GAUGE FIXING : BOSONIC STRING IN MINKOWSKI SPACE-TIME

$$S = -\int d\tau\, d\sigma \sqrt{\dot{X}^2 X'^2 - (\dot{X}\cdot X')^2}$$

$$\dot{X}^\mu = \frac{\partial}{\partial\tau} X^\mu \qquad X^{\mu\prime} = \frac{\partial}{\partial\sigma} X^\mu$$

I. CONFORMAL GAUGE

$$S_{CONF} = \int d\tau\, d\sigma \left(\dot{X}^2 - X'^2\right)$$

$$\underbrace{\ddot{X}_\mu - X''_\mu = 0}_{\text{eqs.}} \qquad \underbrace{X_{,+}^2 = 0 \qquad X_{,-}^2 = 0}_{\text{CONSTRAINTS}}$$

$$\partial_\pm = \partial_\tau \pm$$

$$X_\mu = X_\mu^L + X_\mu^R$$

$$X_\mu^L(\tau+\sigma) = \sum_n a_n^\mu e^{in\tau_+} + a_n^+ e^{-in\tau_+}$$

$$X_\mu^R(\tau-\sigma) = \sum_n \tilde{a}_n e^{in\tau_-} + \tilde{a}_n^+ e^{-in\tau_-}$$

- STILL NEED TO FULLFIL THE TWO (VIRASORO) CONSTRAINTS

NONTRIVIAL: INFINITE SET OF QUADRATIC EQUATIONS FOR $a, \tilde{a}$ COEFFICIENTS!

VERY FEW SOLUTIONS KNOWN

- ENERGY: GENERATOR OF TIME TR.

$$P^t_\tau = \frac{\partial \mathcal{L}}{\partial \dot{X}^0} = \dot{X}^0$$

$$E = \int d\sigma \, \frac{\partial X^0(\tau,\sigma)}{\partial \tau}$$

ANGULAR MOMENTUM

$$J = \int d\sigma \, \frac{\partial \Theta(\tau,\sigma)}{\partial \tau}$$

$$= \int d\sigma \, \left(X_1 \dot{X}_2 - X_2 \dot{X}_1 \right)$$

IV. TIME-LIKE / CONFORMAL GAUGE

$\boxed{X^0 = \tau}$ $\qquad \vec{X}(t,\sigma)$

$$\mathcal{L} = - \sqrt{-(1-\dot{\vec{X}}^2)\vec{X}'^2 - (\dot{\vec{X}}\cdot\vec{X}')^2}$$

$$\underline{\mathcal{H} = \vec{P}^2 + \vec{X}'^2}$$

CONST. $\underline{G = \vec{P}\cdot\vec{X}'}$

Gauge COND. $\underline{\chi = \vec{P}^2 + \vec{X}'^2 - 1}$

RESULT :

$$\underline{\ddot{\vec{X}} - \vec{X}'' = 0} \qquad \begin{cases} \vec{X}_{,+}^2 = 1 \\ \vec{X}_{,-}^2 = 1 \end{cases}$$

- CONFORMAL gauge $+ X^0 = \tau$ additional

- QUESTION 1 ENERGY $\mathcal{H} = 1$

$$E = \int d\sigma \quad - \text{just lenght} \quad \sigma \in (0, L)$$

EXAMPLE 1.

