Interactions of Massless Higher Spin Fields from String Theory

SFT 09

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- Gauge field theory describing interacting particles of higher spins (with $s > 2$) is a fascinating and complicated subject that has attracted a profound interest over many years since the 30s.

- Despite strong efforts by some leading experts in recent years there are still key issues about these theories that remain unresolved (even for the non-interacting particles; much more so in the interacting case)

- There are several reasons why the higher spin theories are so complicated

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1. In order to be physically meaningful, these theories need to possess sufficiently strong gauge symmetries, powerful enough to ensure the absence of unphysical (negative norm) states. For example, in the Fronsdal's description (Fronsdal (1978)) the theories describing symmetric tensor fields of spin s are invariant under gauge transformations with the spin $s-1$ traceless parameter:

$$\begin{aligned} H_{a_1 \ldots a_s}(x) \rightarrow H_{a_1 \ldots a_s}(x) \\ + \partial_{(a_s} \Lambda_{a_1 \ldots a_{s-1})}(x) \\ Tr(\Lambda) \equiv \eta^{a_i a_j} \Lambda_{a_1 \ldots a_i \ldots a_j \ldots a_s} = 0 \\ 1 \leq i < j \leq s \end{aligned} \quad (1)$$

where H and Λ are the symmetric spin s field and spin $s-1$ gauge parameter with H satisfying the on-shell Fierz-Pauli conditions:

$$\begin{aligned} \partial_a \partial^a H_{a_1 \ldots a_s} &= 0 \\ \partial^{a_1} H_{a_1 \ldots a_s} &= 0 \\ Tr(H) \equiv \eta^{a_i a_j} \Lambda_{a_1 \ldots a_i \ldots a_j \ldots a_s} &= 0 \\ 1 \leq i < j \leq s \end{aligned} \quad (2)$$

- Theories with the vast gauge symmetries as this are not trivial to construct even in the non-interacting case,when one needs to introduce a number of auxiliary fields and objects like non-local compensators; much more so in the interacting case

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In the flat space things are further complicated because of the no-go theorems (such as Coleman-Mandula theorem) imposing strong restrictions on conserved charges in interacting theories with a mass gap, limiting them to the scalars and those related to the standard Poincare generators. Thus Coleman-Mandula theorem in $d = 4$ makes it hard to construct consistent interacting theories of higher spin, at least as long as the locality is preserved

String theory is a particularly effective and natural framework to approach the problem of higher spins at least in the massive case, since the higher spin modes naturally appear in the massive sector of the theory. Thus one can hope to obtain the higher field spin theories in the low energy limit of string theory, by analyzing the worldsheet correlators of the appropriate vertex operators.

However, physical vertex operators for HS fields in string theory are constrained by the spin to mass relations. Thus only vertex operator in open string theory, decoupled from superconformal ghost degrees of freedom (and therefore existing at zero ghost picture) has spin 1. Therefore the massless operators for the higher spins are inevitably those that couple to the worldsheet ghost degrees of freedom and violate the picture equivalence.

- In this talk I discuss the construction of physical vertex operators describing massless higher spin fields and the computation of their scattering amplitudes, by using the ghost cohomology approach.

- The Pauli-Fierz on-shell conditions (2) for the massless higher spin fields follow from the BRST-invariance conditions for the constructed open string operators. The gauge transformations (1) stem from the BRST nontriviality constraints for these operators. For this reason, the interaction terms of the higher spin fields, determined by the worldsheet correlation functions of the vertex operators for the higher spins, are gauge-invariant by construction.

Thus string theory provides an efficient and natural framework to build the consistent gauge-invariant interacting theories of higher spin fields.

We start from presentingh the expressions for the vertex operators for the higer spin fields with the spin values $3 \leq s \leq 9$:

$$V_{s=3}(p) = H_{a_1 a_2 a_3}(p)$$
$$\times c e^{-3\phi} \partial X^{a_1} \partial X^{a_2} \psi^{a_3} e^{i\vec{p}\vec{X}}$$
$$V_{s=4}(p) = H_{a_1 ... a_4}(p)$$
$$\times \eta e^{-4\phi} \partial X^{a_1} \partial X^{a^2} \partial \psi^{a_3} \psi^{a_4} e^{i\vec{p}\vec{X}}$$
$$V_{s=5}(p) = H_{a_1 ... a_5}(p)$$
$$\times e^{-4\phi} \partial X^{a_1} ... \partial X^{a^3} \partial \psi^{a_4} \psi^{a_5} e^{i\vec{p}\vec{X}}$$
$$V_{s=6}(p) = H_{a_1 ... a_6}(p)$$
$$\times c\eta e^{-5\phi} \partial X^{a_1} ... \partial X^{a^3} \partial^2 \psi^{a_4} \partial \psi^{a_5} \psi^{a_6} e^{i\vec{p}\vec{X}}$$
$$V_{s=7}(p) = H_{a_1 ... a_7}(p)$$
$$\times c e^{-5\phi} \partial X^{a_1} ... \partial X^{a^4} \partial^2 \psi^{a_5} \partial \psi^{a_6} \psi^{a_7} e^{i\vec{p}\vec{X}}$$
$$V_{s=8}(p) = H_{a_1 ... a_8}(p)$$

$$\times c\eta e^{-5\phi}\partial X^{a_1}...\partial X^{a^7}\psi^{a_8}e^{i\vec{p}\vec{X}}$$
$$V_{s=9}(p) = H_{a_1...a_9}(p)$$
$$\times ce^{-5\phi}\partial X^{a_1}...\partial X^{a^8}\psi^{a_9}e^{i\vec{p}\vec{X}} \quad (3)$$

where X^a and ψ^a are the RNS world-sheet bosons and fermions ($a = 0, ..., d-1$), the ghost fields are bosonized as usual, according to

$$b = e^{-\sigma}$$
$$c = e^{\sigma}$$
$$\gamma = e^{\phi-\chi} \equiv e^{\phi}\eta$$
$$\beta = e^{\chi-\phi}\partial\chi \equiv \partial\xi e^{-\phi}$$
$$(4)$$

For simplicity, we shall concentrate on the totally symmetric polarization tensors $H_{a_1 \dots a_s}(p)$, although it should be relatively straightforward to generalize the vertices (1) to less symmetric cases.

