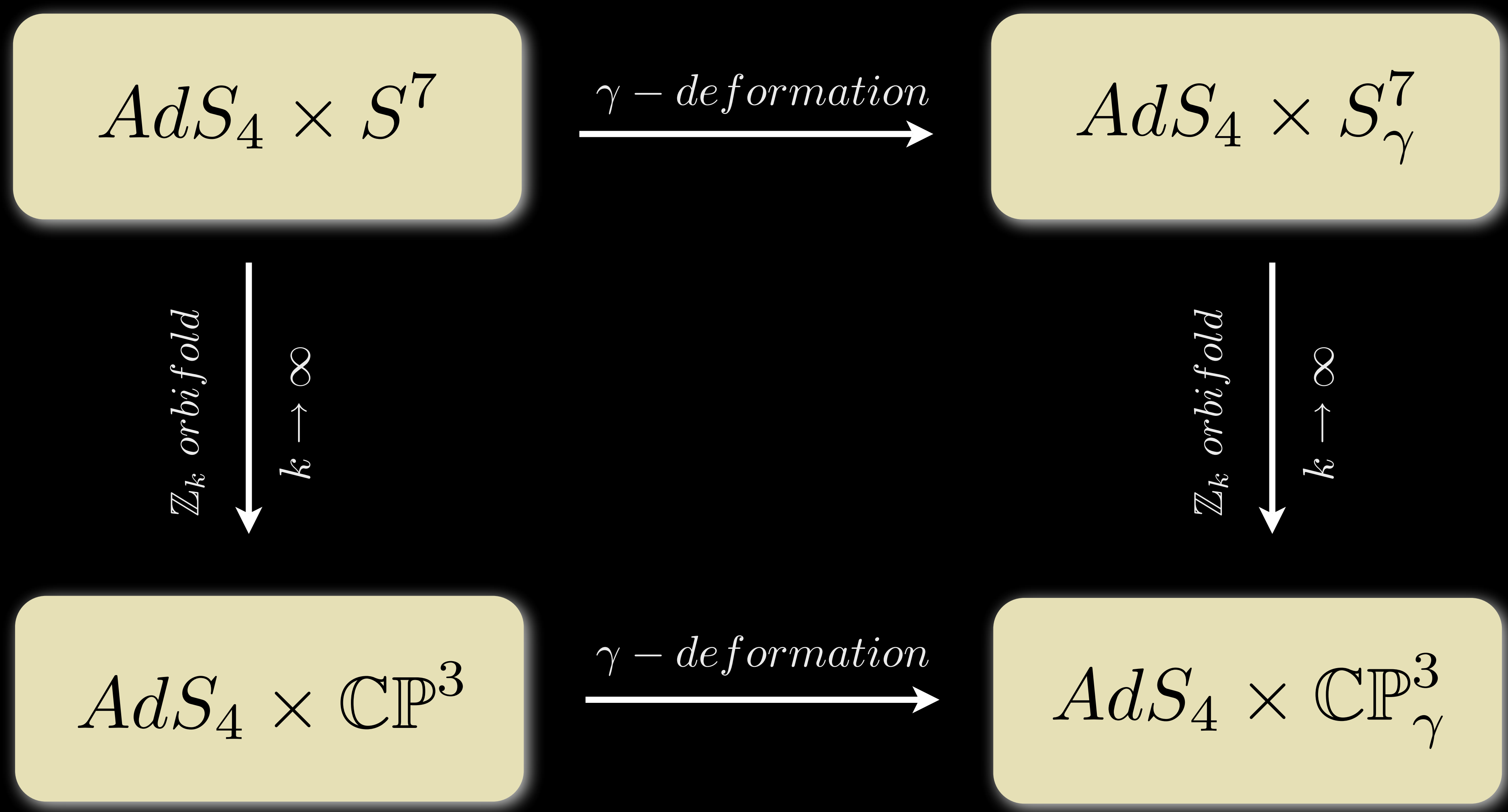
  - * Take the limit $k \to \infty$

M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$

↓

Type IIA superstring theory on $AdS_4 \times \mathbb{CP}^3$

All roads lead to ...



