1+1+2 Covariant Approach to Gravitational Lensing in *f(R)* gravity

Anne Marie Nzioki

University of Cape Town

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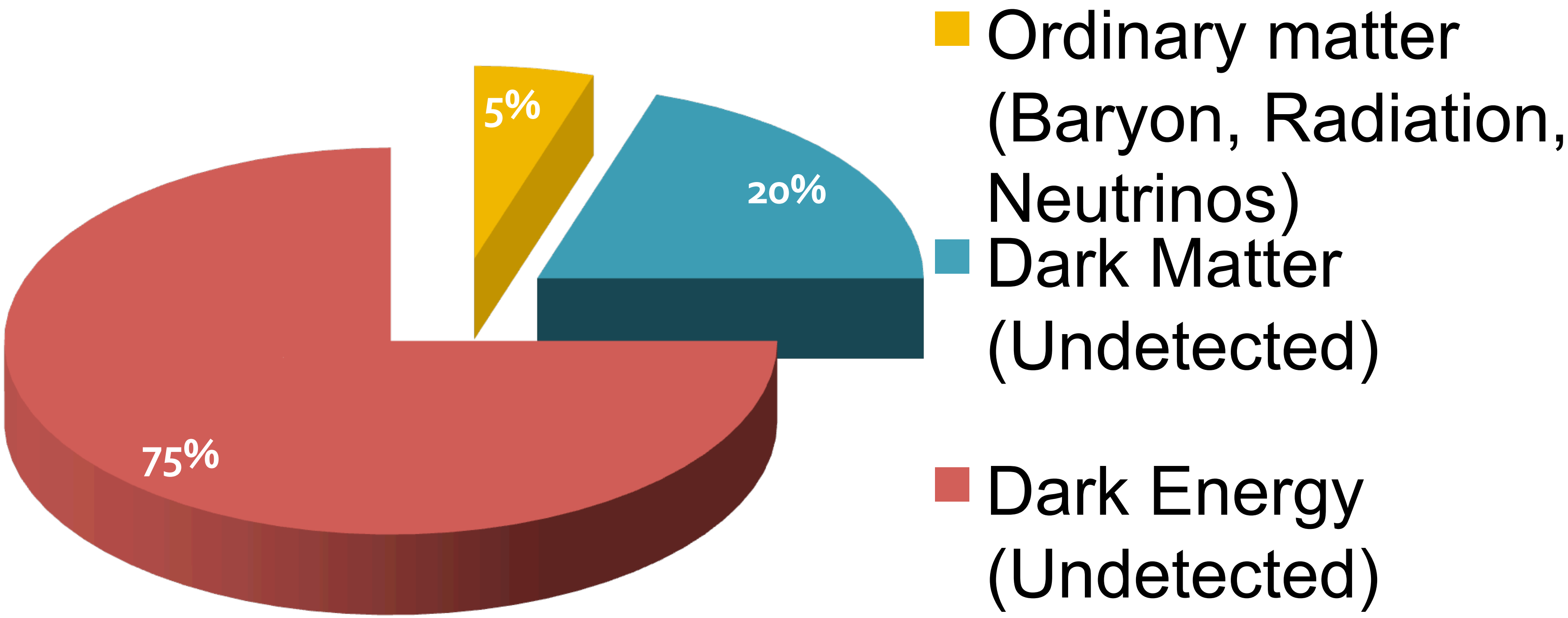
COSMOLOGY & GRAVITY GROUP

Why Fourth Order Gravity?

- ***Early Motivation:***
  - To question Einstein's GR [Weyl 1918, Eddington1923]
  - Account for GR's limitations at high energy regimes [Starobisnky 1980]
- ***Contemporary Motivation:*** Observational evidence:
  - CMB
  - SnIa
  - Weak Lensing

Why Fourth Order Gravity?

Observational evidence:
Flat, accelerating universe driven by a negative pressure component – *dark energy*



Why Fourth Order Gravity?

Einstein's GR

- Cosmological Constant Λ model (Dark Energy) + CDM = ΛCDM Universe (flat & homogenous)
- Quintessence
- Incomplete! Coincidence problem + Discrepancy in Λ value

Modified Gravity

- Geometric origin for dark energy
- Proposals: brane world theories, higher order curvature invariants → *fourth order gravity*

Why Fourth Order Gravity?

f(R) class of models:

- Fourth order theory of gravity
- Must account for observational tests like *bending of light* corresponding to:
  - Null geodesics
  - Knowledge of physically viable spherically symmetric solutions as in GR

Fourth order gravity

- *f(R)* class of models can be derived from the classical action: $$A = \frac{1}{2}\int d^4x\sqrt{-g}\left[f(R) + 2\mathcal{L}_m\right]$$

Fourth order gravity

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General function of curvature *R*

Fourth order gravity

- *f(R)* class of models can be derived from the classical action:

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Standard matter term

Fourth order gravity

- *f(R)* class of models can be derived from the classical action:

$$A = \frac{1}{2}\int d^4x\sqrt{-g}\left[f(R) + 2\mathcal{L}_m\right]$$

- Varying the action with respect to the metric gives the following field equations:

$$f'G_{ab} = f'\left(R_{ab} - \frac{1}{2}g_{ab}R\right) = T^m_{ab} + \frac{1}{2}g_{ab}(f - Rf') + \nabla_b\nabla_a f' - g_{ab}\nabla_c\nabla^c f'$$

- where

$$G_{ab} = \tilde{T}^m_{ab} + T^R_{ab} = T^{tot}_{ab}, \qquad f = f(R), \qquad f' = {df(R)}/{dR}$$

Fourth order gravity

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4th order gravity as standard GR in the presence of two effective fluids.

