What to do if N just isn't big enough

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The AdS/CFT Correspondence

Quantum gravity on asymptotically $AdS_5 \times S^5$ spacetime

$$\updownarrow$$

$\mathcal{N} = 4$ super Yang-Mills theory with gauge group $U(N)$

In the 't Hooft limit of the gauge theory ($N \to \infty$ and $\lambda = g_{YM}^2 N =$fixed)

$$\frac{R^2}{\alpha'} = \sqrt{\lambda}$$

$$g_s = \frac{\lambda}{N}$$

bosonic $SO(2,4) \times SO(6)$ symmetry matches

Dual Map of Field Theory Parameter Space

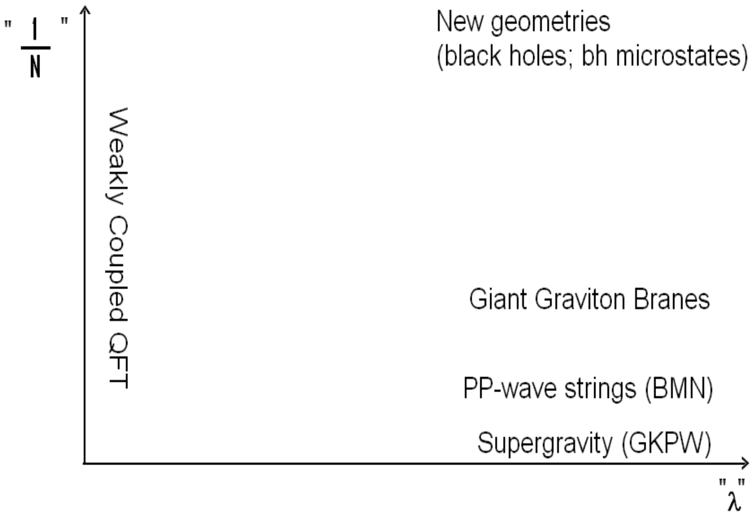

Figure: We are most interested in finite N regions of parameter space.

$\mathcal{N} = 4$ SYM theory

- Study $\mathcal{N} = 4$ SYM theory on $R \times S^3$.
- Consider the complex combinations $X = \phi_1 + i\phi_2$, $Y = \phi_3 + i\phi_4, \quad Z = \phi_5 + i\phi_6$, from the s-wave components of the 6 adjoint scalars.
- In our normalization, the free two point function is

$$\langle Z^i{}_j (Z^\dagger)^k{}_l \rangle = \delta^i_l \delta^k_j$$

- We will consider operators from the $\frac{1}{2}$ BPS sector (built only from Zs) or nearly $\frac{1}{2}$ BPS (built mostly from Zs)
- For these $\mathcal{R}$-charge $=$ dimension

Dual Picture of Parameter Space

- Objects polarize; velocity dependent force doing polarization (Myers)
- $$R = \sqrt{\frac{J}{N}} R_{\mathrm{AdS}} = \sqrt{\frac{J}{N}} (N g_{YM}^2)^{\frac{1}{4}} l_s$$

$\mathcal{R}$-charge	Size (R)	Interpretation
~ 1	~ 0	Graviton
$\sim \sqrt{N}$	$\sim l_s$	String
$\sim N$	$\sim R_{\mathrm{AdS}}$	Brane
$\sim N^2$	R diverges	New Geometry

Table: The size of objects in the dual string theory as a function of $\mathcal{R}$-charge

- Take Away idea: theory is organized by $\mathcal{R}$-charge

The simplicity of $N = \infty$

- Non planar diagrams can be discarded which implies huge simplifications.

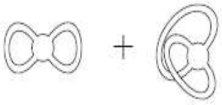

Figure: The second diagram is non-planar.

