A proposal for 3d quantum gravity & its bulk factorisation

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based on 2210.14196 in collaboration with
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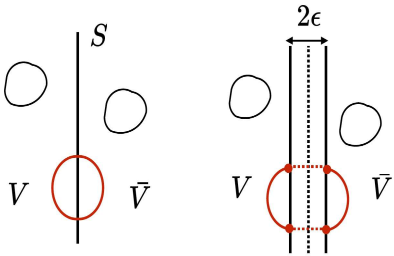
Holographic entanglement entropy formula

$$S_{\mathrm{CFT}} = S_{\text{gen}} = \frac{\mathrm{A}(\gamma)}{4G_{\mathrm{N}}} + S_{\text{bulk}}$$

- Bulk microscopic interpretation of the area term ?
- Is black hole entropy = gravitational entanglement entropy ?
- Why are euclidean gravity path integrals so effective ?
- Expectation : gravity *regularises* entanglement entropy
  - S_{gen} = entanglement entropy of bulk quantum gravity
  - If holographic principle holds, it should be *finite*
- What glues spacetime ? Entanglement, but
  - how do we factorise the bulk Hilbert space ?
  - Bulk diffeomorphism invariance ⇒ no local degrees of freedom

Factorization of Wilson loops



Factorization $\sim$ embedding

$$i : \mathcal{H}_{\text{physical}} \to \mathcal{H}_{\mathrm{V}} \otimes \mathcal{H}_{\overline{\mathrm{V}}}$$

$\mathcal{H}_{\mathrm{V}} \supset$ entanglement edge modes charged under G_{S} (large gauge trafos)
$\mathcal{H}_{\text{physical}} \sim$ projection to subspace invariant under G_{S}

$$\mathcal{H}_{\text{physical}} = \mathcal{H}_{\mathrm{V}} \otimes_{\mathrm{G}_{\mathrm{S}}} \mathcal{H}_{\overline{\mathrm{V}}}$$

Factorization of Wilson loops

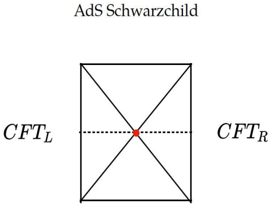
$\mathcal{H}_{\mathrm{V}} \otimes \mathcal{H}_{\overline{\mathrm{V}}}$ allows to define a reduced density matrix

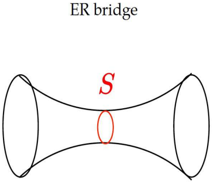
$$\rho_{\mathrm{V}} = \mathrm{tr}_{\overline{\mathrm{V}}} |\psi\rangle\langle\psi|$$
$$S_{\mathrm{V}} = -\mathrm{tr}\rho_{\mathrm{V}} \log \rho_{\mathrm{V}} = S_{\text{bulk}} + S_{\text{edge}}$$

Questions

- Can we use these methods in gravity ? Analogies, differences ?
- If gravitational edge modes are relevant, their existence is independent from $\exists$ gravity propagating dof
  - consider JT or 3d gravity ?

Bulk gauge theory factorization problem (Harlow)





Boundary CFT factorizes, but bulk Wilson lines do not naively do so

Bulk charges must exist allowing the split of the Wilson line into gauge invariant operators

In the low energy EFT, these are entanglement edge modes

$$S_{\text{edge}} \sim \log \dim \text{a}$$

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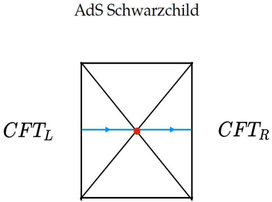

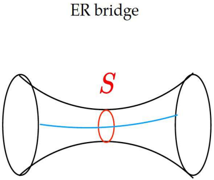

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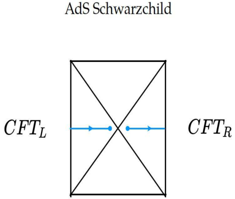

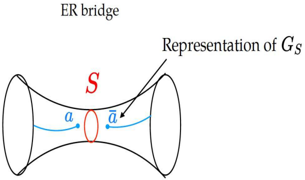

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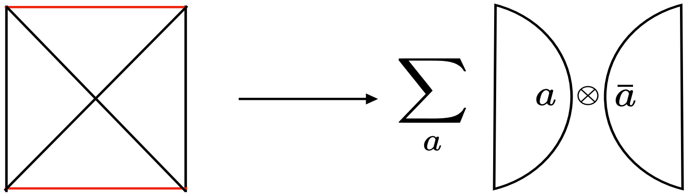
The factorization problem in bulk quantum gravity



Perhaps holographic entanglement entropy is the entanglement entropy of quantum gravity edge modes gluing spacetime together [Lin; Harlow; Donnelly,Freidel; Donnelly, Wong;...]

Questions

- Which factorisation maps ? Any relevant constraints ?
- What determines G_S & its spectrum of representations ?

JT and 3d gravity : lack of factorization

1. In JT gravity [Harlow & Jafferis]

$$\omega_{\mathrm{JT}} = d L \wedge dP \,, \quad H_{\mathrm{JT}} = \frac{P^2}{2\phi_{\mathsf{b}}} + \frac{2}{\phi_{\mathsf{b}}}\, e^{-L}$$

   - L $\sim$ regularised geodesic length connecting both boundaries [lack of factorization]

2. In 3d gravity, perturbative quantisation around eternal BTZ BH [Cotler & Jensen; Henneaux, Merbis & Ranjbar]

   - the radial Wilson line $\mathcal{C} \equiv \mathcal{P}\exp\left[-\int_{\mathrm{L}}^{\mathrm{R}} A_r^{+}(\varphi = 0, r)\, dr\right]$ links the holonomy on the two asymptotic boundaries

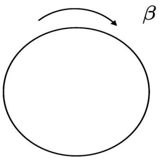
$$\mathcal{P}\exp\left[-\oint_{\mathrm{R}} (L_{-} + \mathcal{L}^{+}(\varphi)\, L_{+})\, d\varphi\right] = \mathcal{C}\, \mathcal{P}\exp\left[-\oint_{\mathrm{L}} (L_{+} + \mathcal{M}^{+}(\varphi)\, L_{-})\, d\varphi\right] \mathcal{C}^{-1}$$

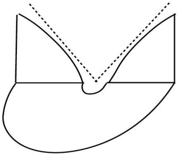
3. Banados, Teitelboim & Zanelli extended the existence of such quantum mechanical conjugate pair responsible for the lack of factorization of the two-sided BH in arbitrary dimensions

Factorization as a path integral

Idea : *locality* should constrain factorisation map i

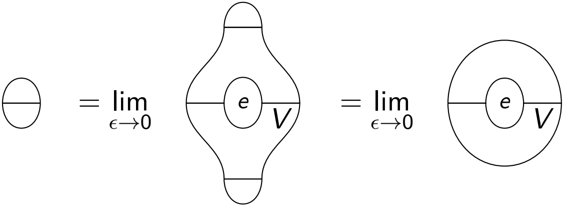
- Define $i \sim$ euclidean path integral





introducing an *stretched* entangling surface S_ϵ

- Set *shrinkable* boundary conditions at S_ϵ [Donnelly, Wong]