1+3 Covariant Approach

- Space-time as an **ideal fluid** with a 1+3 threading of the manifold
  - w.r.t **time-like** congruence e.g., the fluid flow of galaxy cluster in cosmology or any fluid flow in astrophysics or cosmology.
  - and remaining 3-D **spatial** freedom. [Ehlers 1961, Ellis 1971]
- Gives general set of equations based on Ricci and Bianchi identities.
- Physical covariant variables with easy interpretation for co-moving observers.

1+3 Covariant Approach

- From the time-like flow u^a we construct the projection onto 3-D surfaces orthogonal to the flow: $h_{ab} = g_{ab} + u_a u_b$; $\quad u^a u_a = -1$
- Covariant convective derivative : $\dot{T}^{a\ldots b}_{c\ldots d} = u^e \nabla_e T^{a\ldots b}_{c\ldots d}$
- Spatial covariant derivative:

$$D_e T^{a\ldots b}_{c\ldots d} = h_e^f h_g^a \ldots h_h^b h_c^i \ldots h_d^j \nabla_f T^{g\ldots h}_{i\ldots j}$$

- Kinematics of u^a gives information about the space -time geometry.

$$\nabla_a u_b = -u_a \dot{u}_b + \frac{1}{3}\Theta h_{ab} + \sigma_{ab} + \omega_{ab}$$

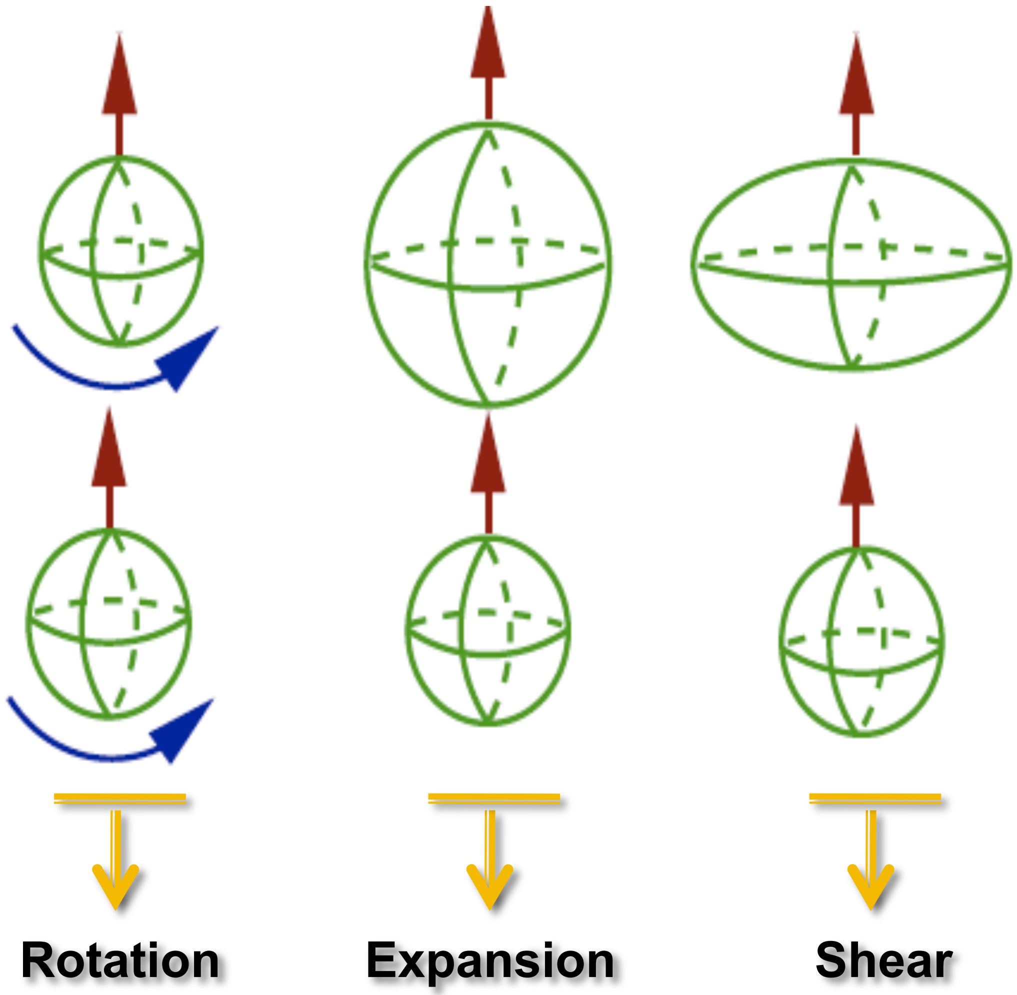
acceleration expansion shear vorticity

1+3 Covariant Approach

Ideal fluid flow:

Observer rest space

Kinematic effects:



Effective Thermodynamics

- The **total energy-momentum tensor** (EMT) is:

$$T_{ab}^{tot} = T_{ab}^{m} + T_{ab}^{R}$$

where

- T^m is the **matter** EMT
- T^R is the **curvature** EMT

$$T_{ab}^{R} = \frac{1}{2} g_{ab}(f - Rf') + \nabla_b \nabla_a f' - g_{ab} \nabla_c \nabla^c f'$$

Effective Thermodynamics

- The **total EMT** can be decomposed as usual relative to u^a giving:

$$T^{tot}_{ab} = \mu^{tot} u_a u_b + p^{tot} h_{ab} + 2q^{tot}_{(a} u_{b)} + \pi^{tot}_{ab}$$