(JM & Mohammed-09)

Deforming the IIA background

* Strategy:

$$AdS_4 \times \mathbb{CP}^3 \xrightarrow{TsT} AdS_4 \times \mathbb{CP}^3_\gamma$$

* The resulting background has:

$$\begin{aligned} ds^2_{\mathbb{CP}^3_\gamma} &= d\xi^2 + G\cos^2\xi\sin^2\xi\left(d\psi + \frac{1}{2}\cos\vartheta_1\,d\phi_1 - \frac{1}{2}\cos\vartheta_2\,d\phi_2\right)^2 \\ &\quad + \frac{1}{4}\cos^2\xi(d\vartheta_1^2 + G\sin^2\vartheta_1\,d\phi_1^2) + \frac{1}{4}\sin^2\xi(d\vartheta_2^2 + G\sin^2\vartheta_2\,d\phi_2^2) \\ &\quad + \hat{\gamma}^2 G\cos^4\xi\sin^4\xi\sin^2\vartheta_1\sin^2\vartheta_2\,d\psi^2 \end{aligned}$$

$$e^{2\Phi} = G\frac{R^3}{k^3}$$

$$B_{NS} \neq 0$$

* Interesting subspaces:

$$\mathbb{R}\times S^2 \qquad \mathbb{R}\times\mathbb{RP}^2_\gamma \qquad \mathbb{R}\times\mathbb{RP}^3_\gamma \qquad \mathbb{R}\times\mathbb{CP}^2_\gamma$$

(Imeroni-08
Schimpf & Rashkov-09
JM & Mohammed-09)

Long strings - giant magnons

* What is the string state dual to an elementary magnon? (Hofman & Maldacena - 2006)

1. Equations of Motion:

$$\partial_+\partial_-\vec{X} + \left(\partial_+\vec{X}\cdot\partial_-\vec{X}\right)\vec{X} = 0$$

2. Constraints:

$$\partial_+\vec{X}\cdot\partial_-\vec{X} = \cos\,\varphi$$

3. The solution:

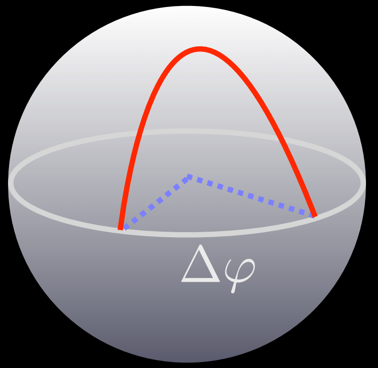
$$\varphi = \varphi_{kink}\left(\frac{x-vt}{\sqrt{1-v^2}}\right)$$

$$W = X_1 + iX_2 = \sqrt{1-v^2}e^{it}\left[\tanh\left(\frac{x-vt}{\sqrt{1-v^2}}\right) - \frac{iv}{\sqrt{1-v^2}}\right]$$

* Properties of the giant magnon?

1. The solution is an open string with endpoints on a 2-sphere moving at the speed of light with separation

$$\delta\phi = 2\tan^{-1}\left(\frac{\sqrt{1-v^2}}{v}\right)$$



2. Dispersion relation: $\Delta - J = \frac{\sqrt{\lambda}}{\pi}|\sin\left(\frac{p}{2}\right)|$

An example

* An example: Classical strings on $\mathbb{R} \times \mathbb{RP}^3_\gamma$ (JM & Asadig Mohammed - 2009)

1. Polyakov action:

$$S = \frac{R^3}{16\pi k\alpha'} \int d\tau d\sigma \left(\dot{t}^2 + 4\left(\dot{\xi}^2 - \xi'^2\right) + G\cos^2\xi\left(\dot{\varphi}_1^2 - \varphi_1'^2\right) + G\sin^2\xi\left(\dot{\varphi}_2^2 - \varphi_2'^2\right) \right.$$
$$\left. + \; 2\hat{\gamma}^2 G\cos^2\xi\,\sin^2\xi\left(\dot{\varphi}_1\varphi_2' - \dot{\varphi}_2\varphi_1'\right)\right)$$

