S_n representation theory computer tutorial

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1 The code

The Matlab code that you have been provided includes a function called: generatesnrep. The function specification is as follows:

```
generatesnrep(youngd,cycle)
```

```
The function generatesnrep returns the matrix representing
the group element "cycle" in the irreducible representation
specified by the Young diagram "youngd". Note that cycle must
be specified as a permutation of the indices 1,...,n.
```

As an example, we obtain the matrix representing group element (12) in the irreducible representation specified by $\yng(3,2,1)$ by utilizing:

```
matrep = generatesnrep([3,2,1],[2 1 3 4 5 6])
```

The character $\chi_{\yng(3,2,1)}(\sigma)$ is obtained by tracing this matrix i.e.

```
charac = trace(generatesnrep([3,2,1],[2 1 3 4 5 6]))
```

2 The exercise

Consider the S_6 irrep

$$R = \yng(3,2,1) \; .$$

Can you count 20 possible S_3 subgroups of S_6?

The code you are using generates irreps in the Yamanouchi basis. The rep subduced from R onto S_3 is reducible. For which S_3 subgroup do you expect the matrices to have block-diagonal form? Check some rep matrix to confirm your answer. Compare the submatrices on the diagonal with S_3 irreps.

What does the block-diagonal structure look like? How many copies of which irreps? Compare with what you would expect by applying the Littlewood-Richardson rule. Think about how the matrix structure is related to the subduction chain $S_3 \subset S_4 \subset S_5 \subset S_6$.

When we define Schur polynomials we sometimes trace over off-diagonal blocks. Check that these can be non-zero. For which group elements? For two off-diagonal group elements, compute

$$\sum_{\sigma \in S_6} \Gamma_R(\sigma)_{ij} \Gamma^*_R(\sigma)_{rs}.$$

Is this proportional to $\delta_{is}\delta_{jr}$? What is the proportionality constant?

What is the dimension of irrep R? Does it match the size of the matrices?

There are six $S_5 \subset S_6$. The subduced rep decomposes into

$$\ydiagram{3,1,1} \to \ydiagram{2,2,1} + \ydiagram{3,1,1} + \ydiagram{3,2}\,.$$

Construct a Casimir with the same eigenvalue for every vector in R. Does it commute with all $\sigma \in S_6$? What does it look like? Why?

To project into the S_5 subspaces we need a Casimir on $S_5 \in S_6$. Which Casimir should we use if we want to easily see the block-diagonal structure. Build this Casimir. Does it have any obvious structure? Interpret the structure. How does it act on the basis vectors? What about if you build it out of a different S_5 subgroup? Think about how these Casimirs are related.

Construct a projector onto each of the S_5 irreps in R. Look at the form of $P\Gamma P$ for a few elements of S_6 and compare with Γ. Does $P^2 = P$?

Try to project onto the $\ydiagram{2,1}$ subspace of R. What happened?

Compare a few matrix elements for any irreps you like with strand diagram calculations.