BRST-invariance conditions for the H.S. vertices:

For simplicity, consider the $s=3$ vertex operator first, all other operators can be analyzed similarly. For our purposes it is convenient to cast the BRST operator as

$$Q_{brst}=Q_1+Q_2+Q_3 \tag{5}$$

where

$$\begin{aligned}Q_1&=\oint\frac{dz}{2i\pi}\{cT-bc\partial c\}\\ Q_2&=-\frac{1}{2}\oint\frac{dz}{2i\pi}\gamma\psi_a\partial X^a\\ Q_3&=-\frac{1}{4}\oint\frac{dz}{2i\pi}b\gamma^2\end{aligned} \tag{6}$$

where T is the full stress-energy tensor. It is easy to demonstrate that all the vertex operators (1) commute with

Q_2 and Q_3 of Q_{brst}. The commutation with Q_1, however, requires the constraints on the on-shell fields. Since all the operators (1) are the worldsheet integrals of operators of conformal dimension 1, they commute with Q_1 if the integrands are the primary fields, i.e. their OPEs with T don't contain singularities stronger than double poles (along with the on-shell $(\vec{p})^2 = 0$ condition). Since $H_{a_1a_2a_3}$ is fully symmetric, the OPE is given by

$$
\begin{aligned}
T(z)\partial X^{(a_1}\partial X^{a_2}\psi^{a_3)}e^{i\vec{p}\vec{X}}(w)H_{a_1a_2a_3}(p) \\
\sim -\frac{\eta^{(a_1a_2}\psi^{a_3)}e^{i\vec{p}\vec{X}}(w)H_{a_1a_2a_3}(p)}{(z-w)^4} \\
+i\frac{p^{(a_1}\partial X^{a_2}\psi^{a_3)}e^{i\vec{p}\vec{X}}(w)H_{a_1a_2a_3}(p)}{(z-w)^3} \\
+O((z-w)^{-2})
\end{aligned}
\tag{7}
$$

Therefore the BRST-invariance conditions for the $s=3$ vertex:

$$
\begin{aligned}
H^{a_1}_{a_1a_3}(p) &= 0 \\
p^{a_1}H_{a_1a_2a_3}(p) &= 0 \\
p^2H_{a_1a_2a_3}(p) &= 0
\end{aligned}
\tag{8}
$$

are precisely the Pauli-Fierz conditions for the symmetric massless higher spins.

BRST Nontriviality of the Higher Spin Vertex operators

We look for the conditions to ensure that V_s cannot be represented as a BRST commutators with operators in small Hilbert space,i.e. for a given V_s there is no operator W_s such that $V_s = \{Q_{brst}, W_s\}$. We start with the operators for massless fields with odd spin values ($s = 3, 5, 7, 9$) that have the following structure if taken at minimal negative ghost pictures $-n$ ($n = 3$ for $s = 3$, $n = 4$ for $s = 5$ and $n = 5$ for $s = 7, 9$):

$$V_s = ce^{-n\phi}F_{\frac{n^2}{2}-n+1}(X,\psi) \qquad (9)$$

where $F_{\frac{n^2}{2}-n+1}(X,\psi)$ is the primary matter field of conformal dimension $\frac{n^2}{2} - n + 1$ (suppressing all the indices). Then there are only two possible sources of W_s. The first possibility is that W_s is proportional to the ghost factor $\partial cc\partial\xi\partial^2\xi e^{-(n+2)\phi}$. Then there is a possibility that V_s could be obtained as a BRST commutator with

$$W_s = \partial cc\partial\xi\partial^2\xi e^{-(n+2)\phi} \times G^{(2n-3)}(\phi,\chi,\sigma)F_{\frac{n^2}{2}-n+1}(X,\psi) \tag{10}$$

where $G^{(2n-3)}(\phi,\chi,\sigma)$ is the conformal dimension $2n-3$ polynomial in the derivatives of the bosonized ghost fields ϕ, χ and σ that must be chosen so that

$$[Q_1, W_s] = 0 \tag{11}$$

Provided that $G^{(2n-3)}(\phi, \chi, \sigma)$ are chosen to satisfy (9), it is easy to check that the W_s-operators also satisfy

$$[Q_2, W_s] = 0$$
$$[Q_3, W_s] = \alpha_n V_s \qquad (12)$$

and therefore

$$[Q_{brst}, W_s] = \alpha_n V_s \qquad (13)$$

where α_n are the numerical coefficients that depend on the structure of $G^{(2n-3)}(\phi, \chi, \sigma)$. A lengthy but straightforward computation shows, however, that for all the choices of $G^{(2n-3)}(\phi, \chi, \sigma)$, consistent with the condition (9) for $n = 3, 4, 5$ (that are relevant for the higher spin operators (1) with $3 \le s \le 9$) one has

$$\alpha_n = 0$$
$$n = 3, 4, 5 \qquad (14)$$

The second, and the only remaining possibility for V_s to be written as BRST commutators stems from the W_s-operators with the ghost structure $\sim c\partial\xi e^{-(n+1)\phi}$,satisfying

$$
\begin{aligned}
[Q_1, W_s] &= 0 \\
[Q_2, W_s] &\sim V_s \\
[Q_3, W_s] &= 0
\end{aligned}
\tag{15}
$$

The only possible construction for W_s with such a structure is given by

$$
W_s = c\partial\xi e^{-(n+1)\phi} F_{\frac{n^2}{2}-n+1}(X,\psi)(\psi_a \partial X^a)
\tag{16}
$$

The operators of this type always commute with Q_3 and produce V_s when commuted with Q_2. Therefore V_s are trivial as long as W_s commute with Q_1.