Energy density **Scalar pressure** **Energy flux** **Anisotropic pressure**

$$\mu^{tot} = T^{tot}_{ab} u^a u^b = \tilde{\mu}^m + \mu^R ,$$

$$q^{tot}_a = -T^{tot}_{bc} h^b_a u^c = \tilde{q}^m_a + q^R_a ,$$

$$p^{tot} = \frac{1}{3} T^{tot}_{ab} h^{ab} = \tilde{p}^m + p^R ,$$

$$\pi^{tot}_{ab} = -T^{tot}_{cd} h^c_{<a} h^d_{b>} = \tilde{\pi}^m_{ab} + \pi^R_{ab} .$$

$$\sim \;\Rightarrow\; \frac{1}{f'}$$

Effective Thermodynamics

- where the **total thermodynamic quantities** are:

$$\mu^{tot} = \frac{1}{f'}\left[\mu^{m} + \frac{1}{2}\left(Rf' - f\right) + f'''D^{a}RD_{a}R + f''D^{2}R - \Theta f''\dot{R}\right],$$

$$p^{tot} = \frac{1}{f'}\left[p^{m} + \frac{1}{2}\left(f - Rf'\right) - \frac{2}{3}f''D^{2}R - \frac{2}{3}f'''D^{a}RD_{a}R\right.$$

$$\left. + \frac{2}{3}\Theta f''\dot{R} + f'''\dot{R}^{2} + f''\ddot{R} - \dot{u}_{c}\left(\nabla^{c}f'\right)\right],$$

$$\pi^{tot}_{ab} = \frac{1}{f'}\left[\pi^{m}_{ab} + f'''D_{<a}RD_{b>}R + f''D_{<a}D_{b>}R - \sigma_{ab}\dot{f}'\right],$$

$$q^{tot}_{ab} = -\frac{1}{f'}\left[q^{m}_{ab} + f'''\dot{R}D_{a}R + f''D_{a}\dot{R} - \frac{1}{3}\Theta f''D_{a}R\right].$$

Null geodesics

- Light propagates as null geodesics
- **Null tangent vector** k^a

$$k^a k_a = 0, \quad \frac{\delta \mathrm{k}^{\mathrm{a}}}{\delta v} = k^b \nabla_b k^a = 0$$

- Light propagation is received in the direction determined by unit spatial vector n^a

$$n^a n_a = 1, \quad n^a u_a = 0$$

- k^a is split as: $k^a = E\left(u^a + n^a\right)$

- where $E = - u_a k^a$: **energy associated with the rays**

1+1+2 Covariant Approach

- Involves 1+1+2 splitting of space-time into the time direction u^a and a further slicing of space into a **preferred direction** e^a and **sheet** [Clarkson & Barrett 2003]

$$e^a e_a = 1, \quad e_a u^a = 0.$$

- The projection onto the 2-D sheet,

$$N_a^b = h_a^b - e^b e_a = h_a^b + u^b u_a - e^b e_a$$

orthogonal to e^a and u^a

$$u^a N_{ab} = e^a N_{ab} = 0$$

1+1+2 Covariant Approach

- Spatial derivatives:
  - derivative in the e^a direction:

$$\hat{\psi}_{a\ldots b}{}^{c\ldots d} = e^e D_e \psi_{a\ldots b}{}^{c\ldots d}$$

  - projected derivative onto the sheet:

$$\delta_e \psi_{a\ldots b}{}^{c\ldots d} = N_e^{\;j} N_a^{\;f} \ldots N_b^{\;g} N_h^{\;c} \ldots N_i^{\;d} D_j \psi_{f\ldots g}{}^{h\ldots i}$$

- Spatial derivative of e^a results in new variables:

$$D_a e_b = -e_a \dot{u}_b + \frac{1}{2}\phi N_{ab} + \xi \varepsilon_{ab} + \varsigma_{ab}$$

acceleration (expansion shear vorticity of the sheet)

Dynamics

- Irreducible set of variables:

$$\left\{\Theta,\ \mathcal{A},\ \Omega,\ \Sigma,\ \phi,\ \xi,\ \mathcal{E},\ \mathcal{H},\ \mathcal{A}^a,\ \Omega^a,\ \Sigma^a,\ \alpha^a,\ a^a,\ \mathcal{E}^a,\ \mathcal{H}^a,\ \Sigma_{ab},\ \varsigma_{ab},\ \mathcal{E}_{ab},\ \mathcal{H}_{ab}\right\}$$

  together with:

$$\left\{\ \mu^{tot},\ p^{tot},\ Q^{tot},\ \Pi^{tot},\ Q_a^{tot},\ \Pi_{ab}^{tot}\right\}$$

  wholly characterize the space-time.

- The dynamical equations constitute those from decomposition of the 1+3 equations + dynamical equations for kinematic variables.

LRS Spacetimes

- **Locally rotationally symmetric** space-times (LRS), exhibit locally a unique preferred spatial direction constituting a local axis of symmetry
- The **kinematical** & **thermodynamic** variables $\left\{\mathcal{A},\, \Theta,\, \phi,\, \Sigma,\, \Omega,\, \mathcal{E},\, \mathcal{H},\, \mu^{tot}, p^{tot}, Q^{tot}, \Pi^{tot}\right\}$ fully describe LRS space-times
- The space-times are **isotropic** and therefore all the vector and tensor 1+1+2 variables are zero.