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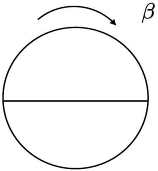

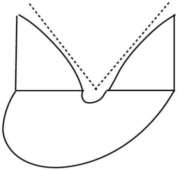

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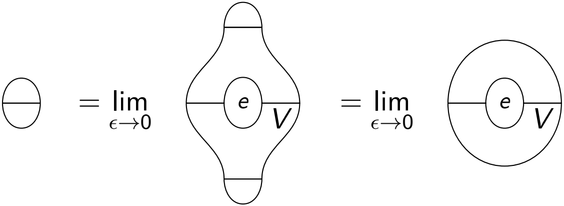

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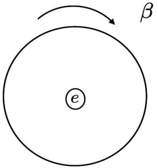

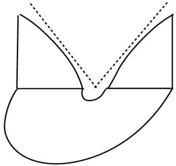

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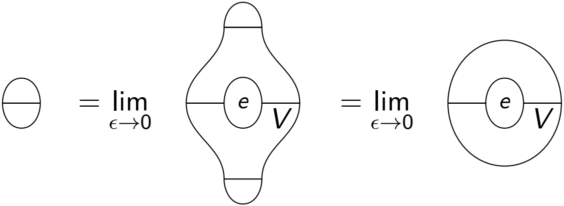

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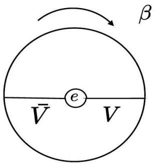

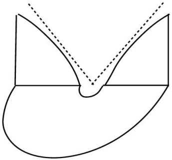

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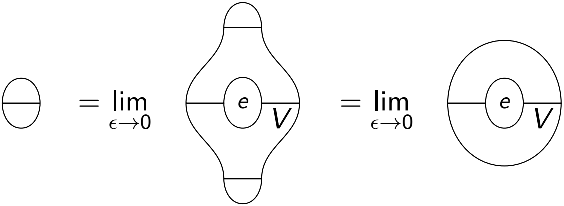

More pragmatic approach

- Given success of JT gravity : $\exists$ an analogue of JT/Schwarzian action for 3d gravity (without any UV completion) ?
- Classical AdS_3 gravity = Chern-Simons theory with $\mathrm{PSL}(2,\mathbb{R}) \times \mathrm{PSL}(2,\mathbb{R})$ gauge group
  - Entanglement entropy is understood in Chern-Simons theory
  - Topological entanglement entropy = EE of anyon edge modes
  - Anyons are collective dof described by a TQFT associated with a modular tensor category Rep(LG) or $\mathrm{Rep}(\mathrm{U_q}(G))$
- Can gravitational anyons provide an explanation for bulk factorization and black hole entropy in 3d gravity ?
- Is there a bulk TQFT ?

3d gravity as a topological phase

McGough & Verlinde

- 3d gravity is a topological phase
- BTZ (M,J) entropy = "topological EE"

$$\boxed{\frac{\mathrm{A}(M,J)}{4G_{\mathrm{N}}} = \log S_0^a \leftarrow \text{ Virasoro S-matrix}}$$

Puzzles

1. Standard CS edge modes give

$$S_{\mathrm{EE}} = \frac{\text{"Area"}}{\epsilon} + \log S_a^0 \leftarrow S_{\mathrm{edge}} = \text{topological EE}$$

   But $S_a^0 = 0$ for the Virasoro S-matrix
2. BH entropy is finite

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Puzzles ⇒ gravity must modify CS calculation, how ?

1. Standard CS edge modes give

$$S_{\mathrm{EE}} = \frac{\text{"Area"}}{\epsilon} + \log S_a^0 \leftarrow S_{\mathrm{edge}} = \text{topological EE}$$

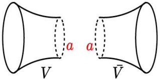
   But $S_a^0 = 0$ for the Virasoro S-matrix

2. BH entropy is finite

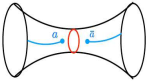
Takeaway message

1. Propose an effective 3d quantum gravity $\sim$ theory of "*vacuum Virasoro blocks in the dual channel*"
2. Propose bulk theory $\sim$ extended TQFT associated to the representation category of $\mathrm{SL}^+_{\mathsf{q}}(2,\mathbb{R}) \otimes \mathrm{SL}^+_{\mathsf{q}}(2,\mathbb{R})$

$a \in \mathrm{SL}^+_q(2,\mathbb{R}) \otimes \mathrm{SL}^+_q(2,\mathbb{R})$



Gravity



Gauge theory

$a \in \mathrm{PSL}(2,\mathbb{R}) \otimes \mathrm{PSL}(2,\mathbb{R})$

3. Bulk edge modes (anyons) determined by the *shrinkable b.c.* are localised on the entangling surface (event horizon)
   - density of edge mode states $=$ Plancherel measure for $\mathrm{SL}^+_{\mathsf{q}}(2,\mathbb{R})$
4. Contrary to CS, no descendants exist at the entangling surface $\Rightarrow$ *finite* EE

Outline

Part 1 : Proposal for an effective 3d gravity theory

- "Universal" high temperature description of a parent AdS_3/CFT_2
- Main features

Part 2 : Bulk factorization

- Bulk Hilbert space
- Shrinkable boundary condition
- $\mathrm{SL}^+_{\mathsf{q}}(2,\mathbb{R})$ and extended Hilbert space factorization

Parent AdS_3/CFT_2

Modular invariant 2d irrational CFT with torus partition function

$$Z(\tau) = \sum_{h,\bar{h}} M_{h,\bar{h}}\, \chi_h(\tau)\chi_{\bar{h}}(\bar{\tau}) = \sum_{h,\bar{h}} M_{h,\bar{h}}\, \chi_h(-1/\tau)\chi_{\bar{h}}(-1/\bar{\tau})$$

Virasoro characters

$$\chi_0(\tau) = \frac{(1-q)}{\eta(\tau)} q^{-\frac{c-1}{24}}, \qquad \chi_h(\tau) = \frac{1}{\eta(\tau)} q^{h-\frac{c-1}{24}}$$

Modular parameters & central charge

$$q \equiv e^{2\pi i\tau} = e^{\frac{\beta}{\ell}(i\mu-1)}, \qquad \bar{q} \equiv e^{-2\pi i\bar{\tau}} = e^{-\frac{\beta}{\ell}(i\mu+1)}$$

$$c = \frac{3\ell}{2G_{\rm N}}$$

Universal high T 2d CFT

- 2d irrational CFT with sufficiently sparsed low-energy spectrum
- High temperature $\left[\tilde{q} \equiv e^{-2\pi i/\tau} = e^{-4\pi^2 \frac{\ell}{\beta} \frac{(\mu+i)}{(\mu^2+1)}}\right]$

$$\beta/\ell \ll \Delta_{\text{gap}}, \quad \text{with} \quad \Delta_{\text{gap}} \equiv \min\left\{\Delta = h + \bar{h}\right\}$$

$$\Rightarrow \quad \frac{\chi_h(-1/\tau)\chi_{\bar{h}}(-1/\bar{\tau})}{\chi_0(-1/\tau)\chi_0(-1/\bar{\tau})} = \frac{1}{(1-\tilde{q})(1-\bar{\tilde{q}})}\tilde{q}^h\bar{\tilde{q}}^{\bar{h}} \to 0$$