So V_s are physical operators only if the commutator $]Q_1, W_s]\neq 0$, which, in turn, imposes constraints on the space-time fields and entails the gauge symmetries for the higher spins. Consider the particular case of $s = 3$, other operators are analyzed similarly. The W_s-operator of the type for $V_{s=3}$ is

$$W_{s=3}(p) = c\partial\xi e^{-4\phi}\partial X^{a_1}\partial X^{a_2}\psi^{a_3} (\vec{\psi}\partial\vec{X})e^{i\vec{p}\vec{X}}H_{a_1a_2a_3}(p) \tag{17}$$

where, as previously, the H three-tensor is symmetric and satisfies the Fierz-Pauli on-shell conditions (6) We easily find $W_{s=3}$ to satisfy:

$$[Q_1, W_{s=3}(p)]$$
$$= -\frac{i}{2}\partial^2 cc\partial\xi e^{-4\phi}\partial X^{a_1}\partial X^{a_2}\psi^{a_3}$$
$$\times(\vec{p}\vec{\psi})e^{i\vec{p}\vec{X}}H_{a_1a_2a_3}(p)$$
$$[Q_2, W_{s=3}(p)] = \frac{d}{2}V_{s=3}(p)$$
$$[Q_2, W_{s=3}(p)] = 0 \quad (18)$$

So the nontriviality of $V_{s=3}$ requires that the right hand side of the commutator $[Q_1, W_{s=3}(p)]$ is nonzero. This leads to the following nontriviality conditions on the H-tensor:

$$p_{[a_4}H_{a_3]a_1a_2} \neq 0 \quad (19)$$

The analysis of the nontriviality constraints for all other higher spin operators (both odd and even spin values

$4 \leq s \leq 9$) is totally similar and leads to the same conditions on $H_{a_1 \ldots a_s}(p)$:

$$p_{[a_{s+1}} H_{a_s] a_1 \ldots a_{s-1}} \neq 0. \tag{20}$$

These nontriviality constraints entail the gauge symmetry transformations for the higher spin fields. It is convenient to consider first more general case case when $H_{a_s||a_1 \ldots a_{s-1}}$ is symmetric in $a_1, \ldots, a_{s-1}$ but not in a_s. Due to the nontriviality constraints () the generic $H_{a_s||a_1 \ldots a_{s-1}}$-tensor can be shifted as (without changing any correlation functions)

$$H_{a_s|a_1 \ldots a_{s-1}} \to p_{a_s} \Lambda_{a_1 \ldots a_{s-1}} \tag{21}$$

where $\Lambda_{a_1 \ldots a_{s-1}}$ is symmetric and must be traceless due to the BRST-invariance conditions (). Renaming the indices:

$$\begin{aligned} a_s &\leftrightarrow a_1 \\ a_s &\leftrightarrow a_2 \\ &\dots \\ a_s &\leftrightarrow a_{s-1} \end{aligned} \tag{22}$$

we get the chain:

$$H_{a_1|a_2...a_{s-1}a_s} \to H_{a_1|a_2...a_{s-1}a_s} + p_{a_1}\Lambda_{a_2...a_s}$$
$$H_{a_2|a_1a_3...a_s} \to H_{a_2|a_1a_3...a_s} + p_{a_2}\Lambda_{a_1a_3...a_s}$$
$$H_{a_{s-1}|a_1a_2...a_{s-2}a_s} \to H_{a_{s-1}|a_1a_2...a_{s-2}a_s} + p_{a_{s-1}}\Lambda_{a_1a_2...a_{s-2}a_s} \quad (23)$$

summing () together, we get the gauge transformations for the fully symmetric tensor $H_{a_1...a_s}$, implied by the nontriviality conditions ():

$$H_{a_1...a_s} \to H_{a_1...a_s} + p_{(a_1}\Lambda_{(a_2...a_s)} \quad (24)$$

where Λ is traceless. Thus the BRST invariance and nontriviality conditions on the higher spin vertex operators () altogether impose Fierz-Pauli on-shell constraints and the gauge symmetries analogous to those in the Fronsdal's approach. For this reason, the correlation functions of these vertex operators, computed below, shall by construction produce the gauge-invariant interaction terms for the massless higher spin fields in space-time, satisfying all the standard on-shell conditions and gauge symmetries for the higher spins...

Before we proceed to the calculation of the gauge-invariant interaction terms determined by the vertex operators (), it is important to analyze the physical reasons behind the appearance of the higher spin vertex operators () in the superstring spectrum. The higher spin vertex operators () are closely related to the surprising nonlinear global space-time symmetries (α-symmetries) in string theory, mixing matter and ghost degrees of freedom, and to the hidden space-time dimensions. Consider the RNS superstring action in the superconformal gauge:

$$S_{RNS} = S_{matter} + S_{bc} + S_{\beta\gamma}$$
$$S_{matter} = \frac{1}{2\pi}\int d^2z(\partial X_m \bar{\partial} X^m + \psi_m \bar{\partial}\psi^m + \bar{\psi}_m \partial \bar{\psi}^m)$$
$$S_{bc} = \frac{1}{2\pi}\int d^2z(b\bar{\partial}c + \bar{b}\partial\bar{c})$$
$$S_{\beta\gamma} = \frac{1}{2\pi}\int d^2z(\beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma})$$

It turns out that, apart from the standard global Poincare symmetries (such as rotations and translations) the action () is also invariant is invariant under the following transformations (with α being a global parameter):

$$\delta X^m = \alpha\{2e^\phi \partial\psi^m + \partial(e^\phi\psi^m)\}$$
$$\delta\psi^m = -\alpha\{e^\phi\partial^2 X^m + 2\partial(e^\phi\partial X^m)\}$$
$$\delta\gamma = \alpha e^{2\phi-\chi}\{\psi_m\partial^2 X^m - 2\partial\psi_m\partial X^m\}$$
$$\delta\beta = \delta b = \delta c = 0$$

so that

$$\delta S_{matter} = -\delta S_{\beta\gamma}$$
$$= \frac{1}{2\pi}\int d^2z(\bar\partial e^\phi)(\psi_m\partial^2 X^m - 2\partial\psi_m\partial X^m)$$
$$\delta S_{bc} = \delta S_{RNS} = 0$$

The generator of these transformations is given by

$$L^{\alpha+} = \oint \frac{dz}{2i\pi} e^{\phi} F(X, \psi)$$
$$\equiv \oint \frac{dz}{2i\pi} e^{\phi} (\psi_m \partial^2 X^m - 2\partial\psi_m \partial X^m)$$

where it is convenient to introduce the notation for the dimension $\frac{5}{2}$ primary field:

$$F(X, \psi) = \psi_m \partial^2 X^m - 2\partial\psi_m \partial X^m$$

along with the matter worldsheet supercurrent

$$G = -\frac{1}{2}\psi_m \partial X^m$$

and the dimension 2 primary

$$L(X,\psi) = 2\partial\psi_m\psi^m - \partial X_m \partial X^m$$

which is the w.s. superpartner of F, i.e.