- Large N is the classical limit (expectation values factorize) of N eigenvalues.
- Observeables admit a double expansion

$$O = \sum_h \sum_n o_{h,n} N^{2-2h} \lambda^n$$

Gravitons

- Suitable operators are $O = \mathrm{Tr}(Z^n)$ with $n \sim O(1)$
- $$\left\langle \frac{\mathrm{Tr}(Z^n)}{\sqrt{n}N^{\frac{n}{2}}} \frac{\mathrm{Tr}(Z^{\dagger\, m})}{\sqrt{m}N^{\frac{m}{2}}} \right\rangle = \delta_{nm}$$

  the orthogonality present here is due to conservation of $\mathcal{R}$-charge
- $$\left\langle \frac{\mathrm{Tr}(Z^n)}{\sqrt{n}N^{\frac{n}{2}}} \frac{\mathrm{Tr}(Z^{\dagger\, n_1})}{\sqrt{n_1}N^{\frac{n_1}{2}}} \frac{\mathrm{Tr}(Z^{\dagger\, n-n_1})}{\sqrt{n-n_1}N^{\frac{n-n_1}{2}}} \right\rangle = \frac{\sqrt{nn_1(n-n_1)}}{N}$$

  this correlator is not forced to vanish - $\mathcal{R}$-charge is conserved; it does vanish at large N
- In field theory: *supression of nonplanar diagrams is responsible*
- In string theory: number of traces is identified with number of gravitons; the above correlator vanishes because particle number is conserved; interactions are weak

Strings

- Suitable operators are $O = \mathrm{Tr}(Z^{n_1} Y Z^{n_2} Y \cdots Y Z^{n_l})$ with $\sum_i n_i = J \sim \sqrt{N}$, $L \sim 1$ (BMN)
- Non-planar diagrams are enhanced
- The large N expansion can be reorganized - $\frac{1}{N}$ is replaced by $\frac{J^2}{N}$ (Kristjansen, Plefka, Semenoff, Staudacher)

$\mathcal{R}$-charge	Size (R)	Interpretation	Coupling
~ 1	~ 0	Graviton	$1/N$
$\sim \sqrt{N}$	$\sim l_s$	String	J^2/N
$\sim N$	$\sim R_{\mathrm{AdS}}$	Brane	???
$\sim N^2$	R diverges	New Geometry	???

Table: The size of objects in the dual string theory as a function of $\mathcal{R}$-charge

Recall the basic contraction: $\langle Z^i_j (Z^\dagger)^k_l \rangle = \delta^i_l \delta^k_j$

To compute

$$\left\langle Z^{i_1}_{j_1} Z^{i_2}_{j_2} Z^{i_3}_{j_3} (Z^\dagger)^{k_1}_{l_1} (Z^\dagger)^{k_2}_{l_2} (Z^\dagger)^{k_3}_{l_3} \right\rangle$$

sum over the $3! = 6$ possible contractions, each of which can be associated with an element of S_3. For example, if $\sigma = (123)$:

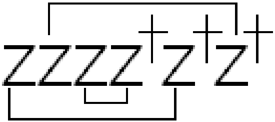

Figure: One of 6 contractions

$$= \delta^{i_1}_{l_2} \delta^{k_2}_{j_1} \delta^{i_2}_{l_3} \delta^{k_3}_{j_2} \delta^{i_3}_{l_1} \delta^{k_1}_{j_3} = \delta^{i_1}_{l_{\sigma(1)}} \delta^{k_2}_{j_{\sigma^{-1}(2)}} \delta^{i_2}_{l_{\sigma(2)}} \delta^{k_3}_{j_{\sigma^{-1}(3)}} \delta^{i_3}_{l_{\sigma(3)}} \delta^{k_1}_{j_{\sigma^{-1}(1)}}$$

$$= \langle I|\sigma|L\rangle \langle K|\sigma^{-1}|J\rangle$$

For $n = 2$, $\sigma = \mathbf{1}$ has $\sigma(1) = 1$ and $\sigma(2) = 2$; $\sigma = (12)$ has $\sigma(1) = 2$ and $\sigma(2) = 1$