ROTATING

II. AN EXPLICIT SOLUTION : SPIKY CONFIG. IN 3D MINKOWSKI

D = 3 FLAT MINKOWSKI

$$X^\mu = (t, X, Y)$$

- CONFORMAL GAUGE : $X_{,+}^2 = X_{,-}^2 = 0$

$$X = a_{n-1} \cos(n-1)\tau_+ + \tilde{a}_1 \cos \tau_-$$

$$Y = a_{n-1} \sin(n-1)\tau_+ + \tilde{a}_1 \sin \tau_-$$

$$t = 2(n-1)\tau = (n-1)(\tau_+ + \tau_-)$$

$$X_{,+}^2 = (n-1)^2 - a_{n-1}^2 (n-1)^2 = 0 \Rightarrow \boxed{a_n = 1}$$

$$X_{,-}^2 = (n-1)^2 - a_1^2 \Rightarrow \boxed{\tilde{a}_1 = n-1}$$

$$K_L = n-1 \qquad K_R = 1$$

$$n_L = a_{n-1}^+ a_{n-1} (n-1) = n-1 \qquad n_R = 1 \cdot (n-1)^2$$

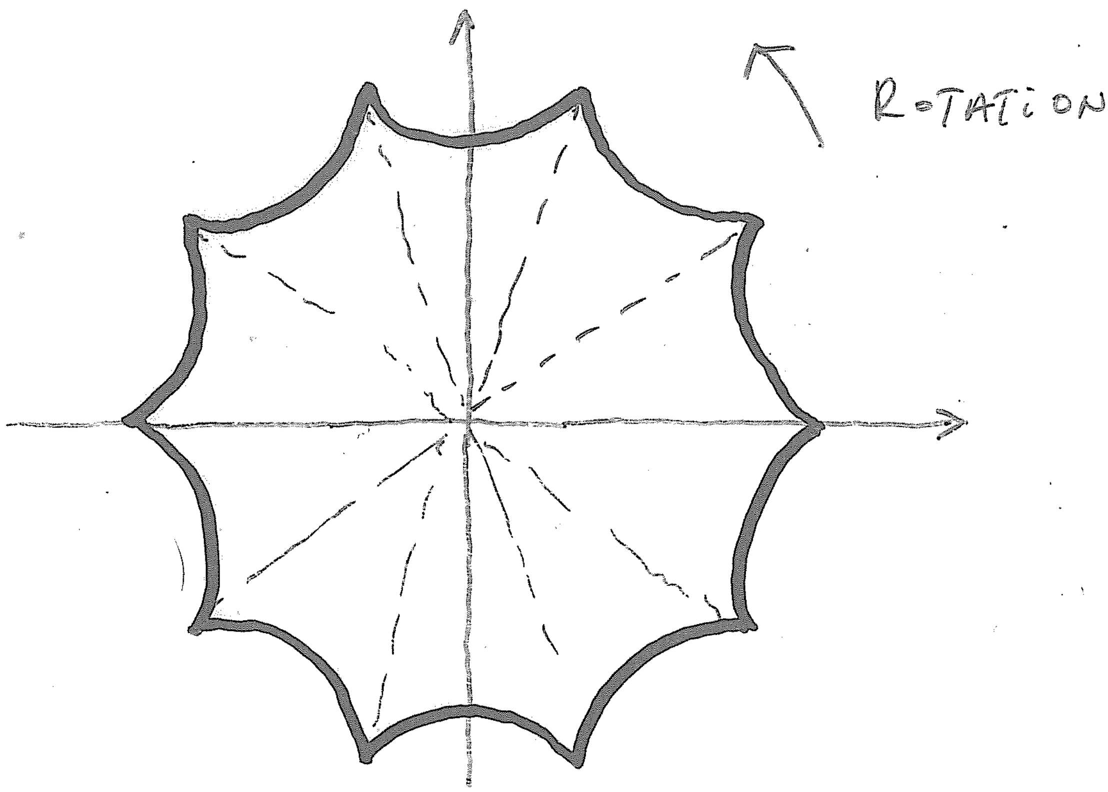
- ANGULAR MOMENTUM

$$J = \int d\sigma \left(\dot{X} Y - \dot{Y} X \right) = n_L + n_R = n(n-1)$$

- ENERGY :

$$E^2 - \left(K_L n_L + K_R n_R \right) = 0$$

$$E^2 = 2(n-1) \rightarrow \boxed{E = 2\sqrt{\frac{n-1}{n}\, J}}$$



$$\theta_m = \sigma_m = 2\pi \left(\frac{m}{n}\right) - \frac{n-2}{n}\tau \quad - \text{ m-th Spike}$$

$$m = 0, 1, \ldots n-1$$

EXAMPLE 2 : ONE AND TWO SOLITON / STRING

Ex. 2 $\longleftrightarrow$ $\boxed{AdS_3}$ SOLUTION

- ONE SOLITON
$$\alpha_s(\tau,\sigma) = \ln\left[2\,\mathrm{th}^2\,\frac{\sigma - v\tau}{\sqrt{1-v^2}}\right]$$

- STRING :
$$\zeta = \sqrt{\frac{1-v}{1+v}} \qquad \tau_\pm = \tau \pm \sigma$$

$\downarrow$

AdS_3
$$-1 = \underbrace{-Z_{-1}^2 - Z_0^2}\ \underbrace{+Z_1^2 + Z_2^2} \equiv -\bar{Z}_+ Z_+ + \bar{Z}_- Z_-$$

complex coor.