$$G(z)L(w) \sim \frac{F(w)}{z-w}$$

L^α-generator is the element of the ghost cohomology H_1. The integrand of the $L^{\alpha+}$ -generator is a primary field of dimension 1, i.e. a physical operator. Its distinctive property is that there are no versions of this generator at ghost pictures below 1 (higher pictures can be obtained from the standard picture-changing transformation). This means that this symmetry generator exists at the minimal ghost picture +1. The negative picture version of this generator can be be obtained by replacing $\phi \to -3\phi$ in the expressions (),(). Using the inverse picture changing operator $\Gamma^{-1} = 4c\partial\xi e^{-2\phi}$ one can obtain pictures $-4, -5, ...$ of the symmetry generator ().

The negative picture versions of the α-symmetry transformations () exist at ghost picture -3 and below, but not above -3. Thus the $L^{\alpha+}$-generator is the element of positive ghost cohomology H_1 and of the negative ghost cohomology H_{-3} (the accurate definition will be given below).

In d-dimensional RNS string theory there are $d+1$ additional α-symmetry generators of minimal ghost number 1 (elements of $H_1 \sim H_{-3}$) which also induce global space-time symmetries :

$$L^{m\alpha} = \oint \frac{dz}{2i\pi} e^{\phi} \times \{\partial^2 \varphi \psi^m$$
$$-2\partial\varphi\partial\psi^m + \partial^2 X^m \lambda - 2\partial X^m \partial\lambda\} \quad (25)$$

and

$$L^{\alpha -} = \oint \frac{dz}{2i\pi} e^{\phi} \times \{\partial^2 \varphi \lambda$$
$$-2\partial\varphi\partial\lambda\} \quad (26)$$

where, as previously, $m = 0, ..., d-1$ and ϕ, λ are the components of the super Liouville field.

Combined together with the $\frac{(d+1)(d+2)}{2}$ standard Poincare generators (including the Liouville direction) the $d+2$ α-generators $L^{\alpha\pm}, L^{\alpha m}$ extend the specetime isometry group from $SO(1, d+1)$ to $SO(2, d+1)$ bringing in an extra dimension and changing the space-time isometry from flat to AdS_{d+2}

Definition

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Positive ghost cohomologies H_n ($n > 0$) consist of picture-inequivalent physical operators, existing at pictures n and above, annihilated by inverse picture changing transformation at minimal positive picture n.

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Negative ghost cohomologies H_{-n} consist of picture-ineguivalent physical operators, existing at pictures $-n$ and below, annihilated by direct picture changing at minimal negative picture $-n$.

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An isomorphism holds between positive and negative cohomologies:

$$H_n \sim H_{-n-2}$$

H_0 by definition consists of picture-equivalent operators existing at all pictures (including picture 0), while H_{-1} and H_{-2} are empty.

Thus the space-time α-symmetry generators of $H_1 \sim H_{-3}$ bring in an extra dimension to the theory, with the intex α labelling the extra dimension. The α-symmetry generators of higher ghost cohomologies can be constructed as well. The $d+3$ generators of $H_2 \sim H_{-4}$ are given by

$$\begin{aligned}
L^{\beta+} &= \oint \frac{dz}{2i\pi} e^{-4\phi} F_1(X,\psi) F_1(\varphi,\lambda)(z) \\
L^{\beta-} &= -\oint \frac{dz}{2i\pi} e^{-4\phi} F_{1m}(X,\lambda) F_1^m(\varphi,\psi)(z) \\
L^{\beta m} &= \oint \frac{dz}{2i\pi} e^{-4\phi} (F_1^m(X,\lambda) F_1(\varphi,\lambda) \\
&\quad - F_1(X,\psi) F_1^m(\varphi,\psi))(z) \\
L^{\alpha\beta} &= \oint \frac{dz}{2i\pi} e^{-4\phi} (\frac{1}{2} F_2(\lambda,\varphi) \\
&\quad + L_1(X,\psi) \partial L_1(\varphi,\lambda) \\
&\quad - \partial L_1(X,\psi) L_1(\varphi,\lambda))(z)
\end{aligned} \tag{27}$$

with the matter+Liouville structures L and F (L_1, F_1 and F_1^m) being the primary fields of dimensions 2 and $\frac{5}{2}$:

$$\begin{aligned}
F_1(X,\psi) &= \psi_m \partial^2 X^m \\
&\quad -2\partial\psi_m \partial X^m \\
F_1(\varphi,\lambda) &= \lambda\partial^2\varphi - 2\partial\lambda\partial\varphi \\
F_1^m(X,\lambda) &= \lambda\partial^2 X^m - 2\partial\lambda\partial X^m \\
F_1^m(\varphi,\psi) &= \psi^m\partial^2\varphi - 2\partial\psi^m\partial\varphi \\
L_1(X,\psi) &= \\
&\quad \partial X_m \partial X^m - 2\partial\psi_m\psi^m \\
L_1(\varphi,\lambda) &= (\partial\varphi)^2 - 2\partial\lambda\lambda
\end{aligned} \tag{28}$$

and $F_2(\lambda,\varphi)$ being the primary field of dimension 5:

$$F_2(\varphi,\lambda) = \frac{1}{4}(\partial\varphi)^5 - \frac{3}{4}\partial\varphi(\partial^2\varphi)^2$$
$$+\frac{1}{4}(\partial\varphi)^2\partial^3\varphi + \lambda\partial\lambda(\partial^3\varphi - (\partial\varphi)^3)$$
$$-\frac{3}{2}\lambda\partial^2\lambda\partial^2\varphi + 3\partial\lambda\partial^2\lambda\partial\varphi\}$$
$$\equiv i : (\oint e^{-i\varphi}\lambda)^3 e^{3i\varphi}\lambda : \quad (29)$$