$$\mathrm{Tr}(\sigma Z^{\otimes 2}) = \delta^{i_1}_{j_{\sigma(1)}} \delta^{i_2}_{j_{\sigma(2)}} Z^{j_1}_{i_1} Z^{j_2}_{i_2} = Z^{j_1}_{j_{\sigma(1)}} Z^{j_2}_{j_{\sigma(2)}}$$

$$\mathrm{Tr}(1 \cdot Z^{\otimes 2}) = \mathrm{Tr}(Z)^2 \qquad \mathrm{Tr}(\,(12) \cdot Z^{\otimes 2}) = \mathrm{Tr}(Z^2)$$

$$\mathrm{Tr}(AZ^{\otimes 3}) = A^{i_1 i_2 i_3}_{j_1 j_2 j_3} Z^{j_1}_{i_1} Z^{j_2}_{i_2} Z^{j_3}_{i_3}$$

Recall the basic contraction: $\langle Z^i_j (Z^\dagger)^k_l \rangle = \delta^i_l \delta^k_j$
To compute

$$\left\langle Z^{i_1}_{j_1} Z^{i_2}_{j_2} Z^{i_3}_{j_3} (Z^\dagger)^{k_1}_{l_1} (Z^\dagger)^{k_2}_{l_2} (Z^\dagger)^{k_3}_{l_3} \right\rangle$$

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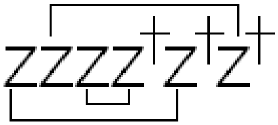


Figure: One of 6 contractions

$$= \delta^{i_1}_{l_2} \delta^{k_2}_{j_1} \delta^{i_2}_{l_3} \delta^{k_3}_{j_2} \delta^{i_3}_{l_1} \delta^{k_1}_{j_3} = \delta^{i_1}_{l_{\sigma(1)}} \delta^{k_2}_{j_{\sigma^{-1}(2)}} \delta^{i_2}_{l_{\sigma(2)}} \delta^{k_3}_{j_{\sigma^{-1}(3)}} \delta^{i_3}_{l_{\sigma(3)}} \delta^{k_1}_{j_{\sigma^{-1}(1)}}$$

$$= \langle I|\sigma|L\rangle \langle K|\sigma^{-1}|J\rangle$$

After summing all contractions:

$$\left\langle Z^{i_1}_{j_1} Z^{i_2}_{j_2} Z^{i_3}_{j_3} (Z^\dagger)^{k_1}_{l_1} (Z^\dagger)^{k_2}_{l_2} (Z^\dagger)^{k_3}_{l_3} \right\rangle = \sum_{\sigma \in S_3} \langle I|\sigma|L\rangle \langle K|\sigma^{-1}|J\rangle$$

If we have n fields

$$\left\langle \langle I|Z^{\otimes n}|J\rangle \langle K|(Z^\dagger)^{\otimes n}|L\rangle \right\rangle = \sum_{\sigma \in S_n} \langle I|\sigma|L\rangle \langle K|\sigma^{-1}|J\rangle$$

and

$$\left\langle \mathrm{Tr}(AZ^{\otimes n}) \mathrm{Tr}(BZ^{\dagger \otimes n}) \right\rangle = \sum_{\sigma \in S_n} \mathrm{Tr}(\sigma^{-1} A \sigma B)$$

Schur Polynomials

$$\chi_R(Z) = \mathrm{Tr}(P_R Z^{\otimes n}), \qquad P_R = \frac{1}{n!}\sum_{\sigma\in S_n}\chi_R(\sigma)\sigma$$

$$\left\langle \chi_R(Z)\chi_S(Z)^\dagger \right\rangle = \sum_{\sigma\in S_n}\mathrm{Tr}(\sigma^{-1}P_R\sigma P_S)$$

$$P_R\sigma = \sigma P_R \qquad P_R P_S = \frac{\delta_{RS}}{d_R}P_R$$

These two properties follow because P_R is (up to a factor) a *projector* - it projects onto an irrep of S_n.

$$\mathrm{Tr}(P_R) = \chi_R(1) = \mathrm{Dim}_N R$$

which follows because $\chi_R(U)$ is the character of the unitary group element U.