$$\Rightarrow \quad Z(\tau) \approx |\chi_0(-1/\tau)|^2$$

Our proposal

Define an effective theory by truncating to the vacuum block in the dual channel

$$\boxed{Z_{3\text{d}}(\tau,\bar{\tau}) \equiv |\chi_0(-1/\tau)|^2}$$

Grand-canonical interpretation

Using the Virasoro modular S-matrices

$$Z_{3d}(\tau,\bar\tau) = |\chi_0(-1/\tau)|^2 = \sum_{p_+,p_-} {S_0}^{p_+}{S_0}^{p_-}\chi_{p_+}(\tau)\chi_{p_-}(\bar\tau)$$

our proposal has a grand canonical partition function interpretation

$$\begin{aligned} Z(\beta,\mu) &\equiv \mathrm{Tr}\left[e^{-\beta H + i\mu\frac{\beta}{\ell}J}\right] \\ &= \int_0^\infty\int_0^\infty dp_+ dp_-\, \frac{{S_0}^{p_+}{S_0}^{p_-}}{|\eta(\tau)|^2}\, e^{-\frac{\beta}{\ell}(p_+^2+p_-^2)}e^{i\mu\frac{\beta}{\ell}(p_+^2-p_-^2)} \\ S_0^{p_\pm} &= \sqrt{2}\sinh(2\pi b p_\pm)\sinh(2\pi b^{-1}p_\pm) \end{aligned}$$

where we used Liouville notation (though our theory is NOT)

$$h = p_+^2 + \frac{Q^2}{4}\,,\quad \bar h = p_-^2 + \frac{Q^2}{4}\,,\quad Q = b + b^{-1}\,,\quad c = 1 + 6Q^2$$

Grand-canonical interpretation

Remark 1: S_0^p is the quantum dimension of $\mathrm{SL}_\mathrm{q}^+(2,\mathbb{R})$ in the representation p with $q = e^{i\pi b^2}$

$$S_0^p = \dim_\mathrm{q} p \quad \left(\&\ S_0^{\bar{p}} = \dim_\mathrm{q} \bar{p}\right)$$

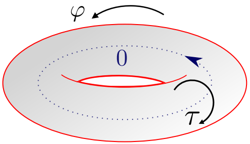
Remark 2: High T and large c $\Rightarrow S_0^p \sim e^{2\pi b p} = \exp\left(\sqrt{\frac{cL_0}{6}}\right)$

$$\boxed{S = (1-\beta\partial_\beta) Z_{3\mathrm{d}}(\tau,\bar{\tau}) \to \log S_0^{p_\star} S_0^{\bar{p}_\star} = \frac{\mathrm{Area}(M^\star, J^\star)}{4G_\mathrm{N}}}$$

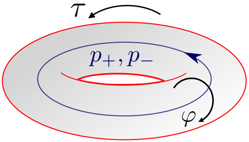
where $M^\star \ell = (p^\star)^2 + (\bar{p}^\star)^2$ and $J^\star = (p^\star)^2 - (\bar{p}^\star)^2$

- This explains McGough & Verlinde's observation
  - it does not have an entanglement interpretation

Trace interpretation in the dual channel

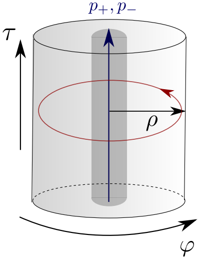


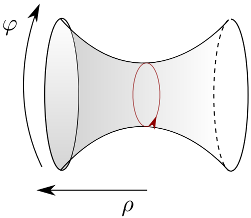
$$= \sum_{p_+,p_-} {S_0}^{p_+}\, {S_0}^{p_-}$$



Interpretation : Off-shell black holes, with a non-trivial measure ${S_0}^{p_+}\,{S_0}^{p_-}$ together with a thermal bath of boundary gravitons for fixed $p_\pm$

- On a solid cylinder, ∃ unique classical gravity solution with hyperbolic monodromies (p_+, p_-)





Observation 3

- Its spatial slice

$$ds^2_{\text{spatial}} = R^2 \frac{d\rho^2 + d\varphi^2}{\cos^2(R\rho/\ell)}, \qquad R^2 = 8H\ell^2,$$

- For fixed $p_\pm$, one finds a thermal partition function of boundary gravitons

$$\frac{1}{\eta(\tau)q^{-1/24}} = \frac{1}{\prod_{m=1}^{\infty}(1-q^m)} = \sum_{n=0}^{+\infty} p(n)q^n$$

$$p(n) = \#\,\text{partitions of } n$$

Low temperature remark

- $Z_{\rm JT} \sim$ universal near-extremal sector 2d irrational CFTs [Ghosh, Maxfield, Turiaci]
- Double-scaling regime : $c \gg 1, \qquad \beta/\ell \sim c$

$$Z(\beta,\mu) \overset{\beta\gg\ell}{\approx} (2\pi b^2)^2 \left(\prod_{\pm} \int_0^\infty dp_\pm \, p_\pm \sinh(2\pi p_\pm) \, e^{-\frac{b^2\beta}{\ell} p_\pm^2 (1\pm i\mu)} \right)$$

$$= (2\pi^3 b^2)^2 Z_{\rm JT}\left(\frac{b^2\beta}{\ell}(1+i\mu)\right) Z_{\rm JT}\left(\frac{b^2\beta}{\ell}(1-i\mu)\right)$$

since μ is arbitrary

- requires $\Delta_{\rm gap} \gtrsim \beta(1+\mu^2)/\ell$, which only holds numerically from a microscopic perspective, since $\Delta_{\rm gap} \leq c/12$, or holds for arbitrary low temperatures in a 3d effective gravity theory with no matter.

Geometric actions

- Cotler & Jensen identified Alekseev-Shatashvili geometric actions $\sim$ fluctuations around a given background
  - Diff S^1 reparametrization $\phi(\tau,\varphi)$ satisfying

$$\phi(\tau,\varphi+2\pi)=\phi(\tau,\varphi)+2\pi\,,\qquad \partial_\varphi\phi\geq 0 \quad \text{contractible spatial cycle}$$

- Swap gauge connection b.c. $\Leftrightarrow$ same action, using a time reparameterisation

$$\begin{cases} f(\tau+2\pi,\sigma) & = f(\tau,\sigma)+2\pi, \\ f(\tau+2\pi\Re(\tau),\sigma+2\pi\Im(\tau)) & = f(\tau,\sigma) \end{cases}$$

for both chiral $f_{L,R}$ satisfying $\dot{f}_{L,R}\geq 0$ modulo independent $\mathrm{SL}(2,\mathbb{R})$ Möbius transformations

  - our proposal satisfies same properties : one-loop exact, single saddle, ...