Combined with the matter + Liouville Poincare generators of $SO(2,d)$ and the α-generators (4) - (6) of $H_1 \sim H_{-3}$, the α-generators of $H_2 \sim H_{-4}$ enlarge the symmetry group to SO(2,d+2), bringing in the second extra dimension to the theory Finally, the (d+4) α-generators at the level $H_3 \sim H_{-5}$ (bringing in the third hidden dimension labelled by γ)

are constructed as

$$
\begin{aligned}
L^{\gamma+} = \oint \frac{dz}{2i\pi} e^{-5\phi}\{2\partial F_1(X,\psi) \\
-F_1(X,\psi)\partial F_2(\varphi,\lambda)\} \\
L^{\gamma\beta} = [L^{\gamma+}, L^{\beta-}] \\
L^{\gamma\alpha} = [L^{\gamma+}, L^{\alpha-}] \\
L^{\gamma m} = [L^{\gamma\alpha}, L^{\alpha m}] \\
L^{\gamma-} = [L^{\gamma\alpha}, L^{\alpha-}] \quad (30)
\end{aligned}
$$

extending the space-time isometry group to $SO(2, d+3)$ At this point, we still lack an explicit construction for the generators of $H_n \sim H_{-n-2}$ for $n \geq 4$, but the conjecture is that each ghost cohomology of the order n (combined with the operators of cohomologies of lower orders) extends the dimensionality of space-time by one unit.

Supersymmetric Extensions of α-Symmetries and Higher Spin Vertex Operators

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The supersymmetric extension of the α-symmetry generators can be obtained by applying the standard space-time supercharge:

$$Q_A = \oint \frac{dz}{2i\pi} e^{-\frac{1}{2}\phi} \Sigma_A(z) \qquad A = 1, ..., 16 \tag{31}$$

to $L^{\alpha\pm}$, $L^{\beta\pm}$, $L^{\gamma\pm}$ in the negative picture representations $(-3, -4, -5)$ Then the higher spin vertex operators () appear as the central terms in the space-time superalgebra of the α-extended supercharges. The central charges in the

SUSY algebra are (by no-go theorem) always related to the appearance of non-perturbative solutions (such as p-branes) in the strongly coupled limit of string theory and to the Wess-Zumino terms of the p-branes with nontrivial topological configurations. Such a connection between higher spin fields and branes is, in a sense, not surprising and is directly related to the role of the higher spin field operators in the AdS/CFT correspondence

Higher Spin Vertices in Positive Picture Representation

Because the higher spin operators () violate picture equivalence, higher picture versions cannot be obtained by straightforward picture-changing transformation (which simply annihilates these operators). Moreover, there are no local (unintegrated) analogues of the operators at higher ghost pictures, so all of their higher picture versions always appear in the integrated form. In particular, we shall need to use, in addition to unintegrated higher spin vertex operators (1) at negative ghost pictures $-n-2$ with $n = 1, 2, 3$, their integrated counterparts at positive ghost pictures n. These counterparts can be constructed by using the K-transformation procedure, defined as follows. Consider one of unintegrated vertex operators (1) for odd spins at minimal negative picture $-n-2$ (the even spin case is considered analogously). Such an operator has a structure

$$V_{-n-2} = ce^{-(n+2)\phi}F_{\frac{n^2}{2}+n+1}(X,\psi) \tag{32}$$

where, as previously, $F_{\frac{n^2}{2}+n+1}(X,\psi)$ the is matter primary field of conformal dimension $\frac{n^2}{2}+n+1$. Using the fact that the operators $e^{-(n+2)\phi}$ and $e^{n\phi}$ have the same conformal dimension $-\frac{n^2}{2}-n$, one starts with constructing the charge

$$\oint V_n \equiv \oint dz e^{n\phi} F_{\frac{n^2}{2}+n+1}(X,\psi) \quad (33)$$

This charge commutes with Q_1 since it is a worldsheet integral of dimension 1 and $b-c$ ghost number zero but doesn't commute with Q_2 and Q_3. To make it BRST-invariant, one has to add the correction terms by using the following procedure: We write

$$[Q_{brst}, V_n(z)] = \partial U(z) + W_1(z) + W_2(z) \quad (34)$$

and therefore

$$[Q_{brst}, \oint dz V_n] = \oint dz (W_1(z) + W_2(z)) \quad (35)$$

where

$$U(z) \equiv cV_n(z)$$

$$[Q_1, V_n] = \partial U$$
$$W_1 = [Q_2, V_n]$$
$$W_2 = [Q_3, V_n] \tag{36}$$

Introduce the dimension 0 K-operator:

$$K(z) = -4ce^{2\chi - 2\phi}(z) \equiv \xi\Gamma^{-1}(z) \tag{37}$$

satisfying

$$\{Q_{brst}, K\} = 1 \tag{38}$$

It is easy to check that this operator has a non-singular operator product with W_1:

$$K(z_1)W_1(z_2) \sim (z_1 - z_2)^{2n} Y(z_2) + O((z_1 - z_2)^{2n+1}) \tag{39}$$

where Y is some operator of dimension $2n + 1$. Then the complete BRST-invariant operator can be obtained from

$\oint dzV_n(z)$ by the following transformation:

$$\oint dzV_n(z) \to A_n(w)$$
$$= \oint dzV_n(z) + \frac{1}{(2n)!} \oint dz(z-w)^{2n}$$
$$\times : K\partial^{2n}(W_1 + W_2) : (z)$$
$$+ \frac{1}{(2n)!} \oint dz\partial_z^{2n+1}$$
$$\times [(z-w)^{2n} K(z)] K\{Q_{brst}, U\} \quad (40)$$

where w is some arbitrary point on the worldsheet. It is then straightforward to check the invariance of A_n by using some partial integration along with the relation (34) as well as the obvious identity

$$\{Q_{brst}, W_1(z) + W_2(z)\}$$
$$= -\partial(\{Q_{brst}, U(z)\}) \quad (41)$$

Although the invariant operators $A_n(w)$ depend on an arbitrary point w on the worldsheet, this dependence is irrelevant in the correlators since all the w derivatives of A_n are BRST exact - the triviality of the derivatives ensures that there will be no w-dependence in any correlation functions involving A_n. Equivalently, the positive picture representations A_n (36) for higher spin operators can also be obtained from minimal negative picture representations V_{-n-2} by straightforward, but technically more cumbersome procedure by using the combination of the picture-changing and the Z-transformation (the analogue of the picture-changing for the $b-c$-ghosts).