Take Away Message

$$\left\langle \mathrm{Tr}(AZ^{\otimes n})\mathrm{Tr}(BZ^{\dagger\otimes n})\right\rangle = \sum_{\sigma\in S_n} \mathrm{Tr}(\sigma^{-1}A\sigma B)$$

$$\chi_R(Z) = \mathrm{Tr}(P_R Z^{\otimes n})$$

$$P_R\sigma = \sigma P_R \qquad P_R P_S = \frac{\delta_{RS}}{d_R}P_R \qquad \mathrm{Tr}P_R = \mathrm{known}$$

$$\left\langle \chi_R(Z)\chi_S(Z)^\dagger\right\rangle = \frac{n!\mathrm{Dim}_N R}{d_R}$$

(Corley, Jevicki, Ramgoolam)

Two Matrices

$$\langle I|Z^{\otimes n}Y^{\otimes m}|J\rangle = Z^{i_1}_{j_1}\cdots Z^{i_n}_{j_n}Y^{i_{n+1}}_{j_{n+1}}\cdots Y^{i_{n+m}}_{j_{n+m}}$$

$$\langle Z^i_j(Z^\dagger)^k_l\rangle = \delta^i_l\delta^k_j = \langle Y^i_j(Y^\dagger)^k_l\rangle$$

$$\Big\langle \langle I|Z^{\otimes n}Y^{\otimes m}|J\rangle\langle K|(Z^\dagger)^{\otimes n}(Y^\dagger)^{\otimes m}|L\rangle\Big\rangle$$

$$= \sum_{\sigma\in S_n\times S_m}\langle I|\sigma|L\rangle\langle K|\sigma^{-1}|J\rangle$$

Can we again diagonalize the two point function using projectors?

We can again diagonalize the two point function using projectors, O_R

$$\left[O_{R,r},\sigma\right]=0,\qquad \sigma\in S_n\times S_m$$

$$O_{R,r}O_{S,s}=\delta_{RS}\delta_{rs}\alpha O_{R,r},\qquad \mathrm{Tr}(O_{R,r})=\beta$$

$$O_{R,r}=\frac{1}{n!m!}\sum_{\sigma\in S_{n+m}}\mathrm{Tr}_r(\Gamma_R(\sigma))\sigma$$

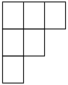
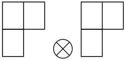
(Balasubramanian, Berenstein, Feng, Huang; Bhattacharyya, Collins, dMK)

$\alpha=\frac{(n+m)!}{n!m!d_R},\qquad \beta=d_r f_R$

(dMK, Smolic, Smolic)

Counting

If a particular $S_n \times S_m$ irrep is subduced more than once, we have interesting possibilities.

 subduces  twice

Call the states in these two irreps $|i, 1\rangle$ and $|i, 2\rangle$. There are four possible definitions for the restricted trace

$$\langle 1, i|\Gamma_R(\sigma)|1, i\rangle \quad \langle 2, i|\Gamma_R(\sigma)|1, i\rangle$$

$$\langle 1, i|\Gamma_R(\sigma)|2, i\rangle \quad \langle 2, i|\Gamma_R(\sigma)|2, i\rangle$$

$4 = (2)^2 =$ the Littlewood-Richardson number squared

Counting

$$Z = \sum_{n,m} \sum_{R \vdash n+m} \sum_{r_1 \vdash n} \sum_{r_2 \vdash m} (g_{r_1\, r_2\, R})^2 z^n y^m$$

When the sum is unrestricted we reproduce the counting formula from Polya theory.
When each partition is restricted to have at most N parts we recover the $g_{YM} = 0$ partition function for $1/4$ BPS states. (Collins)

Schur-Weyl Duality

Basic Idea: You can diagonalize operators which commute. (S_n action and $U(N)$ action on $V^{\otimes n}$ commute so that finding the irreps of S_n gives you irreps of $U(N)$)
Consider an algebra A acting of a vector space V:

$$A : V \to V,$$

so that a rep (in general not an irrep) of A is a map

$$\rho : A \to \mathrm{End}(V).$$

Inside $\mathrm{End}(V)$ there is a subalgebra of all things that commute with A, denoted $\mathrm{com}(A)$ called the *commutant* of A. States in V can be organized into irreps of A:

$$|V_\mu^A, m_\mu, i\rangle$$

where i is a multiplicity label.