LRS Spacetimes

- **LRS class II:**
  - admit spherically symmetric solutions
  - vanishing vorticity Ω ,ξ and consequently magnetic Weyl curvature *H,* vanishes

$$\left\{ \mathcal{A},\ \theta,\ \phi,\ \Sigma,\ \varepsilon,\ \mu^{tot}, p^{tot}, Q^{tot}, \Pi^{tot} \right\}$$

LRS

- **LRS c**
  - adm
  - vani
    mag

Propagation equations

$$
\begin{aligned}
\hat{\phi} &= -\frac{1}{2}\phi^2 + \left(\frac{1}{3}\Theta + \Sigma\right)\left(\frac{2}{3}\Theta - \Sigma\right) - \frac{2}{3}\mu - \frac{1}{2}\Pi - \mathcal{E}\,, \\
\hat{\Sigma} - \frac{2}{3}\hat{\Theta} &= -\frac{3}{2}\phi\Sigma - Q\,, \\
\hat{\mathcal{E}} - \frac{1}{3}\hat{\mu} + \frac{1}{2}\hat{\Pi} &= -\frac{3}{2}\phi\left(\mathcal{E} + \frac{1}{2}\Pi\right) + \left(\frac{1}{2}\Sigma - \frac{1}{3}\Theta\right)Q\,.
\end{aligned}
$$

Evolution equations

$$
\begin{aligned}
\dot{\phi} &= -\left(\Sigma - \frac{2}{3}\Theta\right)\left(\mathcal{A} - \frac{1}{2}\phi\right) + Q\,, \\
\dot{\Sigma} - \frac{2}{3}\dot{\Theta} &= -\mathcal{A}\phi + 2\left(\frac{1}{3}\Theta - \frac{1}{2}\Sigma\right)^2 + \frac{1}{3}(\mu + 3p) - \mathcal{E} + \frac{1}{2}\Pi\,, \\
\dot{\mathcal{E}} - \frac{1}{3}\dot{\mu} + \frac{1}{2}\dot{\Pi} &= \left(\frac{3}{2}\Sigma - \Theta\right)\mathcal{E} + \frac{1}{4}\left(\Sigma - \frac{2}{3}\Theta\right)\Pi \\
&\quad + \frac{1}{2}\phi Q - \frac{1}{2}(\mu + p)\left(\Sigma - \frac{2}{3}\Theta\right)\,.
\end{aligned}
$$

Propagation/Evolution Equations

$$
\begin{aligned}
\dot{\mu} + \hat{Q} &= -\Theta(\mu + p) - (\phi + 2\mathcal{A})Q - \frac{3}{2}\Sigma\Pi\,, \\
\dot{Q} + \hat{p} + \hat{\Pi} &= -\left(\frac{3}{2}\phi + \mathcal{A}\right)\Pi - \left(\frac{4}{3}\Theta + \Sigma\right)Q - (\mu + p)\mathcal{A}\,, \\
\hat{\mathcal{A}} - \dot{\Theta} &= -(\mathcal{A} + \phi)\mathcal{A} + \frac{1}{3}\Theta^2 + \frac{3}{2}\Sigma^2 + \frac{1}{2}(\mu + 3p)\,,
\end{aligned}
$$

Commutation relations

$$
\hat{\dot{\psi}} - \dot{\hat{\psi}} = -\mathcal{A}\dot{\psi} + \left(\frac{1}{3}\Theta + \Sigma\right)\hat{\psi}.
$$

LRS Spacetimes

- **LRS class II:**
  - admit spherically symmetric solutions
  - vanishing vorticity Ω ,ξ and consequently magnetic Weyl curvature *H*, vanishes

$$\left\{\mathcal{A},\ \theta,\ \phi,\ \Sigma,\ \mathcal{E},\ \mu^{tot},p^{tot},Q^{tot},\Pi^{tot}\right\}$$

- **Static conditions**: expansion Θ, consequently shear Σ and heat flux Q

$$\left\{\mathcal{A},\ \phi,\ \mathcal{E},\ \mu^{tot},p^{tot},\Pi^{tot}\right\}$$

- **Vacuum conditions**:

$$\left\{\mathcal{A},\ \phi,\ \mathcal{E},\ \mu^{R},p^{R},\Pi^{R}\right\}$$

LRS Spacetimes

- **Propagation equations**

$$\hat{\phi} = -\frac{1}{2}\phi^2 - \frac{2}{3}\mu^R - \frac{1}{2}\Pi^R - \varepsilon,$$

$$\hat{\varepsilon} - \frac{1}{3}\hat{\mu}^R + \frac{1}{2}\hat{\Pi} = -\frac{3}{2}\phi\left(\varepsilon + \frac{1}{2}\Pi^R\right),$$

$$0 = -\mathcal{A}\phi + \frac{1}{3}\left(\mu^R + 3p^R\right) - \varepsilon + \frac{1}{2}\Pi^R,$$

$$\hat{p}^R + \hat{\Pi}^R = -\left(\frac{3}{2}\phi + \mathcal{A}\right)\Pi^R - \left(\mu^R + p^R\right)\mathcal{A},$$

$$\hat{\mathcal{A}} = -(\mathcal{A} + \phi)\mathcal{A} + \frac{1}{2}\left(\mu^R + 3p^R\right)$$

LRS II
+
Static
+
Vacuum

LRS Spacetimes

- μ^R, p^R, and Π^R are defined:

$$\mu^R = \frac{1}{f'}\left[\frac{1}{2}\left(Rf' - f\right) + f''\hat{X} + f''X\phi + f''X^2\right],$$

$$p^R = \frac{1}{f'}\left[\frac{1}{2}\left(f - Rf'\right) - \frac{2}{3}f''\hat{X} - \frac{2}{3}f''X\phi - \frac{2}{3}f''X^2 - \mathcal{A}f''X\right],$$

$$\Pi^R = \frac{1}{f'}\left[\frac{2}{3}f''X^2 + \frac{2}{3}f''\hat{X} - \frac{1}{3}f''X\phi\right].$$