Namely, the Z-operator, transforming the $b-c$ pictures (in particular, mapping integrated vertices to unintegrated) given by

$$\begin{aligned} Z(w) &= b\delta(T)(w) \\ &= \oint dz(z-w)^3(bT \\ &+4c\partial\xi\xi e^{-2\phi}T^2)(z) \end{aligned} \tag{42}$$

where T is the full stress-energy tensor in RNS theory. The usual picture-changing operator, transforming the $\beta-\gamma$ ghost pictures, is given by

$$\Gamma(w) =: \delta(\beta)G : (w) =: e^{\phi}G : (w)$$

. Introduce the *integrated* picture-changing operators $R_n(w)$ according to

$$R_n(w) = Z(w) : \Gamma^n : (w) \tag{43}$$

where $:\Gamma^n:$ is the nth power of the standard picture-changing operator:

$$:\Gamma^n:(w) =: e^{n\phi}\partial^{n-1}G...\partial GG:(w)$$
$$\equiv: \partial^{n-1}\delta(\beta)...\partial\delta(\beta)\delta(\beta) \quad (44)$$

Then the positive picture representations for the higher spin operators A_n can be obtained from the negative ones V_{-n-2} by the transformation:

$$A_n(w) = (R_2)^{n+1}(w)V_{-n-2}(w) \quad (45)$$

Since both Z and Γ are BRST-invariant and nontrivial, the A_n-operators by construction satisfy the BRST-invariance and non-triviality conditions identical to those satisfied by their negative picture counterparts V_{-2n-2} and therefore lead to the same Pauli-Fierz on-shell conditions (6) and the gauge symmetries

(22), (23) for the higher spin fields.

Below we shall list some concrete examples of the K-transformation (36) applied to the spin $s = 3$ and $s = 4$ operators that will be used in our calculations. For the $s = 3$ operator the above procedure gives

$$V_{s=3} = ce^{-3\phi}\partial X^{a_1}\partial X^{a_2}$$
$$\times\psi^{a_3}e^{i\vec{p}\vec{X}}H_{a_1a_2a_3}(p)$$
$$\rightarrow \oint dzV_1 = H_{a_1a_2a_3}(p)\oint e^{\phi}\partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}}$$
$$[Q_1, V_1] = \partial U$$
$$= H_{a_1a_2a_3}(p)\partial(ce^{\phi}\partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}})$$
$$[Q_2, V_1] = W_1$$
$$= \frac{1}{2}H_{a_1a_2a_3}(p)e^{2\phi-\chi}\{(-(\vec{\psi}\partial\vec{X})$$
$$+i(\vec{p}\vec{\psi})P^{(1)}_{\phi-\chi} + i(\vec{p}\partial\vec{\psi}))$$

$$\times\partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}}$$
$$+\partial X^{a_1}(\partial^2\psi^{a_2}+2\partial\psi^{a_2}P^{(1)}_{\phi-\chi})\psi^{a_3}$$
$$-\partial X^{a_1}\partial X^{a_2}(\partial^2 X^{a_3}+\partial X^{a_3}P^{(1)}_{\phi-\chi})\}e^{i\vec{p}\vec{X}}$$
$$[Q_3,V_1]$$
$$=W_2=-\frac{1}{4}H_{a_1a_2a_3}(p)e^{3\phi-2\chi}P^{(1)}_{2\phi-2\chi-\sigma}$$
$$\times\partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}} \tag{46}$$

where the conformal weight n polynomials in the derivatives of the ghost fields ϕ,χ,σ are defined according to

$$P^{(n)}_{f(\phi,\chi,\sigma)}=$$
$$e^{-f(\phi,\chi,\sigma)}\frac{\partial^n}{\partial z^n}e^{f(\phi(z),\chi(z),\sigma(z))} \tag{47}$$

where f is some linear function in ϕ,χ,σ. For example, $P^{(1)}_{\phi-\chi}=\partial\phi-\partial\chi$, etc. Note that the product (43) is defined in the algebraic sense (not as an operator product).

Accordingly,

$$
\begin{aligned}
&:K\partial^2 W_1 := 4H_{a_1a_2a_3}(p)c\xi\{(-(\vec{\psi}\partial\vec{X})\\
&+i(\vec{p}\vec{\psi})P^{(1)}_{\phi-\chi}+i(\vec{p}\partial\vec{\psi}))\partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}}\\
&+\partial X^{a_1}(\partial^2\psi^{a_2}+2\partial\psi^{a_2}P^{(1)}_{\phi-\chi})\psi^{a_3}-\\
&\partial X^{a_1}\partial X^{a_2}(\partial^2 X^{a_3}+\partial X^{a_3}P^{(1)}_{\phi-\chi})\}e^{i\vec{p}\vec{X}}\\
&:K\partial^2 W_2 := H_{a_1a_2a_3}(p)\\
&\times\{-\partial^2(e^{\phi}\partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}})\\
&+P^{(2)}_{2\phi-2\chi-\sigma}e^{\phi}\partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}}\}
\end{aligned}
\tag{48}
$$

and

$$
\begin{aligned}
&:\partial^{2n+1}KK\{Q_{brst},U\} :=\\
&-24H_{a_1a_2a_3}(p)\partial cc\partial\xi\xi e^{-\phi}\\
&\times\partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}}\\
&:\partial^m KK\{Q_{brst},U\} := 0
\end{aligned}
$$

$$(m < 2n+1) \tag{49}$$

and therefore, upon integrating out total derivatives, the complete BRST-invariant expression for the $s=3$ operator at picture 1 is

$$\begin{aligned}
A_{s=3}(w) = {} & H_{a_1a_2a_3}(p)\oint dz(z-w)^2 \\
& \times\{\frac{1}{2}P^{(2)}_{2\phi-2\chi-\sigma}e^{\phi}\partial X^{a_1}\partial X^{a_2}\psi^{a_3} \\
& +2c\xi[(-(\vec{\psi}\partial\vec{X})+i(\vec{p}\vec{\psi})P^{(1)}_{\phi-\chi} \\
& +i(\vec{p}\partial\vec{\psi}))\partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}} \\
& +\partial X^{a_1}(\partial^2\psi^{a_2}+2\partial\psi^{a_2}P^{(1)}_{\phi-\chi})\psi^{a_3} \\
& -\partial X^{a_1}\partial X^{a_2}(\partial^2 X^{a_3}+\partial X^{a_3}P^{(1)}_{\phi-\chi})] \\
& -12\partial cc\partial\xi\xi e^{-\phi}\partial X^{a_1}\partial X^{a_2}\psi^{a_3}\}e^{i\vec{p}\vec{X}}
\end{aligned} \tag{50}$$