Schur-Weyl Duality

Double Centralizer Theorem tells you how to organize the multiplicity label i:

$$|V_\mu^A, m_\mu^A, V_\mu^{\mathrm{com}A}, m_\mu^{\mathrm{com}A}\rangle$$

or

$$V = \oplus_\mu V_\mu^A \otimes V_\mu^{\mathrm{com}(A)}$$

(Ramgoolam, arXiv:0804.2764)

Other Bases

Choosing a basis is not unique. Different choices of basis correspond to different choices for the commutant.
For operators built from Z and Z^*, $\mathrm{com}(A)$ is the Brauer algebra (Kimura, Ramgoolam).
We can include in A any global symmetry group G. The basis will have good G quantum numbers. (Brown, Heslop, Ramgoolam)

Probing New Geometries

Schur polynomials labeled by Young diagrams with $O(N^2)$ boxes are dual to LLM geometries. "Probe" the geometries with gravitons (following Balasubramanian, de Boer, Jejjala, Simon)

$$\langle \chi_R(Z)\chi_R(Z)^\dagger \mathrm{Tr}(Z)\mathrm{Tr}(Z)^\dagger \rangle =$$

$$N\langle \chi_R(Z)\chi_R(Z)^\dagger \rangle + \langle \frac{d}{dZ_{ii}}\chi_R(Z)\frac{d}{dZ^\dagger_{jj}}\chi_R(Z)^\dagger \rangle$$

Probing New Geometries

After the derivative acts there are only $n-1$ fields

$$\frac{d}{dZ_{ii}}\chi_R(Z)=\frac{1}{(n-1)!}\sum_{\sigma\in S_n}\chi_R(\sigma)Z^{i_1}_{i_{\sigma(1)}}\cdots Z^{i_{n-1}}_{i_{\sigma(n-1)}}\delta^{i_n}_{i_{\sigma(n)}}$$

$$=\frac{1}{(n-1)!}\sum_{\sigma\in S_{n-1}}\chi_R(\sigma\left[N-\sum_{i=1}^{n-1}(i,n)\right])Z^{i_1}_{i_{\sigma(1)}}\cdots Z^{i_{n-1}}_{i_{\sigma(n-1)}}\delta^{i_n}_{i_{\sigma(n)}}$$

Jucys Murphy Elements

$$\sum_{i=1}^{n-1} \Gamma_R\left((i,n)\right), \qquad \sum_{i=1}^{n-2} \Gamma_R\left((i,n-1)\right), \qquad \sum_{i=1}^{n-3} \Gamma_R\left((i,n-2)\right)\cdots$$

They commute and generate a maximal commutative subalgebra in the group algebra of S_n which is exactly the algebra of elements acting diagonally the Young basis of irreducible representations of S_n

In recent years, they have become allpurpose heavyduty technical tools in representation theory of S_n.

Graviton Correlator: No Background

$$\mathcal{A}(N) = \left\langle \prod_{i,j} \mathrm{Tr}(Z^{n_i}) \mathrm{Tr}(Z^{\dagger\, m_j}) \right\rangle$$

- Use:

$$\prod_i \mathrm{Tr}(Z^{n_i}) = \sum_R \alpha_R \chi_R(Z) \qquad \prod_j \mathrm{Tr}(Z^{\dagger\, m_j}) = \sum_T \beta_T \chi_T(Z)^\dagger$$