- where $X = \hat{R}$

Static spherically symmetric solutions

- The space-time is determined by the scalar functions ***R, X, Φ, A***
- To find solutions to ***R, X, Φ, A*** we include the propagation equations in the direction of e^a:

$$f'\left[\hat{\phi}+\phi\left(\frac{1}{2}\phi-\mathcal{A}\right)\right]=\frac{1}{3}Rf'-\frac{2}{3}f+f''X(\phi+2\mathcal{A}),$$

$$f'\left[\hat{\mathcal{A}}+\mathcal{A}(\mathcal{A}+\phi)\right]=\frac{1}{6}f-\frac{1}{3}Rf'-f''X\mathcal{A},$$

$$\hat{R}=X,$$

$$f''\hat{X}=-\frac{1}{3}Rf'+\frac{2}{3}f-f'''X^2-X(\phi+\mathcal{A})f''.$$

Trace equation

Static spherically symmetric solutions

Covariant results for spherically symmetric system:

- *Condition for existence of solutions with vanishing Ricci scalar*:

  Imposing $|f'(0) < +\infty|,\ |f''(0) < +\infty|,\ |f'''0) < +\infty|\ , f(0) = 0\ \ \&\ \ \mathrm{R} = 0$

  - $f'(0)=0$: Propagation equations are identically satisfied for all $\mathcal{A}$ and Φ that guarantees $R=0$ and hence $X=0$
    - ➢ For models with a vanishing Ricci scalar in GR will be a solution to the system.
    - ➢ Fourth order gravity solutions = **GR solutions + additional** e.g. Reissner Nordstrom

Static spherically symmetric solutions

Covariant results for spherically symmetric system:

- *Condition for existence of solutions with constant curvature* : Imposing $R = R_0 = const \quad \Rightarrow \quad X, \hat{X} = 0$

$$f_0'\left[\hat{\phi} + \phi\left(\frac{1}{2}\phi - \mathcal{A}\right)\right] = \frac{1}{3}R_0 f_0' - \frac{2}{3}f_0,$$

$$f_0'\left[\hat{\mathcal{A}} + \mathcal{A}(\mathcal{A} + \phi)\right] = \frac{1}{6}f_0 - \frac{1}{3}R_0 f',$$

$$0 = -R_0 f_0' + 2f_0.$$ ➔Barrow & Ottewill (1983)

- Solution iff
  - ➢$f'_0 \neq 0,\ f_0 0 \neq 0,\ 2f_0 - R_0 f'_0 = 0$
  - ➢$f'_0 \neq 0,\ f_0 0 = 0,\ 2f_0 - R_0 f'_0 = 0$ (Schwarzschild)
  - ➢$f'_0 = 0,\ f_0 0 = 0,\ R = R_0$

Static spherically symmetric solutions

Covariant results for spherically symmetric system:

- ***The curious case of R^2 gravity***
  - It is expected from $2f(R_0) - R_0 f'(R_0) = 0$ that the conditions for constant curvature connects with universal constants
  - Local constant of integration is obtained here instead for R^2 Schwarzschild/Ads space-time
  - Implies *two bodies behaving as different Schwarzschild/Ads objects with different constant values can't have their masses determined uniquely just by studying their geodesics*

Static spherically symmetric solutions

Coordinate system relation

- Define hat derivative $\hat{M} = \frac{1}{2} r\phi \frac{dM}{dr}$
- Specialize choice of equations to $f(R) = R^n$

$$\frac{1}{2} nr\phi \frac{d\phi}{dr} R^{n-1} = \left(\mathcal{A} - \frac{1}{2}\phi\right)\phi R^{n-1} + \frac{n-2}{3n} R^n + (n-1)R^{n-1} X(\phi + 2\mathcal{A}),$$

$$\frac{1}{2} nr\phi \frac{d\mathcal{A}}{dr} R^{n-1} = -\left(\mathcal{A} + \phi\right)\mathcal{A}R^{n-1} + \frac{1-2n}{6n} R^n - (n-1)R^{n-2} X\mathcal{A},$$

$$\frac{1}{2} r\phi \frac{d\mathcal{R}}{dr} = \mathcal{X},$$

$$\frac{1}{2} nr\phi(n-1) \frac{d\mathcal{X}}{dr} R^{n-2} = \frac{2-n}{3} R^n - n(n-1)(n-2)R^{n-3} X^2 .$$

Static spherically symmetric solutions

Exact R^n solutions:

- ***Schwarzschild solution***:
  - $R=0$, $dR/dr=0$, gives usual Schwarzschild coordinates iff $n=1,2,>3$
- ***Solution with constant non-zero R***:
  - *Substitute* $X=0$, $R=R_0 \neq 0$ *gives a solution iff* $n=2$

Static spherically symmetric solutions

- ***Solution with non-constant Ricci scalar non-vanishing at infinity***:
  - From the ansatz: $\phi = \sqrt{C_1 r^{\alpha} + C_2 r^{\beta}}, \quad R = C_3 / r^{\gamma} \, (\gamma > 0)$
  - We get (provided $n < (1+\sqrt{3})/2$ and for $n \in (1,(1+\sqrt{3}/2)$ *n* is an even rational number)

$$\mathcal{A} = \frac{-C(5-4n)r^{-1-\frac{5-4n}{2-n}} + (2n-2)(2n-1)r^{-1+\frac{(2n-2)(2n-1)}{2-n}}}{2(2-n)\left(Cr^{-\frac{5-4n}{2-n}} + r^{\frac{(2n-2)(2n-1)}{2-n}}\right)\sqrt{\frac{(1+2n-2n^2)(7-10n+4n^2)}{(2-n)^2\left(1+Cr^{-\frac{7-10n+4n^2}{2-n}}\right)}}} \;;\; \phi = \frac{2}{r\sqrt{\frac{(1+2n-2n^2)(7-10n+4n^2)}{(2-n)^2\left(1+Cr^{-\frac{7-10n+4n^2}{2-n}}\right)}}}$$

$$R = \frac{6n(n-1)}{[2n(n-1)-1]r^2} \;;\; X = -\frac{12n(n-1)}{[2n(n-1)-1]r^3\sqrt{\frac{(1+2n-2n^2)(7-10n+4n^2)}{(2-n)^2\left(1+Cr^{-\frac{7-10n+4n^2}{2-n}}\right)}}}$$