To abbreviate notations for our calculations of the correlation functions in the following sections, it is convenient to write the vertex operator $A_{s=3}$ as a sum

$$A_{s=3} = A_0 + A_1 + A_2 + A_3 + A_4 + A_5 + A_6 \quad (51)$$

where

$$A_0(w) = \frac{1}{2} H_{a_1 a_2 a_3}(p) \times \oint dz (z-w)^2 P^{(2)}_{2\phi - 2\chi - \sigma} \times e^{\phi} \partial X^{a_1} \partial X^{a_2} \psi^{a_3} e^{i\vec{p}\vec{X}}(z) \quad (52)$$

and

$$A_6(w) = -12 H_{a_1 a_2 a_3}(p) \times \oint dz (z-w)^2 \partial c c \partial \xi \xi e^{-\phi} \times \partial X^{a_1} \partial X^{a_2} \psi^{a_3} \} e^{i\vec{p}\vec{X}}(z) \quad (53)$$

have ghost factors proportional to e^{ϕ} and $\partial cc\partial\xi\xi e^{-\phi}$ respectively and the rest of the terms carry ghost factor proportional to $c\xi$:

$$A_1(w) = -2H_{a_1a_2a_3}(p)\oint dz(z-w)^2 \times c\xi(\vec{\psi}\partial\vec{X}) \times\partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}}(z)$$

$$A_2(w) = 2iH_{a_1a_2a_3}(p) \times\oint dz(z-w)^2 c\xi(\vec{p}\vec{\psi})P^{(1)}_{\phi-\chi} \partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}}(z)$$

$$A_3(w) = 2iH_{a_1a_2a_3}(p) \times\oint dz(z-w)^2 c\xi(\vec{p}\partial\vec{\psi}) \times\partial X^{a_1}\partial X^{a_2}\psi^{a_3}e^{i\vec{p}\vec{X}}(z)$$

$$A_4(w) = 2H_{a_1a_2a_3}(p) \times\oint dz(z-w)^2 c\xi$$

$$(\partial^2\psi^{a_2} + 2\partial\psi^{a_2}P^{(1)}_{\phi-\chi})\psi^{a_3}e^{i\vec{p}\vec{X}}(z)$$

$$A_5(w) = -2H_{a_1a_2a_3}(p)$$
$$\times \oint dz(z-w)^2 c\xi\partial X^{a_1}\partial X^{a_2}(\partial^2 X^{a_3}$$
$$+\partial X^{a_3}P^{(1)}_{\phi-\chi})e^{i\vec{p}\vec{X}}(z) \quad (54)$$

Analogously, the K-operator procedure applied to the $s=4$ vertex operator in (1) leads to the positive picture representation of the $s=4$ operator given by

$$B_{s=4} = B_0+B_1+B_2+B_3+B_4+B_5+B_6 \quad (55)$$

where

$$B_0(w) = \frac{1}{2}H_{a_1a_2a_3a_4}(p)$$
$$\times \oint dz(z-w)^2 P^{(2)}_{2\phi-2\chi-\sigma}\eta e^{2\phi}$$

$$\times\partial X^{a_1}\partial X^{a_2}\partial\psi^{a_3}\psi^{a_4}e^{i\vec{p}\vec{X}}(z) \quad (56)$$

and

$$B_7(w) = -12H_{a_1a_2a_3a_4}(p)$$
$$\times\oint dz(z-w)^2\partial cc\xi$$
$$\times\partial X^{a_1}\partial X^{a_2}\partial\psi^{a_3}\psi^{a_4}e^{i\vec{p}\vec{X}}(z) \quad (57)$$

carry the ghost factors $\sim \eta e^{2\phi}$ and $\sim \partial cc\xi$ respectively, while the rest of the terms carry the ghost factor $\sim ce^{\phi}$:

$$B_1(w) =$$
$$-2H_{a_1a_2a_3a_4}(p)\oint dz(z-w)^2 ce^{\phi}(\vec{\psi}\partial\vec{X})$$
$$\times\partial X^{a_1}\partial X^{a_2}\partial\psi^{a_3}\psi^{a_4}e^{i\vec{p}\vec{X}}(z)$$
$$B_2(w) = 2iH_{a_1a_2a_3a_4}(p)\oint dz(z-w)^2$$
$$\times ce^{\phi}(\vec{p}\partial\vec{\psi})P^{(1)}_{\phi-\chi}\partial X^{a_1}\partial X^{a_2}\partial\psi^{a_3}\psi^{a_4}e^{i\vec{p}\vec{X}}(z)$$
$$B_3(w) = 2iH_{a_1a_2a_3a_4}(p)\oint dz(z-w)^2$$
$$\times ce^{\phi}(\vec{p}\partial\vec{\psi})\partial X^{a_1}\partial X^{a_2}\partial\psi^{a_3}\psi^{a_4}e^{i\vec{p}\vec{X}}(z)$$

$$
\begin{aligned}
B_4(w) &= 2H_{a_1a_2a_3a_4}(p)\oint dz(z-w)^2 \\
&\times P^{(2)}_{\phi-\chi} ce^{\phi}\partial X^{a_1}\partial^2\psi^{a_2}\partial\psi^{a_3}\psi^{a_4}e^{i\vec{p}\vec{X}}(z) \\
B_5(w) &= 2H_{a_1a_2a_3a_4}(p) \\
&\times \oint dz(z-w)^2 ce^{\phi}\partial X^{a_1}\partial X^{a_2}(\frac{1}{2}\partial^3 X^{a_3} \\
&+\partial^2 X^{a_3}P^{(1)}_{\phi-\chi} + \frac{1}{2}\partial X^{a_3}P^{(2)}_{\phi-\chi})\psi^{a_4}e^{i\vec{p}\vec{X}}(z) \\
B_6(w) &= -2H_{a_1a_2a_3a_4}(p) \\
&\times \oint dz(z-w)^2 ce^{\phi}\partial X^{a_1}\partial X^{a_2}(\partial^2 X^{a_3} \\
&+\partial X^{a_3}P^{(1)}_{\phi-\chi})\partial\psi^{a_4}e^{i\vec{p}\vec{X}}(z)
\end{aligned}
\tag{58}
$$

The procedure is totally similar for the operators in (1) with $s \geq 5$ which positive picture representations can be constructed analogously; however, higher ghost number operators generally consist of bigger number of terms, so the manifest expressions for operators with higher n become quite cumbersome.