Further features

- 1st order formulation
  - A_τ trivial monodromy , A_φ arbitrary monodromy
  - $\Rightarrow$ allows to include arbitrary defects
  - $\Rightarrow$ $Z_{3d}(\tau, \bar{\tau})$ computes a gravity partition function
- NOT modular invariant
  - besides fixing a boundary torus, we specify time and space cycles
  - $\exists$ unique saddle
- Global $AdS_3 \not\exists \mathcal{H}_{3d}$ (in our proposal)
  - just as JT/Schwarzian has extremal Poincaré as vacuum
  - our partition function factorises

$$\text{3d pure gravity } = \text{chiral CFT}_{\rm L} \otimes \text{chiral CFT}_{\rm R}$$

Part 2 : Bulk factorization

Part 1 : Proposal for an effective 3d gravity theory

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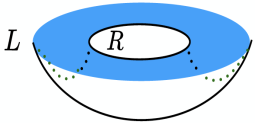
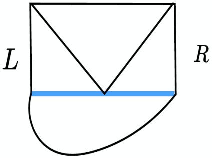
The two-sided bulk Hilbert space





Asymptotic AdS_3 b.c. $\Rightarrow$ Kac-Moody (WZW) $\rightarrow$ Virasoro symmetry

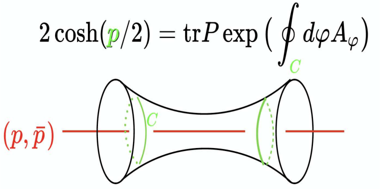
$$A_\varphi = A_\tau = \begin{pmatrix} 0 & \mathcal{L}(\tau,\varphi) \\ 1 & 0 \end{pmatrix} \quad \& \quad \text{2nd chiral sector } \bar{A}_\varphi = \bar{A}_\tau$$

$\exists$ 4 stress tensor components $\mathcal{L}_{\mathrm{L/R}}(\tau,\varphi)$, $\bar{\mathcal{L}}_{\mathrm{L/R}}(\tau,\varphi)$, sharing the stress tensor zero-mode

$$\tfrac{1}{2\pi}\oint d\varphi\,\mathcal{L}^{+}_{\mathrm{L}} = \tfrac{1}{2\pi}\oint d\varphi\,\mathcal{L}^{+}_{\mathrm{R}} = \tfrac{1}{2}(M\ell+J)\,, \qquad \tfrac{1}{2\pi}\oint d\varphi\,\bar{\mathcal{L}}_{\mathrm{L}} = \tfrac{1}{2\pi}\oint d\varphi\,\bar{\mathcal{L}}_{\mathrm{R}} = \tfrac{1}{2}(M\ell-J)$$

The two-sided bulk Hilbert space

Equivalently, the holonomy around φ detects the presence of a wormhole threading Wilson line, parameterized by the $(p, \bar{p})$ black hole quantum numbers



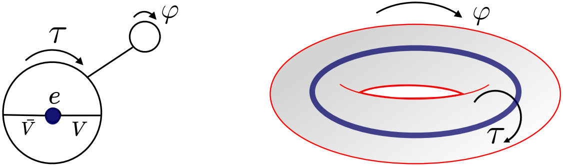
Bulk Hilbert space of Virasoro Representations

$$\mathcal{H}_{\text{bulk}} = \mathcal{H} \otimes \bar{\mathcal{H}}$$
$$\mathcal{H} \equiv \oplus_p V_{\mathsf{p}}^{\mathrm{L}} \otimes V_{\mathsf{p}}^{\star\mathrm{R}}$$
$$V_{\mathsf{p}}^{\mathrm{L}} = \text{span}\left\{|p, \mathfrak{i}_{\mathrm{L}} : m_{\mathrm{L}}\rangle\,,\; m_{\mathrm{L}} \text{ descendant label}\right\}$$

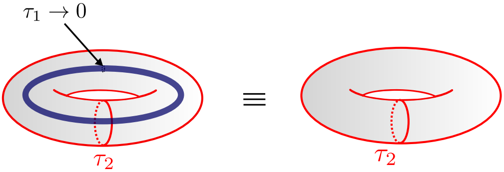
$\mathfrak{i}_{\mathrm{L}}$ labels the vector in the zero mode Kac-Moody (degenerate) subspace [projection by AdS_3 b.c.]

Towards 3d bulk gravity factorization

- Introduce an stretched entangling surface

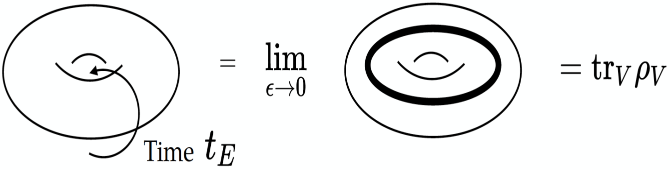


- Impose the *shrinkable* b.c. $\left[\tau_n = \frac{\beta_n}{2\pi\ell}(\mu_n + i)\right]$

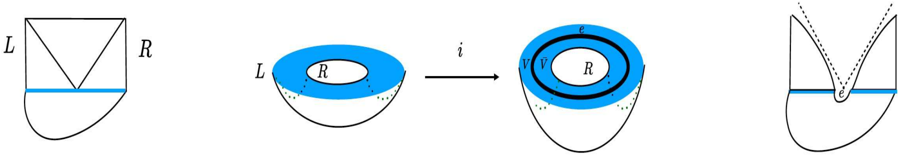


Towards 3d bulk gravity factorization

To get a bulk trace interpretation in the original channel, apply the shrinkable boundary condition

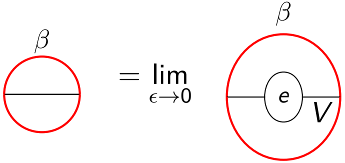


within the extended Hilbert space that will provide a bulk factorization map

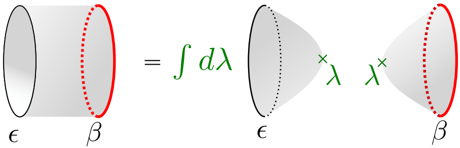


Spelling the shrinkable b.c. in JT gravity

1. $Z_{\text{disk}}(\beta)$ equals the $\epsilon \to 0$ limit of the full annulus



2. ϵ finite, annulus $\sim$ two boundary amplitude ("closed string" channel)



Equivalently,

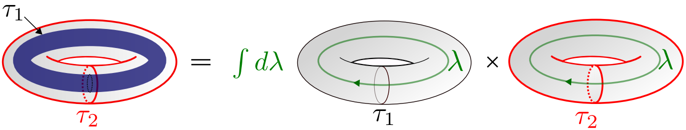
$$Z(\epsilon, \beta) = \int d\lambda \, Z_{\text{inner}}(\epsilon, \lambda) \, Z_{\text{outer}}(\beta, \lambda) \, .$$

after inserting $\mathbf{1} = \int d\lambda \, |\lambda\rangle \, \langle\lambda| \sim$ complete set of defect insertions

Spelling the shrinkable b.c. out

In 3d, the annulus $\mathcal{A} \to \mathcal{A} \times S_1 \equiv \mathbb{T}^2 \times \mathrm{I}$

- Path integral $\sim$ amplitude between inner (entangling surface) & outer (holographic) boundary
- Insert $\mathbf{1} = \int d\lambda \, |\lambda\rangle \langle\lambda|$ $\sim$ inserting Wilson loops labelled by λ



$$Z(\tau_2) = \int d\lambda \, Z_{\text{inner}}(\tau_1 \to 0, \lambda) \, Z_{\text{outer}}(\tau_2, \lambda)$$

$$\stackrel{!}{=} \frac{1}{\eta(\tau_2)} \int dp \, \sinh(2\pi b p) \sinh(2\pi p \, b^{-1}) \, e^{-\beta_2 p^2}$$

Solving the shrinkable b.c.

