The Attractor Mechanism in Gauged Supergravity

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Perspective

Perspectives on Black Holes in String Theory

- Microscopic entropy: supersymmetric indices, ...
- Black hole quantum information, entanglement, complexity ....
- Spacetime properties of black holes.

Introduction

- Objective: “attractor mechanism” for BPS Black holes in **gauged SUGRA**.
- Review: **the** attractor mechanism for BPS black holes in **ungauged** SUGRA.
- **Obvious** differences between gauged and ungauged SUGRA
  $\Rightarrow$ **attractor mechanism is different**.
- The entropy function revisited.
- Conserved charges as flux integrals.

Research “in progress” with **Marina David**, **Nizar Ezroura**. Supported by DoE.

A Black Hole as a Domain Wall

The radial (and angular) evolution interpolates between

- **Distant** region (Minkowski or AdS).
- **Horizon** region (includes an AdS_2).

Highly symmetric asymptotics in both directions.

Radial dependence:
interpolation between vacua = domain wall.

Setting: Extended Supergravity in Five Dimensions

An instructive **theory**: $\mathcal{N} = 2$ supergravity in $D = 5$ dimensions.

Spectrum:

- $\mathcal{N} = 2$ **gravity**: graviton, graviphoton A^0_μ, gravitini.
- $\mathcal{N} = 2$ **vector**: gauge field A^I_μ, **real scalar** X^I, gaugini.

Index $I = 0, \ldots n_V$ ($n_v + 1$ values) for A^I_μ.

Scalars X^I **projective**: defined modulo $X^I \to \lambda X^I$ with $\lambda \in \mathbb{R}$.

Prepotential:

$$\mathcal{F} = \frac{1}{6} c_{IJK} X^I X^J X^K \underset{\text{gauge}}{=} 1$$

Asymptotics of Black Holes in **Ungauged** Supergravity

Moduli: exactly massless scalars X^I.

The solution far away:
Minkowski and **scalars taking any value** X^I_∞.

Black hole **BPS mass** related to the **central charge** $\mathcal{Z}$

$$\{\mathcal{Q}_\alpha, \mathcal{Q}_\alpha\} = 2P_\mu \Gamma^\mu_{\alpha\beta} - \delta_{\alpha\beta}\mathcal{Z} \quad \Rightarrow \quad M \quad = \quad \mathcal{Z}_{\rm infty} \quad = \quad Q_I X^I_\infty \ .$$

Free parameters of black hole: electric charges Q_I and moduli X^I_∞.

The Attractor Mechanism in Ungauged Supergravity

Attractor mechanism:

- Horizon values $X^I_{\rm hor}$ **independent** of X^I_∞.
- Interpretation: black hole interior **independent of couplings**.

Bonus:

- Simple attractor **flow** relates asymptotic and horizon data.
- Flow controlled by **local** central charge:

$$\mathcal{Z} \;=\; Q_I X^I$$

Highlights of the Ungauged Attractor Flow

- Geometry:

$$ds_5^2 = -f^2 dt^2 + f^{-1}\left(dR^2 + R^2 d\Omega_3^2\right)$$

- SUSY requires **monotonic** central charge "flow":

$$\partial_{R^2}\mathcal{Z} = 2f^{-2}G_{IJ}R^4\partial_{R^2}X^I\partial_{R^2}X^J \geq 0$$

- It is **minimized** at the horizon and black hole entropy is

$$S = 2\pi\left(\frac{1}{3}\mathcal{Z}_{\rm min}\right)^{\frac{3}{2}}$$

- Minimization over **moduli space** yields attractor values $X^I_{\rm hor}$.

- Many generalizations: rotation, curvature corrections, ...

Attractor Flow in Gauged Supergravity?

Standard AdS/CFT correspondence:

- Asymptotic AdS, including fall-off encoding BH quantum numbers: UV behavior.
- Flow: renormalization group repackages QFT data.
- Horizon geometry, including approach encoding horizon potentials: IR behavior.

What is the fate of the attractor mechanism from ungauged supergravity?

The AdS$_5$ Vacuum: Asymptotic Behavior

In **gauged** supergravity scalars X^I experience a **potential**:

$$V = \frac{1}{2} G^{IJ} D_I W D_J W - \frac{2}{3} W^2 \; .$$

FI-gauging a simple set-up for $U(1)^{n_V}$ theory.

The superpotential with **FI parameters** ξ_I:

$$W = \xi_I X^I$$

Extremization conditions:

$$D_I W = \xi_I - \frac{1}{3} X_I (\xi \cdot X) = 0$$

$$\Rightarrow \quad \frac{1}{2} c_{IJK} X^J X^K \; \equiv \; X_I \quad \propto \quad \xi_I$$

No Attractor Mechanism for Gauged Supergravity

Asymptotic values for the scalars **fixed by theory**:

$$X^I_{\text{infty}} = \frac{1}{2} c^{IJK} \xi_J \xi_K$$

AdS_5 scale yields **overall** normalization of FI-parameters:

$$(\frac{1}{6} c^{IJK} \xi_I \xi_J \xi_K)^{\frac{1}{3}} \quad = \quad \ell_5 \quad = \quad g^{-1}$$

Scalar values at the **horizon**: functions of Q_I **and** ξ_I.