Gauge-Invariant Interactions of Higher Spin Fields

Here we present the result for the 3-point function describing the cubic gauge-invariant interaction of two $s = 3$ and $s = 4$ particles. In order to satisfy the ghost number anomaly cancellation condition, the overall ϕ-ghost number of the correlator must be equal to -2, $b-c$ ghost number $+3$ and χ-ghost number $+1$. For this reason, two out of 3 vertex operators must be taken at positive picture representations (integrated) and one at negative (unintegrated). Note that the non-standard ghost structure of the h.s. operators leads to deformed

Koba-Nielsen's (SL(2,R)) measure and thus the integrated vertices enter the gam already at the level of 3-point functions, leading to non-localities in the interacting terms. The result of the calculation is

$$
\begin{aligned}
&< V_{s=3}(p_1)V_{s=4}(p_2)V_{s=3}(p_3) > \\
&= \{272\eta^{a_3b_2}\eta^{a_2b_3}\eta^{b_4c_3}T^{a_1|b_1|c_1c_2}_{1,1,2|4}(p_1,p_2,p_3) \\
&+144\eta^{a_3b_2}\eta^{b_3c_2}\eta^{b_4c_3}T^{a_1a_2|b_1|c_1}_{2,1,1|2}(p_1,p_2,p_3) \\
&-128\eta^{a_2b_3}\eta^{a_3c_2}\eta^{b_4c_3}T^{a_1|b_1b_2|c_1}_{1,2,1|2}(p_1,p_2,p_3) \\
&-(16ip_2^{a_3}\eta^{b_3c_2}\eta^{b_4c_3} \\
&+24ip_2^{b_3}\eta^{2_3c_2}\eta^{b_4c_3})T^{a_1a_2|b_1b_2|c_1}_{2,2,1|2}(p_1,p_2,p_3) \\
&-32ip_1^{b_3}\eta^{a_3b_2}\eta^{b_4c_3}T^{a_1a_2|b_1|c_1c_2}_{2,1,2|4}(p_1,p_2,p_3) \\
&+(48ip_1^{c_3}\eta^{a_3b_4}\eta^{a_2b_3} \\
&+72ip_1^{b_3}\eta^{a_2b_4}\eta^{a_3c_3}
\end{aligned}
$$

$$-144ip_2^{a_3}\eta^{a_2b_3}\eta^{b_4c_3})T_{1,2,2|4}^{a_1|b_1b_2|c_1c_2}(p_1,p_2,p_3)$$

$$+((56-20(\vec{p}_1\vec{p}_2))\eta^{a_3b_3}\eta^{b_4c_3}$$

$$-24p_3^{b_3}p_3^{a_3}$$

$$-8p_1^{b_3}p_1^{b_4}\eta^{a_3c_3}$$

$$-20p_1^{b_3}p_1^{c_3}\eta^{a_3b_4})T_{2,2,2|4}^{a_1a_2|b_1b_2|c_1c_2}(p_1,p_2,p_3)\}$$

$$\times I(\vec{p}_1\vec{p}_2)H_{a_1a_2a_3}(p_1)H_{b_1b_2b_3b_4}(p_2)H_{c_1c_2c_3}(p_3)$$

$$\times\delta(p_1+p_2+p_3)$$

$$+\{24\eta^{a_2b_4}\eta^{a_3[b_3}\eta^{b_5]c_3}$$

$$\times T_{1,3,2|4}^{a_1|b_1b_2b_3|c_1c_2}(p_1,p_2,p_3)$$

$$+8\eta^{b_4c_3}(ip_1^{b_3}\eta^{a_3b_5}-ip_1^{b_5}\eta^{a_3b_3})$$

$$T_{2,3,2|4}^{a_1a_2|b_1b_2b_3|c_1c_2}(p_1,p_2,p_3)\}$$

$$\times I(\vec{p}_1\vec{p}_2)H_{a_1a_2a_3}(p_1)H_{b_1b_2b_4b_5}(p_2)H_{c_1c_2c_3}(p_3)$$

$$\times\delta(p_1+p_2+p_3)$$

$$+\{24\eta^{a_3b_3}\eta^{b_4[a_4}\eta^{a_1]c_3}$$

$$\times T^{a_1a_2|b_1b_2|c_1c_2}_{2,2,2|4}(p_1,p_2,p_3)$$

$$+16\eta^{a_3b_3}\eta^{b_4[a_4}\eta^{a_1]c_3}\eta^{b_3c_2}$$

$$T^{a_1a_2a_3|b_1b_2|c_1}_{3,2,1|2}(p_1,p_2,p_3)\}$$

$$\times I(\vec{p}_1\vec{p}_2)H_{a_2a_3a_4}(p_1)H_{b_1b_2b_3b_4}(p_2)H_{c_1c_2c_3}(p_3)$$

$$\times\delta(p_1+p_2+p_3)$$

$$+\eta^{b_5c_3}\eta^{b_3[a_3}\eta^{a_4]b_4}$$

$$\times T^{a_1a_2a_3|b_1b_2b_3|c_1c_2}_{3,3,2|4}(p_1,p_2,p_3)$$

$$\times I(\vec{p}_1\vec{p}_2)H_{a_1a_2a_4}(p_1)H_{b_1b_2b_4b_5}(p_2)H_{c_1c_2c_3}(p_3)$$

$$\times\delta(p_1+p_2+p_3) \tag{59}$$

where

$$I(\vec{p}_1\vec{p}_2) = 4\prod_{n=-3}^{2}\frac{1}{(\vec{p}_1\vec{p}_2)+n} \tag{60}$$