- Outer boundary satisfies coset boundary conditions

$$Z_{\text{outer}}(\tau_2,\lambda) = \chi^{\text{Vir}}_{\lambda}\left(-\frac{1}{\tau_2}\right) = \frac{1}{\eta(\tau_2)}\int_0^{+\infty} dp\, \cos(2\pi\lambda p)\, e^{-\beta_2 p^2}$$

$\sim$ a Wilson loop insertion in the interior of the solid torus

- Inner boundary is an entanglement surface

Edge modes as in CS

$\exists$ Kac-Moody edge modes $\Rightarrow$ Kac-Moody character of $\widehat{\mathrm{SL}(2,\mathbb{R})}$

$$Z_{\text{inner}}(\tau_1,\lambda) \stackrel{?}{=} \chi_{\hat{\lambda}}(-1/\tau_1) = \int dp\, \cos(2\pi\lambda p)\, \chi_{\hat{p}}(\tau_1)$$

where $\chi_{\hat{p}}(\tau_1) \sim 1/\eta(\tau_1)^3$.

- this choice does *not* satisfy the shrinkable b.c.
- $Z_{\text{inner}} \to \infty$ as $\tau_1 \to 0$ due to degeneracy of descendants localised at entangling surface

Solving the shrinkable b.c.

Requiring shrinkable b.c. determines

$$Z_{\text{inner}}(\tau_1,\lambda) = \int dp\, \sinh(2\pi bp) \sinh(2\pi p\, b^{-1}) \cos(2\pi\lambda p)\, e^{-\beta_1 p^2}$$

Interpretation

- Density of states $\dim_q(p) = \sinh(2\pi bp) \sinh(2\pi b^{-1} p)$ counts edge modes living on the bulk entangling surface as $\beta_1 = \epsilon \to 0$
- $\dim_q(p)$ is Plancherel measure on a *quantum group* $\mathrm{SL}_q^+(2,\mathbb{R})$
- $\exists$ connection with Ponsot & Teschner work (later)

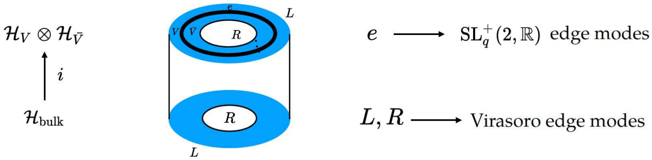
Physics recap : Gauge theory vs 3d gravity

Gauge theory (CS) and 3d gravity measures are different

- Key : shrinkable b.c. excludes bulk geometries with conical defects in the euclidean time direction
- Gauge theory sums over these defects with gauge group $\mathrm{PSL}(2,\mathbb{R}) \otimes \mathrm{PSL}(2,\mathbb{R})$
- Physically, the absence of descendants suggests BTZ entropy $\sim$ topological entanglement entropy [McGough, Verlinde]

The shrinkable route to factorization

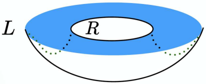
A more abstract perspective on the previous calculation



Question : Given a Hartle-Hawking state compatible with $Z_{3\text{d}}$, can we define a factorization map compatible with the bulk trace interpretation and acknowledging the existence of $\mathrm{G}_\mathrm{S} = \mathrm{SL}_\mathrm{q}^+(2,\mathbb{R}) \otimes \mathrm{SL}_\mathrm{q}^+(2,\mathbb{R})$ acting on the gravitational edge modes ?

Bulk Hartle-Hawking state

The bulk Hartle-Hawking state whose norm gives $Z_{3\text{d}}(\tau, \bar{\tau})$



$$= \quad |\mathrm{HH}_{\beta,\mu}\rangle \otimes \overline{|\mathrm{HH}_{\beta,\mu}\rangle}$$

$$|\mathrm{HH}_{\beta,\mu}\rangle = \int_0^{+\infty} dp \sqrt{\dim_q(p)}\, e^{-\frac{\beta}{\ell}p^2(1-i\mu)} \sum_{m_L=m_R} q^{N/2} |p\, m_L\, m_R\rangle$$

Level

where

$$|p\, m_{\mathrm{L}} m_{\mathrm{R}}\rangle \equiv |p\, \mathfrak{i}_{\mathrm{L}} : m_{\mathrm{L}}\rangle \otimes |p\, \mathfrak{i}_{\mathrm{R}} : m_{\mathrm{R}}\rangle$$

An ansatz for the factorization map

Define subregion states

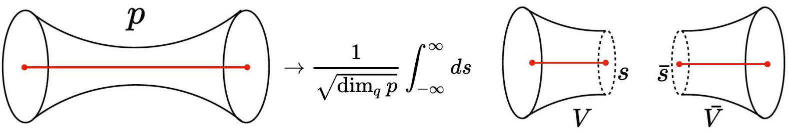
$$|p\, i_{\rm L}\, s : m_{\rm L}\rangle \quad s \in \mathbb{R} \quad p \in \mathbb{R}^+$$

whose projector satisfies the trace relation

$$\mathrm{Tr}_{\rm V}\left(\int_{-\infty}^{\infty} ds\, |p\, i_{\rm L}\, s : m_{\rm L}\rangle\langle p\, i_{\rm L}\, s : m_{\rm L}|\right) = \dim_{\rm q} p$$

The factorization map is the co-product in $\mathrm{SL}^+_{\rm q}(2,\mathbb{R})$ (in each chiral sector)

$$i : |p i_{\rm L} : m_{\rm L}\rangle \otimes |p i_{\rm R} : m_{\rm R}\rangle \to \frac{1}{\sqrt{\dim_{\rm q} p}} \int_{-\infty}^{\infty} ds\, |p i_{\rm L} s : m_{\rm L}\rangle_{\rm V} \otimes |p i_{\rm R} \bar{s} : m_{\rm R}\rangle_{\bar{\rm V}}$$



Black hole state

Entangled Subregion states

What is $\mathrm{SL}_{\mathbf{q}}^{+}(2,\mathbb{R})$?

Definition 1.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad a,b,c,d = \text{operators on } L^2(\mathbb{R}\otimes\mathbb{R}) \text{ with positive spectrum}$$

$$ab = q^{1/2}ba, \quad ac = q^{1/2}ca \qquad bd = q^{1/2}db, \quad cd = q^{1/2}dc$$

$$bc = cb, \quad ad - da = (q^{1/2} - q^{-1/2})bc$$

Definition 2. A quantum (semi) group G is the algebra of functions $\mathrm{L}^2(\mathrm{G})$.