There is **no attractor mechanism** in FI-gauged supergravity:

scalars X^I_{hor} **depend on all asymptotic data**, including X^I_∞.

(Near) Horizon Perspective: the Entropy Function

Attractor mechanism and "entropy extremization" have similarities.

Highlights of entropy extremization:

- **Entropy function**: Lagrangian **density** on AdS_2 as function of all continuous parameters.
- **Extremization** determines continuous parameters **at horizon** and black hole entropy.

Important **differences**:

- Starting point: **horizon geometry** (the IR).
- **Extremal** black holes: $AdS_2 \times \ldots$. **SUSY optional**.

The Entropy Function: Details

- Preserve **entire** AdS_2 symmetry.
- The **microcanonical** ensemble (fixed $Q_I, J, \ldots$).

  Include appropriate Wilson lines or Legendre transform.
- **Angular momenta**: preserve symmetries e.g. **KK reduction**.
- Chern-**Simons** terms: add total derivatives so **bulk flux** is conserved.
- **Exact** construction (includes higher curvature, quantum, ...).

Supersymmetric Black Holes in AdS

The entropy function is a great tool. Challenges:

- FI parameters ξ^I are continuous parameters, do we extremize?

  No, they are **couplings**.

- Supersymmetric black holes in AdS **must** rotate.

  More work and want to **avoid** KK reduction 5D $\to$ 4D (new).

- SUSY black holes exist only when charges satisfy **constraint**:

$$\left(Q_I \cdot \frac{1}{2} c^{IJK} \xi_J \xi_K + \frac{\pi}{4G_5}\right)\left[\xi_I \cdot \frac{1}{2} c^{IJK} Q_J Q_K - \frac{\pi}{4G_5} \cdot 2J\right] = g^3 \left(\frac{1}{6} c^{IJK} Q_I Q_J Q_K + \frac{G_5}{4\pi} J^2\right)$$

Goal: extremize entropy function and study attractor flow in 5D.

Ansatz for the Near Horizon Geometry

The near horizon **geometry** (free parameters: $e^{-U_1}, e^{-U_2}, v, e^0$):

$$ds_5^2 = \underbrace{d\mathrm{AdS}_2^2}_{AdS_2 \text{ scale}=v} + \underbrace{e^{-U_1} d\Omega_2^2}_{\text{horizon scale} \sim e^{-U}} + \overbrace{e^{-U_2}}^{\text{squashing}} (\underbrace{\sigma_3 - e^0 R^2 dt}_{e^0 = \text{angular velocity}})^2$$

Matter: **scalars** X^I and **gauge fields** (free parameters: e^I, v^I):

$$A^I = \underbrace{e^I R^2 dt}_{\text{electric field}} + \overbrace{v^I}^{\text{magnetic field}} \underbrace{(\sigma_3 - e^0 R^2 dt)}_{\text{preserves AdS}_2 \text{ symmetry}}$$

Electric fields e^I and angular velocity e^0 potentials for Q_I, J.

Details of the Entropy Function

Entropy function is **near horizon Lagrangian** after Legendre transform to fixed charges:

$$\mathcal{S} = \frac{4\pi^2}{G_5} \cdot e^{-U_1 - \frac{1}{2}U_2} \Big[1 - \frac{v}{4} e^{U_1} + \frac{v}{16} e^{2U_1 - U_2} + \frac{v}{256} e^{2U_1 + 2U_2} (J - Q_I v^I - \frac{2}{3} c_{IJK} v^I v^J v^K)^2 - \frac{v}{4} (\frac{2}{3} W^2 - \frac{1}{2} G^{IJ} D_I W D_J W) + \frac{v}{512} e^{2U_1 + U_2} G^{IJ} (Q_I + 2c_{IKL} v^K v^L)(Q_J + 2c_{JMN} v^M v^N) + \frac{v}{8} e^{2U_1} G_{IJ} v^I v^J \Big] \ .$$

Conserved charges $Q_I = \partial_{e^I} \mathcal{L}$ and $J = \partial_{e^0} \mathcal{L}$.

Input: conserved charges (Q_I, J) and couplings ξ_I (encoded in W).

Output **determined by extremization**:
near horizon geometry v, U_1, U_2, magnetic fields v^I, scalars X^I.