2. Equations of Motion:

$$\partial_\tau\left[G\cos^2\xi(\dot{\varphi}_1 + \hat{\gamma}\sin^2\xi\varphi_2')\right] - \partial_\sigma\left[G\sin^2\xi(\varphi_1' + \hat{\gamma}\sin^2\xi\dot{\varphi}_2)\right] = 0$$

$$\partial_\tau\left[G\sin^2\xi(\dot{\varphi}_2 - \hat{\gamma}\cos^2\xi\varphi_1')\right] - \partial_\sigma\left[G\sin^2\xi(\varphi_2' - \hat{\gamma}\cos^2\xi\dot{\varphi}_1)\right] = 0$$

3. Rotating string ansatz:

$$\xi = \xi(\eta), \quad \varphi_1 = \omega_1\tau + f_1(\eta), \quad \varphi_2 = \omega_2\tau + f_2(\eta), \quad \eta = \alpha\sigma + \beta\tau.$$

Giant magnons and single spikes

4. Conserved quantities:

$$E = -\int d\sigma \frac{\partial \mathcal{L}}{\partial \dot{t}} = \sqrt{\frac{\lambda}{2}} \int d\sigma\ \dot{t}$$

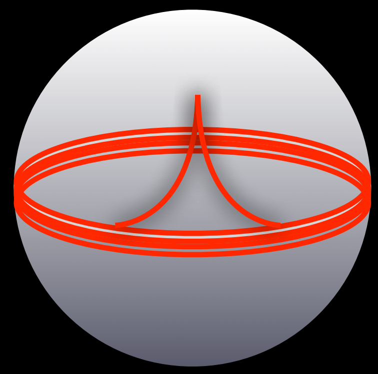
$$J_1 = \int d\sigma \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_1} = \sqrt{\frac{\lambda}{2}} \int d\sigma\ G \cos^2\xi (\dot{\varphi}_1 + \hat{\gamma} \sin^2\xi \varphi_2')$$

$$J_2 = \int d\sigma \frac{\partial \mathcal{L}}{\partial \dot{\varphi}_2} = \sqrt{\frac{\lambda}{2}} \int d\sigma\ G \sin^2\xi (\dot{\varphi}_2 - \hat{\gamma} \cos^2\xi \varphi_1')$$

5. Solutions:

$$E - J_2 = \sqrt{J_1^2 + 2\lambda \sin\left(\frac{p}{4} - \frac{\pi}{2}\hat{\beta}\right)}$$ Giant Magnon

$$E - \sqrt{2\lambda}\frac{\Delta\varphi_2}{4} = \sqrt{2\lambda}\left(\frac{p}{4} - \frac{\pi}{2}\hat{\beta}\right)$$ Single Spike



Short strings - pp-wave limits

* Are there any limits in which the quantum string is tractable?

(Penrose-76,
Guven-00,
Sadri & Sheikh-Jabbari-03)

* Start with $AdS_5 \times S^5$

$$ds^2_{AdS_5\times S^5} = R^2\Big(-dt^2\cosh^2\rho + d\rho^2 + \sinh^2\rho\, d\Omega_3^2$$
$$+\cos^2\theta\, d\psi^2 + d\theta^2 + \sin^2\theta\, d\tilde{\Omega}_3^2\Big)$$

* Zoom in on the null-geodesic:

$$\rho = 0,\ \theta = 0,\quad t = t(\lambda) \quad \psi = \psi(\lambda)$$

* Change coordinates and take the scaling limit $R \longrightarrow \infty$

$$ds^2_{AdS_5\times S^5} \rightarrow \boxed{ds^2_{ppw} = -2dx^+dx^- - \mu^2(x^i)^2(dx^+)^2 + (dx^i)^2}$$

* Properties of the pp-wave:
  * Maximally supersymmetric
  * SO(4) x SO(4) isometry
  * Exact string background
  * Quantum string spectrum can be exactly computed!