- Note that α_R, β_R are independent of N.
-

$$\mathcal{A}(N) = \sum_{R,T} \alpha_R \beta_T \left\langle \chi_R(Z) \chi_T(Z)^\dagger \right\rangle = \sum_R \alpha_R \beta_R f_R$$

Graviton Correlator: Background

$$\langle O \rangle_B \equiv \frac{\left\langle \chi_B(Z)\chi_B(Z^\dagger)O \right\rangle}{\langle \chi_B(Z)\chi_B(Z^\dagger) \rangle}$$

$$= f_B^{-1} \left\langle \chi_B(Z)\chi_B(Z^\dagger)O \right\rangle$$

- A particularly simple example is obtained when B has M columns and N rows
- In this case the product $\chi_B(Z)\chi_R(Z)$ has a single term

Graviton Correlator: Background

$$\mathcal{A}_B(N) = f_B^{-1} \left\langle \chi_B(Z)\chi_B(Z^\dagger) \prod_{i,j} \mathrm{Tr}(Z^{n_i})\mathrm{Tr}(Z^{\dagger\, m_j}) \right\rangle$$

$$= f_B^{-1} \sum_{R,T} \alpha_R \beta_T \left\langle \chi_B(Z)\chi_B(Z^\dagger)\chi_R(Z)\chi_T(Z)^\dagger \right\rangle$$

$$= f_B^{-1} \sum_{R,T} \alpha_R \beta_T \left\langle \chi_{+R}(Z)\chi_{+T}(Z)^\dagger \right\rangle = f_B^{-1} \sum_R \alpha_R \beta_R f_{+R}$$

- Compare $\mathcal{A}(N) = \sum_R \alpha_R \beta_R f_R$ and $\mathcal{A}_B(N) = \sum_R \alpha_R \beta_R \frac{f_{+R}}{f_B}$
- to obtain $\mathcal{A}_B(N) = \mathcal{A}(N+M)$

(dMK, Dey, Ives, Stephanou, arXiv:0905.2273)

Is there a reorganization of the $\frac{1}{N}$ expansion for $\frac{1}{2}$-BPS operators which have an $\mathcal{R}$-charge of $O(N^2)$? What is the effective expansion parameter?

Yes - in the background considered, the $\frac{1}{N}$ expansion is replaced by a $\frac{1}{N+M}$ expansion. This nontrivial renormalization of the effective string coupling is a direct consequence of the fact that many non-planar diagrams have been summed.

What we really want - multimatrix operators (V. Balasubramanian, J. de Boer, V. Jejjala and J. Simon)

One Loop Dilatation Operator

$$D\,\chi_{(R,(r,s))}(Z^{\otimes\, n}, X^{\otimes\, m}) = g_{YM}^2 \mathrm{Tr}\left([X,Z] \left[\frac{d}{dZ}, \frac{d}{dX} \right] \right) \chi_{(R,(r,s))}(Z^{\otimes\, n}, X^{\otimes\, m})$$

$$= g_{YM}^2 \frac{N d_T\, nm}{d_t d_u (n+m)!} \sum_{T,(t,u)} \sum_{\psi \in S_{n+m}, \mathrm{fix}\, n+1} \chi_{R,(r,s)} \left(\Gamma_R((n,n+1)\psi - \psi(n,n+1)) \right)$$

$$\times \chi_{T,(t,u)} \left(\Gamma_R((n,n+1)\psi - \psi(n,n+1)) \right) \chi_{T,(t,u)}(Z^{\otimes\, n}, X^{\otimes\, m})$$

(De Comarmond, dMK, Jeffries)