Entropy Extremization: The AdS_2 Scale

Entropy function:

$$\mathcal{S} = \frac{4\pi^2}{G_5} \cdot e^{-U_1 - \frac{1}{2}U_2} \Big[1 - \frac{v}{4} e^{U_1} + \frac{v}{16} e^{2U_1 - U_2} + \frac{v}{256} e^{2U_1 + 2U_2} (J - Q_I v^I - \frac{2}{3} c_{IJK} v^I v^J v^K)^2$$
$$- \frac{v}{4} (\frac{2}{3} W^2 - \frac{1}{2} G^{IJ} D_I W D_J W) + \frac{v}{512} e^{2U_1 + U_2} G^{IJ} (Q_I + 2 c_{IKL} v^K v^L)(Q_J + 2 c_{JMN} v^M v^N)$$
$$+ \frac{v}{8} e^{2U_1} G_{IJ} v^I v^J \Big] \ .$$

Extremization over AdS_2 scale v $\Rightarrow$ **area law** for black hole entropy:

$$\mathcal{S} = \frac{4\pi^2}{G_5} \cdot e^{-U_1 - \frac{1}{2}U_2}$$

The near horizon geometry (reminder):

$$ds_5^2 = d\mathrm{AdS}_2^2 + e^{-U_1} d\Omega_2^2 + e^{-U_2} (\sigma_3 - e^0 R^2 dt)^2$$

Entropy Extremization: Symplectic Vectors

Extremize over magnetic fields v^I and scalars X^I, then reorganize.

Magnetic fields:

$$v^I = \frac{1}{2} \frac{c^{IJK}\xi_J Q_K \left(\frac{\mathcal{S}}{2\pi}\right)^2 - \left(\frac{1}{2}c^{IJK}Q_J Q_K - \frac{1}{2}c^{IJK}\xi_J\xi_K \left(\frac{\mathcal{S}}{2\pi}\right)^2\right) J}{\left(\frac{\mathcal{S}}{2\pi}\right)^2 + J^2}$$

Scalar fields:

$$X^I = \frac{J c^{IJK}\xi_J Q_K + \frac{1}{2}c^{IJK}Q_J Q_K - \frac{1}{2}c^{IJK}\xi_J\xi_K \left(\frac{\mathcal{S}}{2\pi}\right)^2}{\left(\frac{\mathcal{S}}{2\pi}\right)^2 + J^2} \cdot 4e^{-U_1}$$

Note: dependence on FI-couplings ξ_I all over.

Aside on Units and Notation

In gravity, charges and FI-parameters are dimensionful.

The corresponding **quantum numbers**:

$$Q_I = n_I \ell_5$$

$$\tilde{\xi}_I = \xi_I \ell_5$$

The gravitational coupling:

$$\frac{\pi \ell_5^3}{4 G_5} = \frac{1}{2} N^2$$

Extremization: Geometry as Function of Charges

Extremize over horizon scale e^{-U_1} and squashing parameter e^{-U_2}.

The **horizon scale** (in terms of charges and FI-parameters):

$$R_{S^3}^6 = 64 e^{-3U_1} = \ell_5^6 \cdot \frac{\left(\frac{\mathcal{S}}{2\pi}\right)^2 + J^2}{\frac{1}{2}N^2\left(n_I \cdot \frac{1}{2}c^{IJK}\tilde{\xi}_J\tilde{\xi}_K + \frac{1}{2}N^2 + J\right)}$$

The analogous **squashing parameter**:

$$e^{U_1 - U_2} = \frac{\frac{2}{N^2}\left(n_I \cdot \frac{1}{2}c^{IJK}\tilde{\xi}_J\tilde{\xi}_K + \frac{1}{2}N^2 + J\right)\left(\frac{\mathcal{S}}{2\pi}\right)^2}{\left(\frac{\mathcal{S}}{2\pi}\right)^2 + J^2} \ .$$

The **AdS_2 scale** (relative to the horizon scale):

$$\frac{v}{4}e^{U_1} = \frac{\frac{1}{2}N^2\left(n_I \cdot \frac{1}{2}c^{IJK}\tilde{\xi}_J\tilde{\xi}_K + \frac{1}{2}N^2 + J\right)}{\left(\frac{\mathcal{S}}{2\pi}\right)^2 + \left(n_I \cdot \frac{1}{2}c^{IJK}\tilde{\xi}_J\tilde{\xi}_K + \frac{1}{2}N^2\right)^2}$$

Collecting Results: Black Hole Entropy

From geometry, entropy as function of charges **and** FI-parameters:

$$\boxed{\mathcal{S} = 2\pi\sqrt{\tilde{\xi}_I \cdot \frac{1}{2}c^{IJK}n_J n_K - \frac{1}{2}N^2 \cdot 2J}}$$

Supersymmetric black holes only possible when charges satisfy:

$$\frac{1}{6}c^{IJK}Q_I Q_J Q_K + \frac{1}{2}N^2 J^2 = (Q_I \frac{1}{2}c^{IJK}\xi_J\xi_K + \frac{1}{2}N^2)\left(\frac{\mathcal{S}}{2\pi}\right)^2 ,$$

Status

- Extremization of the entropy function is **principled**.
- Output: **entropy** as function of charges and FI-parameters. Generalizes to **corrections** (curvature, quantum, ...).
- But: computation (as presented) is **un-illuminating**.