How to take the Penrose limit of $AdS_4 \times \mathbb{CP}^3_\gamma$

∗ Step 1. Find a null geodesic: (JM & Mohammed-09)

$$\rho = \theta_1 = \theta_2 = 0 \quad \xi = \frac{\pi}{4}$$

∗ Step 2. Expand around it:

$$t = x^+ + \frac{x^-}{\tilde{R}^2} \qquad \tilde{\psi} = x^+ - \frac{x^-}{\tilde{R}^2} \qquad \theta_i = \sqrt{2}\frac{y_i}{\tilde{R}} \qquad \rho = \frac{r}{\tilde{R}} \qquad \xi = \frac{\pi}{4} + \frac{y_3}{2\tilde{R}}$$

∗ Step 3. Send $\tilde{R} \to \infty$ (and secretly change coordinates) so that

$$ds^2 = -4dx^+dx^- - \left(\sum_{i=1}^{4}(x^i)^2 + \frac{1+\hat{\gamma}^2}{4}\sum_{i=5}^{8}(x^i)^2\right)(dx^+)^2 + \sum_{i=1}^{8}(dx^i)^2$$

∗ Step 4. Do the same for the B-field:

$$B = -\frac{\hat{\gamma}}{2}\left(x^6\, dx^+ \wedge dx^5 - x^5\, dx^+ \wedge dx^6 + x^8\, dx^+ \wedge dx^7 - x^7\, dx^+ \wedge dx^8\right)$$

Strings on the pp-wave

* Lightcone gauge action: (JM & Mohammed-09)

$$S = \frac{1}{4\pi\alpha'}\int d\sigma d\tau \left[\sum_{i=1}^{8}\left((\dot{X}^1)^2 + (X'^i)^2\right) - (2p^+)^2\sum_{i=1}^{4}(X^i)^2 - (1+\hat{\gamma}^2)(p^+)^2\sum_{i=5}^{8}(X^i)^2 \right.$$
$$\left. -\hat{\gamma}p^+\left(X^6X'^5 - X^5X'^6 + X^8X'^7 - X^7X'^8\right)\right]$$

* Equations of motion: $\ddot{X}^i - X''^i + \sum_j h_{ij}X'^j + k_iX^i = 0$

* How to solve:

  * Step 1 - Mode expand: $X^i(\tau,\sigma) = \sum_{n\in\mathbb{Z}} X^i_n(\tau)e^{2in\sigma}$

  * Step 2 - Frequency base ansatz: $X^i_n = a_j^{(n)}A_{ij}^{(n)}e^{i\omega_j^{(n)}\tau}$ (Metsaev-01; Metsaev&Tseytlin-02 de Mello Koch, JM, Smolic,Smolic-05)

  * Step 3 - Solve the determinant: $\det\left[(\omega_n^2 - 4n^2 - k_i^2)\delta_{ij} - 2inh_{ij}\right] = 0$

* Spectrum: $\omega_n^0 = \sqrt{4p^{+2}+4n^2}$ $\qquad \omega_n^{\pm} = \sqrt{(1+\hat{\gamma}^2)p^{+2}+4n^2 \pm 2\hat{\gamma}p^+n}$

Comparison with the IIA pp-wave

* Our light-cone Hamiltonian:

$$2p_{\hat{\gamma}}^{-} = \sum_{n\in\mathbb{Z}} \sqrt{1+\frac{n^2}{(p^+)^2}}N_n + \sqrt{\frac{1+\hat{\gamma}}{4}+\frac{n^2}{(p^+)^2}+\frac{n\hat{\gamma}}{2p^+}}\mathcal{N}_n^{(1,2,5,6)} + \sqrt{\frac{1+\hat{\gamma}}{4}+\frac{n^2}{(p^+)^2}-\frac{n\hat{\gamma}}{2p^+}}\mathcal{N}_n^{(3,4,7,8)}$$

* Hamiltonian of IIA pp-wave: (Nishioka & Takayanagi-08 Gaiotto, Giombi & Yin-08)

$$2p^{-} = \sum_{n\in\mathbb{Z}} \sqrt{1+\frac{n^2}{(p^+)^2}}N_n^{(1)} + \sqrt{\frac{1}{4}+\frac{n^2}{(p^+)^2}}N_n^{(2)}$$

Excellent Matching! (but not unexpected)