Static spherically symmetric solutions

- Solving for the metric definitions of the covariant scalars: $\mathcal{A} = \frac{1}{2A(r)}\frac{dA}{dr}\sqrt{B(r)}, \quad \phi = \frac{2}{r}\sqrt{B(r)}$

  A(r) and *B(r)* are from the metric:

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right)$$

- Gives the solution (also found by T. Clifton 2006):

$$A(r) = r^{(2n-2)\frac{(2n-1)}{(2-n)}} + \frac{C}{r^{\frac{(5-4n)}{(2-n)}}}; \quad \frac{1}{B(r)} = \frac{(2-n)^2}{\left(7-10n+4n^2\right)\left(1+2n-2n^2\right)}\left(1+\frac{C}{r^{\frac{(7-10n+4n^2)}{(2-n)}}}\right)$$

General form of Lensing Angle

- Consider now the geometry of a null congruence using the 1+1+2 formalism.
- The kinematics are based on a **null vector** k^a which can be split as

$$k^a = E(u^a + \kappa e^a + \kappa^a)$$

- where E (*energy of ray*) and κ (*magnitude of radial component*) are the **lensing variables**
- The derivative in the k^a direction is defined

$$f' = k^a \nabla_a f \quad \text{with} \quad f' = E\kappa\hat{f}$$

General form of Lensing Angle

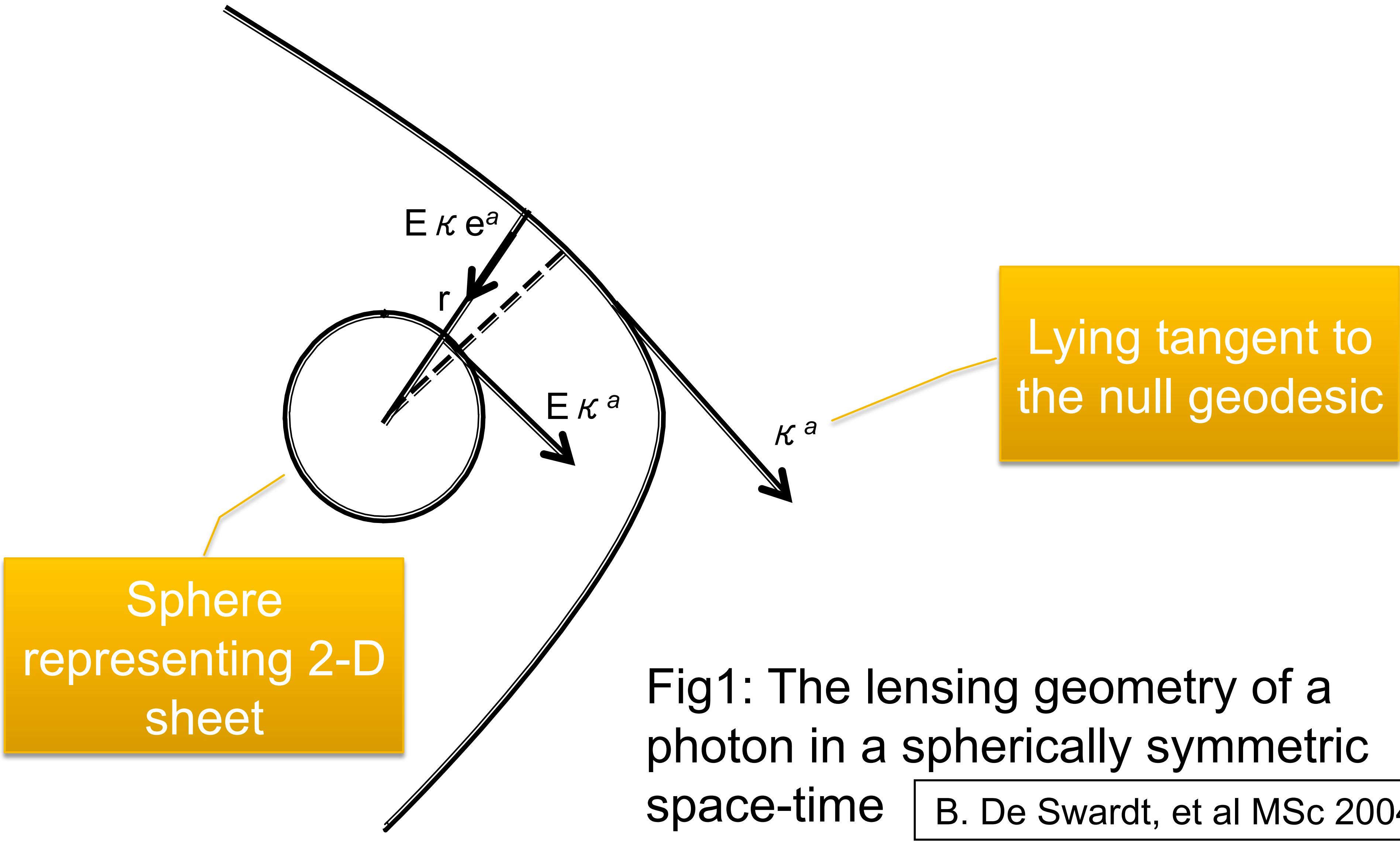


Fig1: The lensing geometry of a photon in a spherically symmetric space-time

B. De Swardt, et al MSc 2004

General form of Lensing Angle

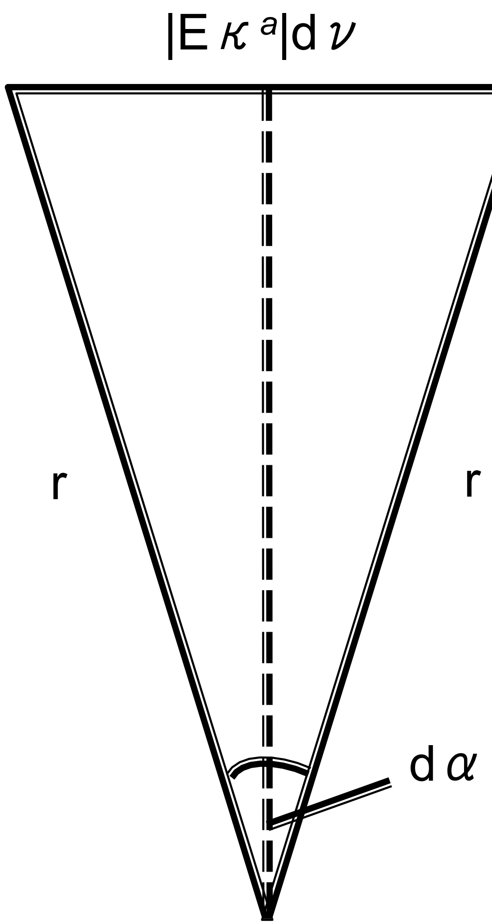


Fig2: geometry of the deflection angle

General form of Lensing Angle

- The **general form** of the deflection angle is given as

$$\alpha = \int_{\nu_1}^{\nu_2} \frac{1}{r} |E| \sqrt{1-\kappa^2}\, d\nu - \alpha_0$$

for any form of gravity

- From:
  - the propagation equations of *E* and κ *(in static LRS II conditions)*: $E' = -E^2 \mathcal{A}\kappa,$

$$\kappa' = E\left(1-\kappa^2\right)\left(\frac{1}{2}\phi - \mathcal{A}\right)$$

  - the solution to $\mathcal{A}$ and Φ

Deflection Angle in f(R)

- The **deflection angle** :

$$\alpha = 2\int_{r_0}^{r_*} L^{-1}\frac{J}{r^2}\left[r^{\frac{(4n^2-6n+2)}{(n-2)}} - J^2\left(r^{-2} + Cr^{\frac{(4n^2-12n+11)}{(n-2)}}\right)\right]^{-\frac{1}{2}} dr - \pi$$

- where:

$$J = \left[\frac{r_0^{\frac{3(2n-3)}{n-2}}}{r_0^{\frac{-4n^2+10n-7}{n-2}} + C}\right]^{\frac{1}{2}}; \quad \mathrm{L} = \sqrt{\frac{(\mathrm{n}-2)^2}{(1+2n-2n^2)(7-10n+4n^2)}}$$