- Natural basis for this non-commutative algebra $\sim$ products of matrix elements $g_{i_1 j_1} \dots g_{i_n j_n}$
- It has a product : $(f_1(g), f_2(g)) \to f_1(g) \cdot f_2(g)$
- It has a co-product : $\Delta : \mathrm{L}^2(\mathrm{G}) \to \mathrm{L}^2(\mathrm{G}) \otimes \mathrm{L}^2(\mathrm{G})$
  - $g_{ij} \to \sum_k g_{ik} \otimes g_{kj}$

What is $\mathbf{L}^2\left(\mathrm{SL}^+_{\mathbf{q}}(2,\mathbb{R})\right)$?

Remark. Any square-integrable function $f(g)$ is mapped to another one $f(h_{\rm L} g h_{\rm R}^{-1})$ furnishing a representation of $G \otimes G$. Its decomposition into irreps is controlled by the Peter-Weyl theorem

$$\mathsf{L}^2(\mathsf{G}) = \oplus_{\rm R} \mathsf{V}_{\rm R} \otimes \mathsf{V}_{{\rm R}^\star}$$

which provides a complete basis

$$\mathsf{R}_{ab}(g) \quad a, b = 1, \ldots \mathrm{dimR} \qquad \delta(g_1, g_2) = \sum_{\mathsf{R,a,b}} \mathsf{R}_{ab}(g_1) \mathsf{R}^\star_{ab}(g_2)$$

Lesson. $\mathsf{L}^2(\mathsf{G})$, and consequently G, can be reconstructed from the set of representations of G, i.e. the representation category Rep(G)

Rep$\left(\mathrm{SL}^+_{\mathsf{q}}(2,\mathbb{R})\right)$

$$\mathrm{L}^2(\mathrm{SL}^+_{\mathsf{q}}(2,\mathbb{R})) = \int_{\oplus_{p\geq 0}} \mathrm{dim}_{\mathsf{q}}(p)\, V_{\mathsf{p}} \otimes V^*_{\mathsf{p}} \quad \text{with} \quad q = e^{\pi i b^2}$$

where V_{p} is a continuous series representation of $\mathrm{SL}_{\mathsf{q}}(2,\mathbb{R})$
$\Rightarrow$ the set V_{p} is a complete set of representations of $\mathrm{SL}^+_{\mathsf{q}}(2,\mathbb{R})$

- Representation matrices R^p_{ab} with measure $\mathrm{dim}_{\mathsf{q}}(p)$ are known [Ip]

Ponsot, Teschner showed Rep $\left(\mathrm{SL}^+_{\mathsf{q}}(2,\mathbb{R})\right)$ solves the modular bootstrap for Liouville theory (a "universal theory" for Virasoro reps)

This means there is a one to one map (a ``functor)

$$\mathrm{Rep}(\mathrm{SL}^+_q(2,\mathbb{R})) \longleftrightarrow \mathrm{Rep}(\mathrm{Vir}) \quad \text{With} \quad c = 1 + 6(b+b^{-1})^2$$

$$V_p^{\mathrm{SL}^+_q(2,\mathbb{R})} \longleftrightarrow V_p^{\mathrm{Vir}}$$

Rep$\left(\mathrm{SL}_q^+(2,\mathbb{R})\right)$

This equivalence allows to identify the zero mode subspace with representation matrices of $\mathrm{SL}_q^+(2,\mathbb{R})$

$$\boxed{\langle g|p_\pm \mathrm{i}_L \mathrm{i}_R\rangle \sim R^{p_\pm}_{\mathrm{i}_L\mathrm{i}_R}(g), \qquad\qquad g \in \mathrm{SL}_q^+(2,\mathbb{R})}$$

- This is a quantum deformation of the same statements established by explicit wave function calculations in JT gravity [Blommaert, Mertens & Verschelde]

Co-product as a factorization map

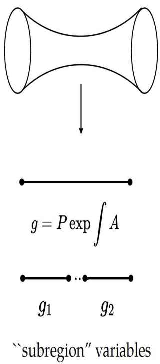
$L^2(G)$ has a natural factorization map given by the co-product

$$i : L^2(G) \to L^2(G) \otimes L^2(G)$$

$$R_{ab}(g) \to R_{ab}(g_1 \cdot g_2) = \sum_{c=1}^{\dim R} R_{ac}(g_1) R_{cb}(g_2)$$

c indices label edge modes (singlets under the diagonal action of G)

$\Rightarrow$ each basis state has EE $\log(\dim R)$



Co-product as a factorization map

For $\mathsf{G} = \mathrm{SL}^+_{\mathsf{q}}(2, \mathbb{R})$,

- $\mathsf{L}^2(\mathsf{G})$ is the zero mode subspace of BH states
- Each BH in representation (p_+, p_-) has EE

$$\boxed{\mathsf{S}_{\mathrm{V}} = \log\left(\dim_{\mathsf{q}} p_+ \dim_{\mathsf{q}} p_-\right)}$$

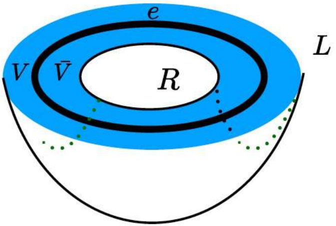
Physics recap

- Within our 3d gravity proposal

$$\boxed{\text{BH entropy} = \text{bulk entanglement entropy}}$$

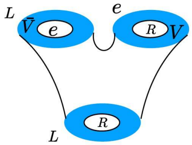
- We defined a shrinkable factorization of the Hartle-Hawking state

$$i|HH(\beta,\mu)\rangle =$$



Physics recap

- The factorization map is a co-product acting on the zero mode subspace, while boundary gravitons are spectators



$$\langle g | p\, \mathrm{i}_L\, s\rangle = \sqrt{\dim_q p}\; R^{p}_{\mathrm{i}_L\, s}(g) \quad \text{Zero mode subregion Wavefunctions}$$

$$i : |p\, \mathrm{i}_L \mathrm{i}_R\rangle \to \frac{1}{\sqrt{\dim_q p}} \int_{-\infty}^{\infty} ds\, |p\, \mathrm{i}_L\, s\rangle_{\bar{V}} \otimes |p\, s\, \mathrm{i}_R\rangle_{V}$$

- Absence of edge mode descendants $\Rightarrow$ finite edge mode EE (in 3d gravity, not in CS !!)

Physics recap

- Within our 3d quantum effective theory (no matter), the full entanglement entropy is quantum

$$S = -\mathrm{tr}_{\mathrm{V}}\left(\rho_{\mathrm{V}} \log \rho_{\mathrm{V}}\right) = S_{\text{gen}}$$

  - its semiclassical limit reproduces Bekenstein-Hawking
- Condensed matter realization. EE calculations performed by collective (edge) modes capturing the long range entanglement structure of the model (despite having a UV description)

Physics recap

- Modularity of the parent CFT_2 theory knows about the existence of this Plancherel measure

shrinkable b.c. $\sim$ modularity in the bulk using euclidean path integral
& open-closed "duality" $\Rightarrow \exists$ edge modes when cutting opened
a co-dimension 2 surface

Connection to extended TQFT

d-dim TQFT Attiyah's axioms

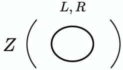
- closed d-1 manifolds $\leftrightarrow$ Hilbert spaces
- bordisms of d-1 manifolds $\leftrightarrow$ complex linear maps
- set of gluing compatibility conditions

Extended TQFT

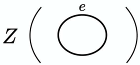
Question : Which mathematical objects should be assigned to higher co-dimension manifolds, i.e. entangling surfaces ?