Disclosure: complex variables makes formulae smoother.

However, objective is improved **spacetime** understanding.

BPS Thermodynamics

BPS mass:

$$M_* = \mathcal{Z} = \Phi^I_* Q_I + 2\Omega_* J$$

The first law **near** extremality:

$$TdS = dM + \Phi^I dQ_I + 2\Omega dJ = d(M - M_*) + (\Phi^I - \Phi^I_*) dQ_I + 2(\Omega - \Omega_*) dJ$$

The first law **at** extremality: entropy depends on charges so that

$$dS = e^I dQ_I + 2e^0 dJ$$

Electric fields e^I, e^0 are **thermal** derivatives:

$$e^I = \lim_{T\to 0} \frac{\Phi^I - \Phi^I_*}{T} \qquad e^0 = \lim_{T\to 0} \frac{\Omega - \Omega_*}{T}$$

Electric Fields in Spacetime

Electric fields e^I, e^0 are **dual** to charges Q, J.

$$dS = e^I dQ_I + 2e^0 dJ$$

Electric fields e^I, e^0 are **radial** derivatives of potentials

$$A^I = \underbrace{e^I \; R^2 \; dt}_{\text{electric field}} \; + \; v^I(\sigma_3 - \underbrace{e^0 \; R^2 \; dt}_{\text{velocity field}})$$

Interpretation: a **physical** temperature **near** the horizon

Radial derivatives and thermal derivatives are **equivalent**.

Towards an Attractor Flow

- So far: all computations **at** horizon.
- Now: return to the attractor **flow**.
- First: generalize effective Lagrangean to **any radial position**.
- Result "only" a little more complicated than entropy function at the horizon.
- But: there are derivatives.

Conserved Currents

- The next step: **conserved charges**.
- Nearly all explicit computations in GR and AdS/CFT correspondence: **asymptotic** expressions.
- The Noether-Wald procedure: conserved charges **anywhere** along flow.
- Variation of Lagrangian:

$$\delta\mathcal{L} = \delta\Phi_i \left[\frac{\partial\mathcal{L}}{\partial\Phi_i} - \partial_\mu \left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\Phi_i} \right) \right] + d\Theta[\Phi_i, \delta\Phi_i] = dJ_\zeta$$

- Presymplectic potential

$$\Theta^\mu = \delta\Phi_i \frac{\delta\mathcal{L}}{\delta\partial_\mu\Phi_i}$$

Conserved Charges

- Careful: the Chern-Simons terms.
- Applications:
  - Gauge symmetry (electric charge)
  - Killing symmetry (angular momentum).
- Results for **radially independent** charges:

$$Q_I = 2Rf^{-1}e^{-U_2}G_{IJ}\left(\partial_R(fX^J) + f^2 we^{U_2}\partial_R b^J\right) - 2c_{IJK}b^J b^K$$
$$J = -2Rf^{-1}e^{-U_2}\partial_R(f^2 we^{U_2}) + Q_I b^I + \frac{2}{3}c_{IJK}b^I b^J b^K$$

- Physical interpretation: Q_I, J are **flux integrals** on any surface surrounding black hole.

Supersymmetry Conditions

Solution Killing spinor equations gives **flow equations**:

$$\left(\partial_{R^2} + \frac{1}{R^2}\right) u^I = \frac{1}{2}\epsilon c^{IJK}(f^{-1}X_J)\xi_K$$

$$\left(\partial_{R^2} + \frac{2}{R^2}\right)(g_m - 1) = \frac{2}{R^2}\epsilon\xi_I u^I$$

$$\left(\partial_{R^2} - \frac{2}{R^2}\right) w = -\frac{1}{2}f^{-1}X_I\left(\partial_{R^2} - \frac{2}{R^2}\right) u^I$$

$$R^4\partial_{R^2}(f^{-1}X_I) = -\frac{1}{g_m}(Q_I + 2c_{IJK}u^J u^K + 2\epsilon w R^2\xi_I)$$

Supersymmetry **insufficient** to solve equations.

Need some **equations of motion**.

The Gauss's law (conservation law) sufficient.

The Routhian and Its Supersymmetric Solutions

- The Routhian: the Lagrangian (anywhere along the flow) with **electric fields** eliminated in favor of conserved charges.
- **Supersymmetric flow equations** guarantee extremization.
- Working hypothesis: such solutions are **gradient flows**.
- Work is still in progress.