Observables

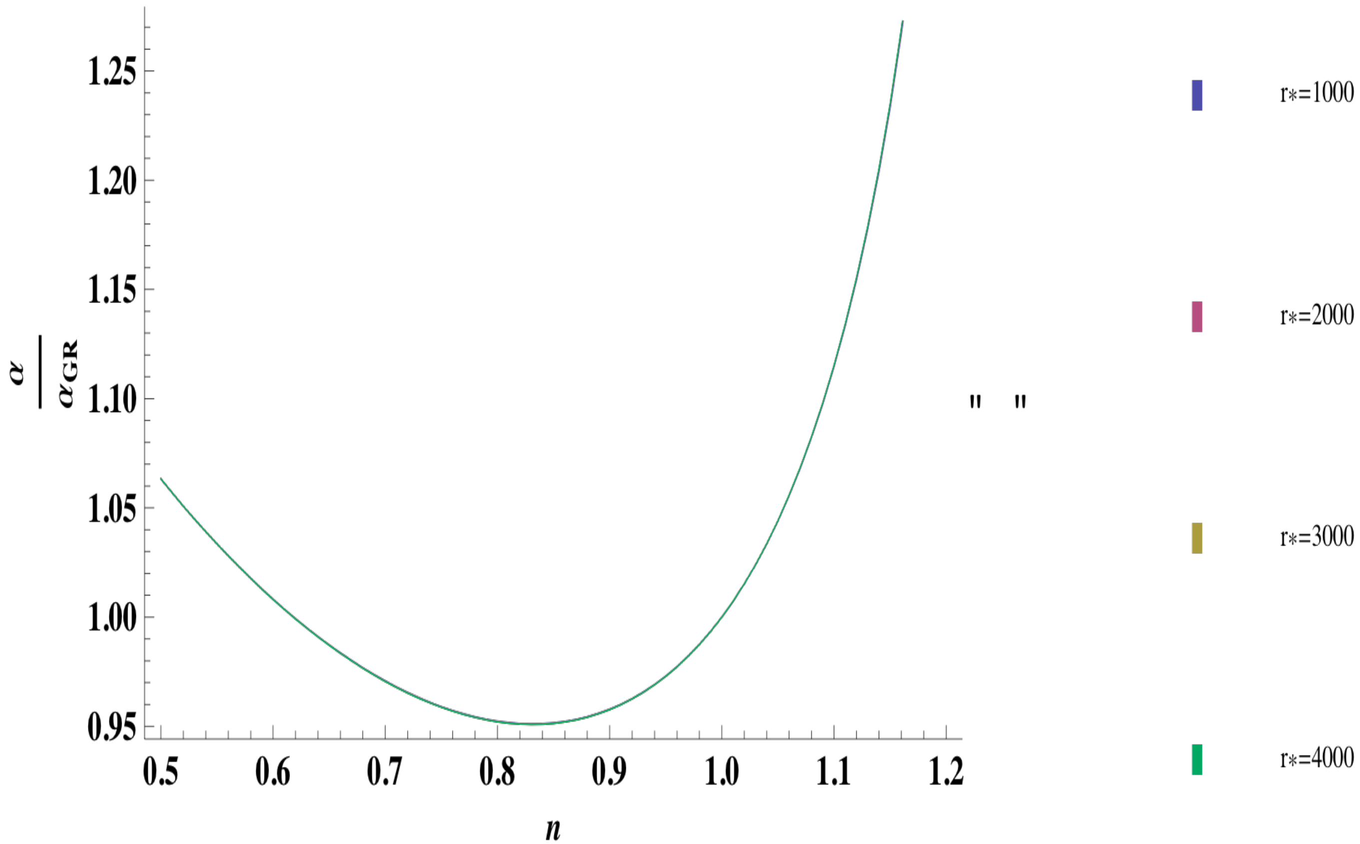


Fig3: plot of comparative bending angle vs n for various distances from source

Observables

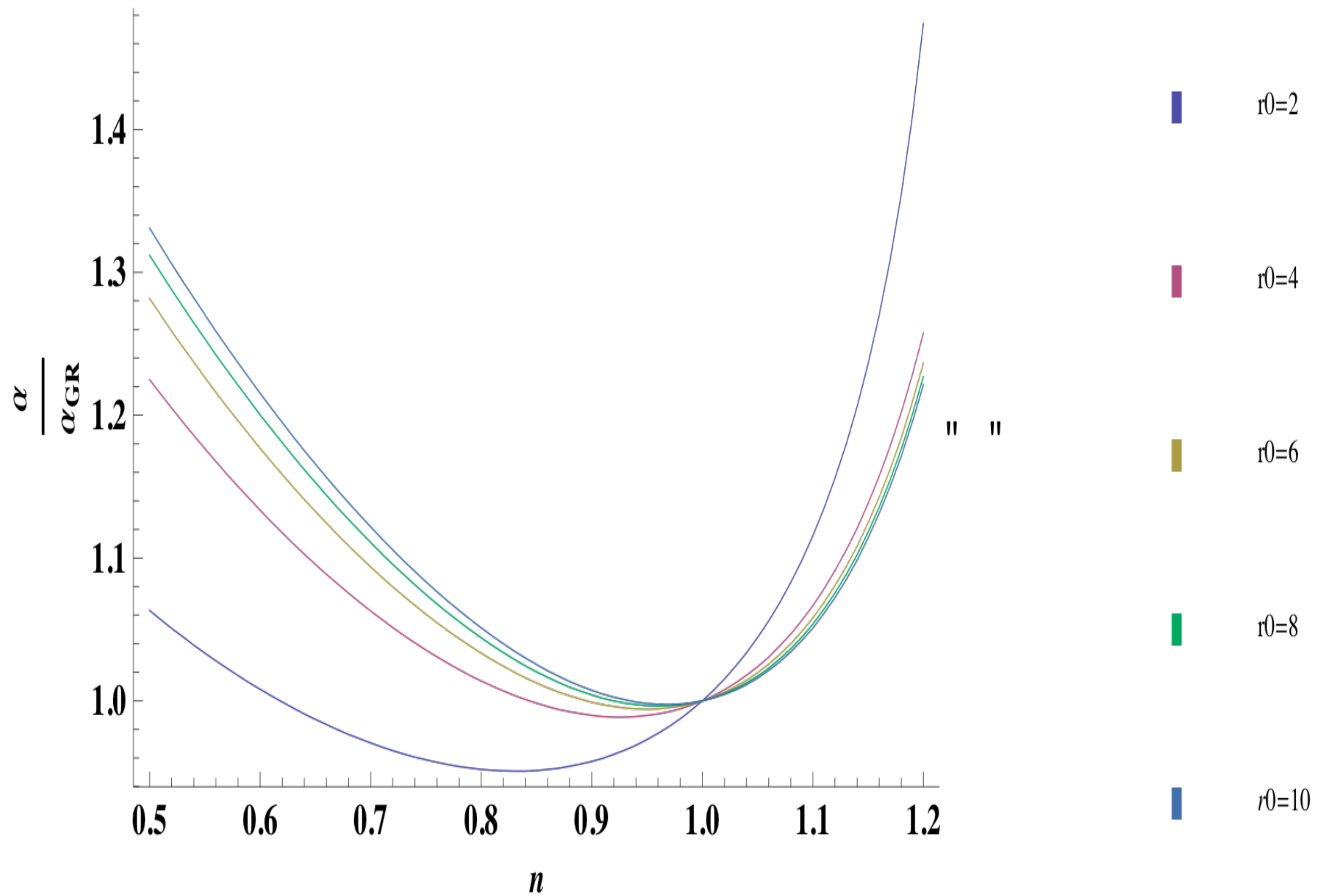


Fig4: plot of comparative bending angle vs n for various distances of closest approach

Conclusion

- *The deflection angle from the metric approach*:

$$\alpha = \int_{r_0}^{r_*} \frac{(AB)^{1/2}\, dr}{r\sqrt{(r/r_0)^2 B(r_0) - B(r)}} - \pi$$

- *Advantage of the covariant approach*:
  - Applies to any LRS space-time whilst metric applies only to spherically symmetric space-times
  - Physically relevant quantities (the *kinematic* & *dynamic variables* & their *propagation equations*) manifesting the underlying physics for better understanding

Conclusion

- A set of master equations that provide important results and conditions for spherically symmetric static solutions in *f(R)* gravity were derived [gr-qc/ 0908.3333]
- A number of exact solutions were found from these equations for $f(R) = R^n$ ➔ indicative of Birkoff's theorem violation

A covariant form of the bending angle for a particular solution was derived where more bending expected for R^n gravity model and could be used to obtain signatures for these models