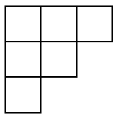
Tutorial

1. Show that the representation matrices of a group form a group themselves.

2. Let V be a vector space carrying a rep of a finite group G and let U and W be orthogonal subspaces of V such that $V = U \oplus W$. Show that if U is closed under G, then so W.

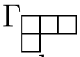
3. Draw all possible Young diagrams of four boxes. How many conjugacy classes are there in S_4? Number the Young diagrams to construct all valid Young tableaux. Order the tableaux.

4. Use the Littlewood-Richardson rule to show that the irrep of S_6 labelled by

   subduces ten irreps when restricted to S_3.

5. Use the Littlewood-Richardson rule to show that the number of orders of removing all boxes from a Young diagram, where at each step you have only valid Young diagrams, equals the dimension of the irrep. How is this related to the numbering used to construct Young Tableaux?

6. Compute the matrix Γ$((1\,2\,3\,4))$ in the Yamanouchi basis. Strand diagrams are your friend.

7. A Casimir is an operator which commutes with all elements in a group. Show that $\sum_{i<j}^{n}(i\,j)$ is a Casimir of S_n. What about a sum of 3-cycles? Generalize.

8. Write out all 5 S_4 Schur polynomials explicitly using the following character table:

	$\ydiagram{4}$	$\ydiagram{3,1}$	$\ydiagram{2,2}$	$\ydiagram{2,1,1}$	$\ydiagram{1,1,1,1}$
e	1	3	2	3	1
$(\bullet\ \bullet)$	1	1	0	-1	-1
$(\bullet\ \bullet\ \bullet)$	1	0	-1	0	1
$(\bullet\ \bullet)\ (\bullet\ \bullet)$	1	-1	2	-1	1
$(\bullet\ \bullet\ \bullet\ \bullet)$	1	-1	0	1	-1

9. Write out the following restricted Schur polynomial explicitly using strand diagrams:
$$\chi_{\ytableaushort{{}1,{},2}}\left(Z, W^{(1)}, W^{(2)}\right).$$

10. Calculate the result of the following reductions (hint: you may have to apply the subgroup swap rule):

    (a) $D_{W^{(3)}}\chi_{\ytableaushort{{}{}1,{}2,3}}|_1|_2|_3\left(Z, W^{(1)}, W^{(2)}, W^{(3)}\right)$

    (b) $D_Z\chi_{\ytableaushort{{}{}1,{}}}\left(Z, W^{(1)}\right)$

11. Write out the following restricted Schur polynomials explicitly:
$$\chi_{\ydiagram{3};\ydiagram{2}\otimes\ydiagram{1}}(Z, X), \qquad \chi_{\ydiagram{1,1,1};\ydiagram{1,1}\otimes\ydiagram{1}}(Z, X).$$

12. Calculate,
$$\langle\chi_{\ydiagram{2}}(Z)\,\chi_{\ydiagram{2}}\left(Z^\dagger\right)\rangle,$$
first by using the explicit expression for the Schur polynomial and implementing the contractions of the matrices. Then check that this matches what you would obtain if you apply the rule for calculating two point functions of Schur polynomials.

13. Calculate:
$$\langle\chi_{\ytableaushort{{}{}1,2}}\left(Z, W^{(1)}, W^{(2)}\right)\chi_{\ytableaushort{{}{}1,2}}\left(Z^\dagger, W^{(1)\dagger}, W^{(2)\dagger}\right)\rangle.$$

14. Calculate:
$$\langle\chi_{\ydiagram{4,3};\ydiagram{3,2}\otimes\ydiagram{2}}(Z, X)\,\chi_{\ydiagram{4,3};\ydiagram{3,2}\otimes\ydiagram{2}}\left(Z^\dagger, X^\dagger\right)\rangle.$$