- In d=3, co-dimension 2 manifolds, i.e. entangling surfaces $\leftrightarrow$ *linear category*
- CS literature and our 3d work suggest : two copies of $\text{Rep}(\mathrm{SL}_q^+(2,\mathbb{R}))$
- Our shrinkable b.c. determines part of the data characterising this category of representations

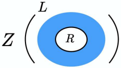
A gravitational extended TQFT ?



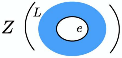
$= \mathrm{Rep}(\mathrm{Vir} \otimes \mathrm{Vir})$



$= \mathrm{Rep}(\mathrm{SL}_q^+(2,\mathbb{R}) \otimes \mathrm{SL}_q^+(2,\mathbb{R}))$

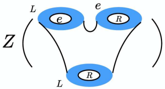


The identity functor on $\mathrm{Rep}(\mathrm{Vir})$



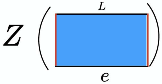
Functor

$= \mathrm{Rep}(\mathrm{Vir} \otimes \mathrm{Vir}) \to \mathrm{Rep}(\mathrm{SL}_q^+(2,\mathbb{R}) \otimes \mathrm{SL}_q^+(2,\mathbb{R})) = \oplus_p V_p^{\mathrm{Vir}} \otimes V_p^{\mathrm{SL}_q^+(2,\mathbb{R})} \otimes (p \to \bar{p})$



Co Product

$$i : |p\, m_L\, m_R\rangle \to \frac{1}{\sqrt{\dim_q(p)}} \int_{-\infty}^{+\infty} ds\, |p\, i_L\, s; m_L\rangle \otimes p_{\pm}\, s\, i_R; m_R\rangle$$



$= \text{Functor from } \mathrm{Rep}(\mathrm{Vir}) \text{ to } \mathrm{Rep}(\mathrm{SL}_q^+(2,\mathbb{R}))$

$= \oplus_p V_p^{\mathrm{Vir}} \otimes V_p^{\mathrm{SL}_q^+(2,\mathbb{R})}$

Conclusions

1. JT/Schwarzian analogue for 3d gravity
   - Universal high temperature sector of holographic irrational 2d CFTs
   - Not a 2d CFT, not modular invariant, does not contain global AdS_3
   - Unique saddle (BTZ) at high temperature & $(JT)^2$ in a double-scaled low temperature
2. Proposal for 3d bulk factorisation
   - Factorisation map (i) must satisfy a shrinkable b.c.
     - ⋆ solving it constrains density of edge modes localised at the entangling surface and the spectrum of its representations
   - In 3d gravity $\Rightarrow$ Rep $\left(\mathrm{SL}^+_q(2,\mathbb{R}) \otimes \mathrm{SL}^+_q(2,\mathbb{R})\right)$
   - i uses a quantum group generalisation of the Peter-Weyl theorem
   - Our work stresses the measure differences between CS and 3d gravity & has important links with extended TQFT

Future directions

1. Extensions
   - RT formula : vacuum/excited states, matter dof, multiple intervals
   - dS_3 or $\mathbb{R}^{1,2}$
2. Relation to other work
   - split property (algebraic QFT) & von Neumann algebra approach
   - Classical description of edge modes (covariant phase space)
3. "Deep waters"
   - Is there any microscopic interpretation of the subregion states ? EOW branes, fuzzballs ?
   - Topological strings have the same math structure : can D-branes be the underlying dof responsible for the gravitational edge modes discussed earlier ?
   - The shrinkable b.c. attempts to "fills holes" : can a UV complete description of these ideas provide some "D-brane"-like picture analogous to the open-closed string picture that we suspect is responsible for AdS/CFT duality ?

Example : 2d YM on an interval $[x_1, x_2]$

- "entanglement" boundary conditions $A_t = 0$ at both ends
- Gauss' law & Peter-Weyl theorem

$$\mathcal{H}_{\text{physical}} = \mathrm{L}^2(\mathrm{G}) = \bigoplus_R \mathcal{P}_R \otimes \mathcal{P}_R^\star$$

provides a representation of $G \otimes G$

$$f(g) \to f(h_L g h_R^{-1}), \qquad f \in \mathrm{L}^2(\mathrm{G}), \qquad (h_L, h_R) \in G \otimes G.$$

- basis of states for the interval Hilbert space

$$\left\{ |R, a, b\rangle = \sqrt{\dim \mathrm{R}}\, R_{ab}(g)\,, \quad a, b = 1, 2, \ldots \dim \mathrm{R} \right\}.$$

Example : 2d YM on an interval $[x_1, x_2]$

"Edge mode" interpretation

- would-be (large) gauge trafos $\rightarrow$ physical trafos at endpoints
  - physical dof $\sim$ edge modes of the physical boundary
- For $[x_1, x_2]$, large gauge trafos $G \times G$
  - acting by left and right multiplication at the left and right endpoints, respectively

$$|g\rangle \rightarrow |h_{\mathrm{L}}^{-1} g\rangle \ , \qquad |g\rangle \rightarrow |g h_{\mathrm{R}}\rangle$$

Example : 2d YM on an interval $[x_1, x_2]$

Extended & Physical Hilbert spaces

- Split $[x_1, x_2]$ into $V = [x_1, y - \epsilon]$ and $\bar{V} = [y + \epsilon, x_2]$
- Using $A_t = 0$ at each regulated entangling surface $\Rightarrow \mathrm{L}^2(\mathrm{G}) \otimes \mathrm{L}^2(\mathrm{G})$
- Surface symmetry at the split entangling surface $\mathrm{G}_\mathrm{S} = \mathrm{G} \otimes \mathrm{G}$
  - with the left copy of G acting by right multiplication on V and vice versa for $\bar{V}$
- Edge modes $\sim$ ungauged large gauge transformations acting at entangling endpoints
- $\mathcal{H}_{\mathrm{extended}} = \mathrm{L}^2(G) \otimes \mathrm{L}^2(G)$
  - $\mathcal{H}_{\mathrm{physical}}$ requires quotienting by the diagonal action of G_S

$$|g_1\rangle \to |g_1 h\rangle\,, \qquad |g_2\rangle \to |h^{-1} g_2\rangle$$

$$\mathcal{H}_{\mathsf{physical}} = \mathrm{L}^2(\mathrm{G}) \otimes_{\mathrm{G}_\mathrm{S}} \mathrm{L}^2(\mathrm{G})$$

Example : 2d YM on an interval $[x_1, x_2]$

Factorisation & Fusion

- $L^2(G)$ has a co-multiplication $\sim$ *factorization map*

$$
\begin{aligned}
i &: L^2(G) \to L^2(G) \otimes L^2(G), \\
i\,|g\rangle &= \frac{1}{|G|} \sum_{g_1, g_2 \in G} \delta(g_1 \cdot g_2, g)\,|g_1\rangle \otimes |g_2\rangle\,,
\end{aligned}
$$

  - i is an isometry, since its adjoint $i^*\left(|g_1\rangle \otimes |g_2\rangle\right) = |g_1 g_2\rangle$ *fuses* back the split intervals, i.e.