$\downarrow$

$$Z_\pm = \frac{e^{(1+i)(i\tau_+ + \tau_-)}}{2\left(e^{-\zeta\tau_+} - e^{-\zeta\tau_-}\right)}\left\{ e^{-\frac{\tau_-}{\zeta}}\left(1 + e^{\tau_+ - \tau_-}\right) - \right.$$
$$\left. - e^{-\zeta\tau_+}\,\frac{(i\zeta - 1)(1-\zeta^2) \pm e^{\tau_+ - \tau_-}(1+\zeta)^2}{(i\zeta+1)(1-\zeta^2)} \right\}$$

$$E - S = \ln\frac{4S(1+\zeta)^2}{1+\zeta^2} - \frac{2\zeta}{(1+\zeta^2)} + O\left(\frac{1}{S}\right)$$

large S

IV. T-DUALITY IN STRING THEORY

SIMPLEST EXAMPLE : STRING ON S_1

$$X^2 + Y^2 = R^2$$

$$X = R_0 \cos \phi(\tau,\sigma)$$

$$Y = R_0 \sin \phi(\tau,\sigma)$$

$$\mathcal{L} = R_0^2 \int d\tau\, d\sigma\, \frac{1}{2}\left((\partial_\tau \phi)^2 - (\partial_\sigma \phi)^2 \right)$$

- interchange $\tau, \sigma \equiv \sigma_\alpha \quad \alpha = 1,2$

$$\partial_\alpha \phi = \varepsilon_{\alpha\beta}\, \partial_\beta \chi$$

$$\Pi_\phi \leftarrow \partial_\tau \phi = \partial_\sigma \chi$$

$$\partial_\sigma \phi = -\partial_\tau \chi - \Pi_\chi$$

$$\Pi_\phi \longrightarrow \partial_\sigma \chi$$

$$\phi \longrightarrow -\Pi_\chi$$

NEW (DUAL) FIELD ESSENTIALLY GIVEN BY Π

CANONICAL

$$\pi_\phi = \frac{\partial L}{\partial \dot\phi} = R_0^2 \, \dot\phi$$

$$\mathcal{L} = \Pi_\phi \dot\phi - \frac{1}{2}\left(\frac{1}{R^2}\Pi^2 + R^2(\partial_\sigma \phi)^2\right)$$

$$+\partial^{-1}\dot\Pi\,\partial\phi - \frac{R^2}{2}(\partial\phi)^2$$

INTEGRATE OUT ϕ!

$$\mathcal{L}_\Pi = \frac{1}{2R^2}\left(\left(\partial_\sigma^{-1}\dot\Pi\right)^2 - \Pi^2\right)$$

$$\Pi \to \partial_\sigma \chi$$

$$\mathcal{L}^{DUAL} = \frac{1}{2R_0^2}\left(\dot\chi^2 - (\partial_\sigma\chi)^2\right)$$

Result $R_0 \to \frac{1}{R_0}$ EXCHANGE

V°. YANG-MILLS AMPLITUDE from ADS

ADS$_5$

$$(ds)^2 = \frac{\overbrace{(-dx^0)^2 + (d\vec{x})^2}^{\text{MINKOWSKI}} + \overbrace{(dz)^2}^{\text{EXTRA 5-th}}}{z^2}$$