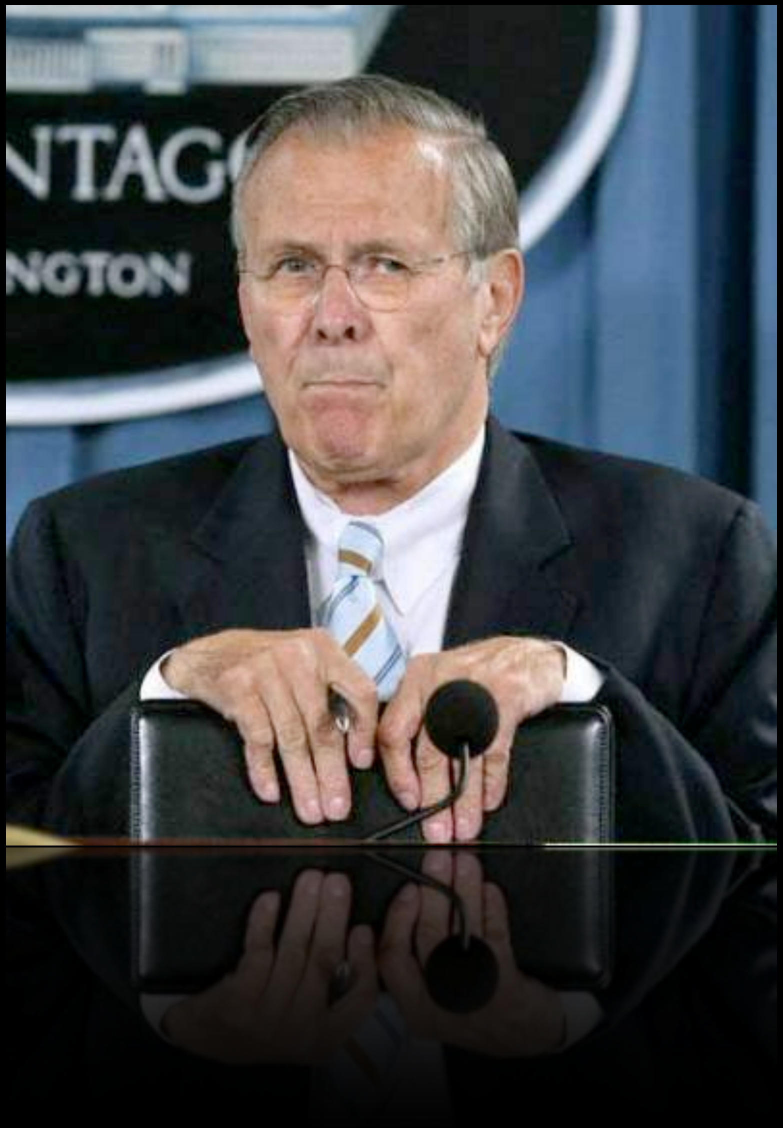
* Some observations:

  * The string spectrum on the deformed pp-wave exhibits a characteristic dependence on the deformation parameter. (cf de Mello Koch, JM, Smolic, Smolic-05 Mateos-05)

  * The fermion spectrum can be argued to have the same form as the bosonic.

  * This provides an important prediction for the spectrum of anomalous dimensions of BMN-like (almost BPS) operators in the ABJM theory.

I miss the Republicans...



"Reports that say that something hasn't happened are always interesting to me, because as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns -- the ones we don't know we don't know."

A Rumsfeldian summary

* Known knowns:
  * Classical strings are important indicators of the **integrability** of string theory.
  * **Integrable deformations** allow us to probe what happens as we take down the scaffold of supersymmetry systematically.
  * **PP-wave limits** allow us to probe full quantum string theory in a controlled way.
  * We have found **new giant magnon** and single spike limits of the rigid spinning string on an integrably deformed $AdS_4 \times \mathbb{CP}^3$
  * We have **quantized the type IIA string** on the deformed background
* Known unknowns:
  * **Finite size corrections** to the giant magnon dispersion relation.
  * Spectrum of anomalous scaling dimensions in the **dual ABJM gauge theory**.
  * What is the **Leigh-Strassler deformation** of ABJM?
* Unknown unknowns:

If I knew what these were...


That's all Folks!