Membrane States

$$O_a(b_0, b_1) = \chi_{\yng(2,2,2,1,1,1)\,;\,\yng(2,2,2,1)\,\yng(1,1)}$$

$$O_b(b_0, b_1) = \chi_{\yng(2,2,2,1,1)\,;\,\yng(2,1,1,1,1)\,\yng(1,1)}$$

$$O_d(b_0, b_1) = \chi_{\yng(2,2,2,1,1)\,;\,\yng(2,2,1,1)\,\yng(2)}$$

$$O_e(b_0, b_1) = \chi_{\yng(2,2,2,1,1)\,;\,\yng(2,2,1,1)\,\yng(1,1)}$$

Membrane States

$$O_f(b_0, b_1) = \chi_{(3,2,2,1,1,1,1);(2,2,2,1,1,1)\;(2)}\,, \qquad O_g(b_0, b_1) = \chi_{(3,2,2,1,1,1,1);(2,2,2,1,1,1)\;(1,1)}\,.$$

$$O_h(b_0, b_1) = \chi_{(3,2,2,2,1,1);(2,2,2,1,1,1)\;(2)}\,, \qquad O_i(b_0, b_1) = \chi_{(3,2,2,2,1,1);(2,2,2,1,1,1)\;(1,1)}\,.$$

Final Result

$$D\hat{O}_a(b_0,b_1) = 4\lambda\frac{(N-b_0-b_1-1)}{N(b_1+2)^2}\hat{O}_a(b_0,b_1) - 2\lambda\frac{\sqrt{(N-b_0-b_1-1)(N-b_0+1)}}{N(b_1+2)}\sqrt{\frac{b_1+3}{b_1+1}}\hat{O}_d(b_0,b_1)$$

$$+2\lambda\frac{\sqrt{(N-b_0-b_1-1)(N-b_0+1)}b_1}{N(b_1+2)^2}\sqrt{\frac{b_1+3}{b_1+1}}\hat{O}_e(b_0,b_1)$$

$$+4\lambda\frac{\sqrt{(N-b_0-b_1-1)(N-b_0+1)}}{N(b_1+2)^2}\hat{O}_b(b_0-1,b_1+2)$$

$$+2\lambda\frac{(N-b_0-b_1-1)}{(b_1+2)}\sqrt{\frac{b_1+1}{N(b_1+3)}}\hat{O}_d(b_0-1,b_1+2) - 2\lambda\frac{(b_1+4)(N-b_0-b_1-1)}{N(b_1+2)^2}\sqrt{\frac{b_1+1}{b_1+3}}\hat{O}_e(b_0-1,b_1+2)$$

(dmk, Mashile, Park)

Compare to BMN

$$\mathcal{O}_l = \mathrm{Tr}(YZ^l YZ^{J-l})$$

$$\langle \mathcal{O}_l \mathcal{O}_k^\dagger \rangle \propto (\delta_{l,k} + \delta_{J,l+k})$$

$$D\mathcal{O}_l \propto \lambda(2\mathcal{O}_l - \mathcal{O}_{l-1} - \mathcal{O}_{l+1})$$

Weak mixing; looks like a second derivative on the lattice of Zs

Final Result

In the limit that $N - b_0 = O(N)$, $b_0 = O(N)$ and $b_1 = O(\sqrt{N})$ we have giants which are separated by a distance of $O(1)$ in string units. In this limit

$$D\hat{O}_a(b_0, b_1) = \lambda \times O(b_1^{-1}), \qquad D\hat{O}_b(b_0, b_1) = \lambda \times O(b_1^{-1}),$$

$$D\hat{O}_d(b_0, b_1) = \lambda\left(1 - \frac{b_0}{N}\right)\left[2\hat{O}_d(b_0, b_1) - \hat{O}_d(b_0 - 1, b_1 + 2) - \hat{O}_d(b_0 + 1, b_1 - 2)\right]$$

$$-\lambda\left(1 - \frac{b_0}{N}\right)\left[2\hat{O}_e(b_0, b_1) - \hat{O}_e(b_0 - 1, b_1 + 2) - \hat{O}_e(b_0 + 1, b_1 - 2)\right] + O(b_1^{-1})$$

$$D\hat{O}_e(b_0, b_1) = \lambda\left(1 - \frac{b_0}{N}\right)\left[2\hat{O}_e(b_0, b_1) - \hat{O}_e(b_0 - 1, b_1 + 2) - \hat{O}_e(b_0 + 1, b_1 - 2)\right]$$

$$-\lambda\left(1 - \frac{b_0}{N}\right)\left[2\hat{O}_d(b_0, b_1) - \hat{O}_d(b_0 - 1, b_1 + 2) - \hat{O}_d(b_0 + 1, b_1 - 2)\right] + O(b_1^{-1})$$