Y-M$_4$ $\quad x^\mu = (x^0, \vec{x})$ MINKOWSKI

$\tilde{A}_\mu(k)^{ij} \leftarrow$ COLOR

$k^2 = 0$ gluons : light like

↓

IR (small momentum) DIVERGENCES

μ - small mass regulates

ADS $\quad z \sim x, t$

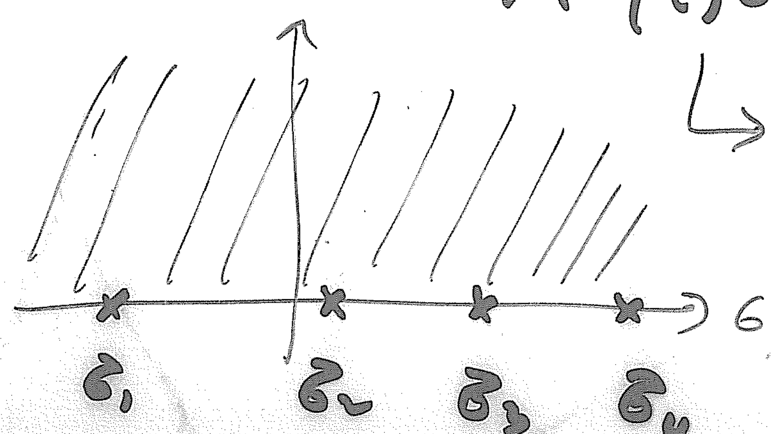
$z \to$ small (distance) UV

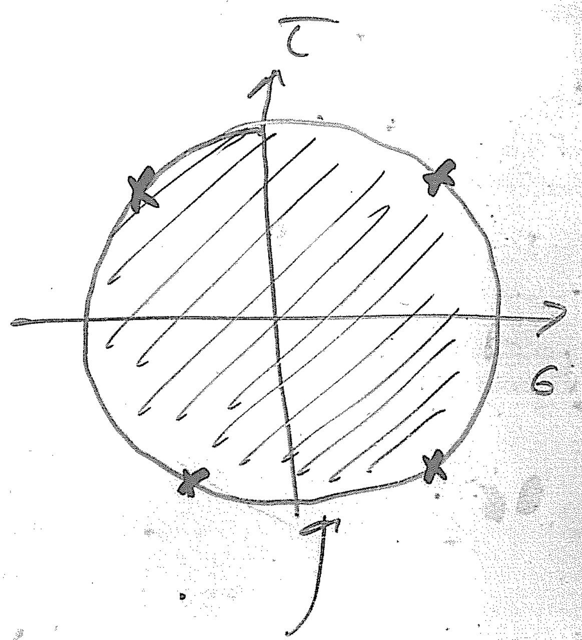
$z \to$ large : IR $\quad z_{IR} = \frac{1}{\mu}$

- STRING THEORY (IN ADS)

SIMULATION of YANG-MILLS Amplit.

$$X^{\mu}(\tau,\sigma)$$





DISK : BOUND.

$$\left\langle e^{ik_1 X(\sigma_1)}\, e^{ik_2 X(\sigma_2)}\, e^{ik_3 X}\, e^{ik_4 X} \right\rangle$$

- PATH INTEGRAL THESE ARE B.C.

ON $p_\mu(\sigma)$

$$\sum_i \underline{k_i}\;\underline{\hat{X}(\sigma_i)} = \int d\sigma\; \overset{\text{source}}{p(\sigma)}\; \hat{X}(\sigma)$$

$$p^{\mu}(\sigma) = \sum k_i^{\mu}\, \delta(\sigma-\sigma_i)$$

$\partial_\sigma Y$

conjugate

$$\to \int d\sigma \; \underbrace{\partial_\sigma^{-1} p(\sigma)}_{y(\sigma)} \; \underbrace{\partial_\sigma X}_{\Pi_y}$$

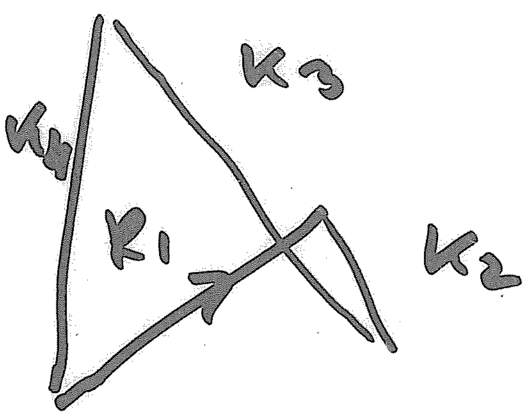
$$y^\mu(\sigma) = \sum k_i \, \theta(\sigma - \sigma_i) \qquad \underline{\partial^{-1}\delta = \theta}$$

BOUNDARY CONDITION ON THE DUAL <u>COORDINATE</u>

$$Y^\mu(\sigma) = y_0^\mu(\sigma)$$

Segments given BY <u>MOMENTA</u>

$Y^\mu(\sigma)$



POLYGON (CLOSED) B.C.