$$i^* \circ i = 1$$

Example : 2d YM on an interval $[x_1, x_2]$

Factorisation & Locality

- Using representation basis

$$
\begin{aligned}
i : L^2(G) &\to L^2(G) \otimes L^2(G) \\
\langle g | R, a, b \rangle &\to \langle g_1 \cdot g_2 | R, a, b \rangle = \sqrt{\dim R}\, R_{ab}(g_1 \cdot g_2) \\
&= \frac{1}{\sqrt{\dim R}} \sum_c \langle g_1 | R, a, c \rangle \langle g_2 | R, c, b \rangle
\end{aligned}
$$

edge modes $\sim$ index c $\to$ entanglement in the state

- Locality $\sim$ Wilson lines

$$
g = \mathrm{P} \exp \left(i \int A \right)
$$

Factorisation $\sim$ splitting Wilson line in each representation R

JT revisited

- Cauchy slice $\sim$ wormhole connecting both asymptotic boundaries

$$\mathcal{H}_{\mathrm{i}_L \mathrm{i}_R} \quad \text{spanned by} \quad |k\,\mathrm{i}_L\,\mathrm{i}_R\rangle\,, \quad k \in \mathbb{R}^+$$

  i_L, i_R satisfy coset boundary conditions
  $k \sim$ momentum related to energy $E = k^2$ (in some units)

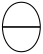
- $k \sim$ representation of a Wilson line crossing the wormhole

$$\langle g|k, \mathrm{i}_L \mathrm{i}_R\rangle = \sqrt{k \sinh 2\pi k}\; R^k_{\mathrm{i}_L \mathrm{i}_R}(g), \quad g \in \mathrm{SL}(2,\mathbb{R})$$

  - wave functions $\sim$ representation matrix elements of the gauge group $\mathrm{SL}(2,\mathbb{R})$

JT revisited

- Disk partition function

$$Z_{\text{disk}}(\beta) =$$  $$\equiv \langle \text{HH}_\beta | \text{HH}_\beta \rangle = \int_0^\infty dk\, (k \sinh 2\pi k)\, e^{-\beta\, k^2}$$

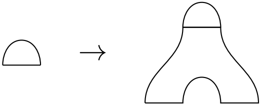
- Hartle-Hawking state

 $$= |\text{HH}_\beta\rangle = \int_0^\infty dk\, \sqrt{k \sinh 2\pi k}\, e^{-\beta\, k^2/2}\, |k\, \mathfrak{i}_L\, \mathfrak{i}_R\rangle$$

- Factorization map i

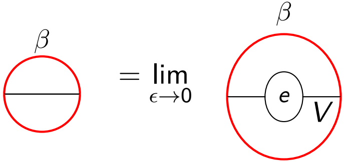
$$\mathcal{H}_{\mathfrak{i}_L \mathfrak{i}_R} \hookrightarrow \mathcal{H}_{\mathfrak{i}_L e} \otimes \mathcal{H}_{e\, \mathfrak{i}_R}$$

when applied to $|\text{HH}_\beta\rangle$, produces a half annulus

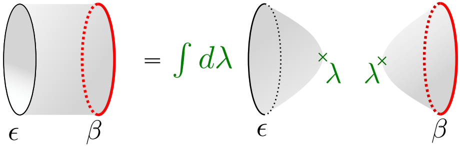


JT & shrinkable b.c.

1. $Z_{\text{disk}}(\beta)$ equals the $\epsilon \to 0$ limit of the full annulus



2. ϵ finite, annulus $\sim$ two boundary amplitude ("closed string" channel)



Equivalently,

$$Z(\epsilon, \beta) = \int d\lambda\, Z_{\text{inner}}(\epsilon, \lambda)\, Z_{\text{outer}}(\beta, \lambda)\,.$$

after inserting $\mathbf{1} = \int d\lambda\, |\lambda\rangle\, \langle\lambda| \sim$ complete set of defect insertions

Solving shrinkable b.c.

1. Using $\cos(2\pi\lambda k) = \langle\lambda|k\rangle$: wavefunction of a boundary state $|\lambda\rangle$

$$Z_{\text{outer}}(\beta_2, \lambda) \equiv \int dk\ \cos(2\pi\lambda k)\, e^{-\beta_2 k^2}$$

2. By definition,

$$Z_{\text{inner}}(\epsilon, \lambda) \equiv \langle e|\, \exp^{-H_{\text{closed}}}\, |\lambda\rangle = \int dk\ \langle e|k\rangle\ \cos(2\pi\lambda k)\, e^{-\epsilon k^2}$$

3. Altogether,

$$Z(\epsilon, \beta) = \int dk\, \langle e|k\rangle\ e^{-(\epsilon+\beta)k^2}$$

$Z_{\text{disk}} = \lim_{\epsilon\to 0} Z(\epsilon, \beta) \ \Rightarrow\ \langle e|k\rangle = k \sinh 2\pi k$, leading to

$$Z_{\text{inner}}(\epsilon, \lambda) = \int_0^\infty dk\, (k \sinh 2\pi k) \cos(2\pi\lambda k) e^{-\epsilon k^2}$$

JT : edge mode interpretation

Comparing

$$Z_{\rm outer}(\beta_2, \lambda) = \int_0^\infty dk\, \cos(2\pi\lambda k)\, e^{-\beta_2 k^2}$$
$$Z_{\rm inner}(\epsilon, \lambda) = \int_0^\infty dk\, (k \sinh 2\pi k)\, \cos(2\pi\lambda k) e^{-\epsilon k^2}$$

- $\cos(2\pi\lambda k) \leftrightarrow$ defect insertion
- the *density of edge states* $\leftrightarrow$ inner entangling boundary
  - counts the zero (modular) energy edge modes at fixed k, which are localized to the entangling surface
  - corresponds to the Plancherel measure for $\mathrm{SL}^+(2,\mathbb{R})$ [Ponsot, Teschner]

$$\mathrm{L}^2(\mathrm{SL}^+(2,\mathbb{R})) = \int_{\oplus_{k\geq 0}} (k \sinh 2\pi k)\, \mathcal{P}_k \otimes \mathcal{P}_k$$

JT conclusion

Further lesson

Solving the shrinkable b.c. in JT $\Rightarrow$

- fixing $\mathrm{G_S} = \mathrm{SL}^+(2,\mathbb{R})$
- edge modes localised at the inner entangling surface belong to continuous series representations

Bulk factorisation completion : Armed with the generalization of the Peter-Weyl theorem for $\mathrm{SL}^+(2,\mathbb{R})$ & its extension in the presence of holographic boundaries

$$\mathcal{H}_{\mathrm{ei}_R} = \mathsf{L}^2(\mathrm{SL}^+(2,\mathbb{R})/\sim) \equiv \int_{\oplus k\geq 0} (k \sinh 2\pi k)\, \mathcal{P}_k \otimes \mathcal{P}_{k,\mathrm{i}_R}$$

- define factorisation map compatible with shrinkable b.c.
- entropy of bulk reduced density matrix reproduces Hawking-Bekenstein entropy [Blommaert, Mertens, Verschelde]