Excited Giant Gravitons

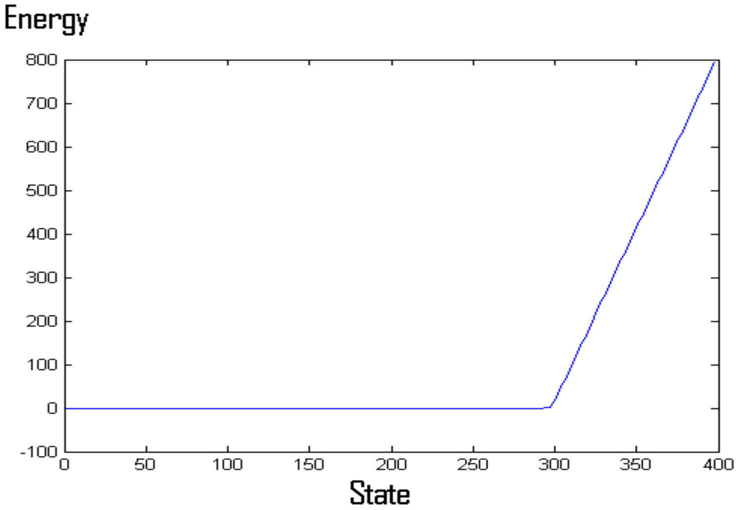


Figure: Numerically obtained spectrum

Dilatation Operator

Deformations of a single giant are all BPS (Das, Jevicki, Mathur, hep-th/0009019)

$$D\,\chi_{(R,(r,s))}(Z^{\otimes n}, X^{\otimes 2})$$

$$= g_{YM}^2 \frac{1}{(n-1)!} \sum_{\psi \in S_{n+2}} \mathrm{Tr}_{(r,s)} \left(\Gamma_R((n,n+2)\psi - \psi(n,n+2)) \right)$$

$$Z^{i_1}_{i_{\psi(1)}} \cdots Z^{i_{n-1}}_{i_{\psi(n-1)}} X^{i_{n+1}}_{i_{\psi(n+1)}} (XZ - ZX)^{i_n}_{i_{\psi(n)}} \delta^{i_{n+2}}_{i_{\psi(n+2)}}$$

Excited Giant Gravitons

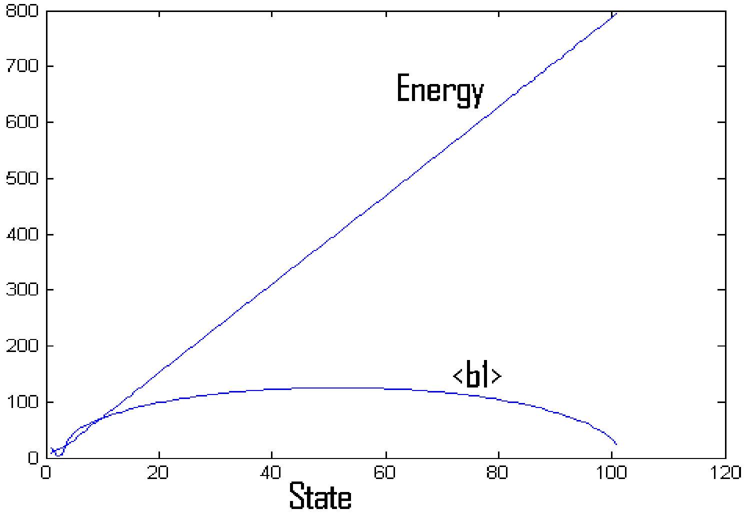


Figure: Numerically obtained spectrum and brane separation