Baryonic symmetries in AdS_4/CFT_3

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w/. N.Benishti & J.Sparks

(Many thanks to N.Benishti, S.Franco, A.Hanany, I.Klebanov, J.Park & J.Sparks!!!)

- Much less is know about AdS_{d+1}/CFT_d for d other than 4. One reason is that finding CFT´s is difficult
- d=3 is an interesting case:
  - AdS_4 factor: quantum gravity in 4d???
  - 3d CFT: relations to condensed matter systems???
- Of course, understanding (super) YM is interesting by itself, and AdS/CFT is a powerful tool

- Let's start with maximally SUSY SYM in 3d. At the abelian level it just has 7 scalars + U(1) gauge field. Upon duality, it is just 8 scalars
- SO(8) is actually an R-symmetry: there is a conformal algebra with such property

Flow to an IR sCFT!

(Seiberg)

- In 3d the YM coupling has $[g_{YM}] = m^{1/2}$. But we can construct a CS (dimensionless) term natural for CFT → ABJM (after BL, G)

(Schwarz)

- The 8 transverse scalars suggest a (2+1)d object in 11d: an M2 brane. Indeed, close to the branes (the "IR flow") the space is AdS

AdS_4/CFT_3

- Obvious question: can we generalize this to more generic (less symmetric) spaces???
- A particularly important role should be played by (global) symmetries: R (CY) and baryonic (non-trivial topology)

This talk´s motivation!

Contents

- Motivation
- AdS_4/CFT_3 and (Betti) vector fields: bulk perspective
- Towards a field theory interpretation
- Resolving the cone: M5 baryonic condensate and Goldstone bosons
- What about 6-cycles?
- Conclusions

AdS/CFT and Betti vectors

- We are interested in CY4 compactifications of 11d SUGRA: at least N=2 SUSY, such that there is an R-charge
- The CY4 X will be a cone over a 7d SE base Y. We will assume toric CY´s
- The standrd Freund-Rubin ansatz is

$$
\begin{aligned}
ds^2_{11} &= h^{-2/3}\, ds^2_{1,2} + h^{1/3} ds^2(X)\,; \qquad h = 1 + \frac{R^6}{r^6} \\
G &= d^3x \wedge dh^{-1} \\
& \Downarrow \\
ds^2_{11} &= ds^2(AdS_4) + R^2 ds^2(Y)\,; \qquad (\rho_{AdS_4} \sim r^2)
\end{aligned}
$$

- Generically Y will have non-trivial topology: we can reduce the 6-form on b_2 5-cycles to get b_2 vectors in AdS (Betti)

$$\delta C_6 = \frac{2\pi}{T_5} \sum_{i=1}^{b_2} \mathcal{A}_i \wedge \alpha_i$$

- Vectors in AdS admit the two possible falloffs

$$\mathcal{A}_\mu \sim a_\mu + \frac{j_\mu}{\rho^{d-2}}\,, \qquad \rho \to \infty$$

- This raises the possibility of choosing either fall-off: each will be dual to a DIFFERENT boundary CFT.

- In order to properly define the variational problem in AdS one has to take care of the boundary terms: this requires some boundary terms in the action depending on what b.c. one wishes to impose (Klebanov & Witten, Witten, Marolf & Ross)
- The outcome is that the two b.c. (either a_μ or j_μ fixed at the boundary) are Legendre transformed one of the other, and thus cannot be imposed at the same time

Choice of b.c. ⇔ Choice of th. with dynamical/global symm.

- Working the details:
  - Fixed j_μ : $\vec{E}=0$ No electric sources = M5
  - Fixed a_μ: $\vec{B}=0$ No magnetic sources = M2
- In the bdy. this amounts to gauging a global symm. with a backgr. field (S-operation). A T-operation is also defined: SL(2,Z) algebra

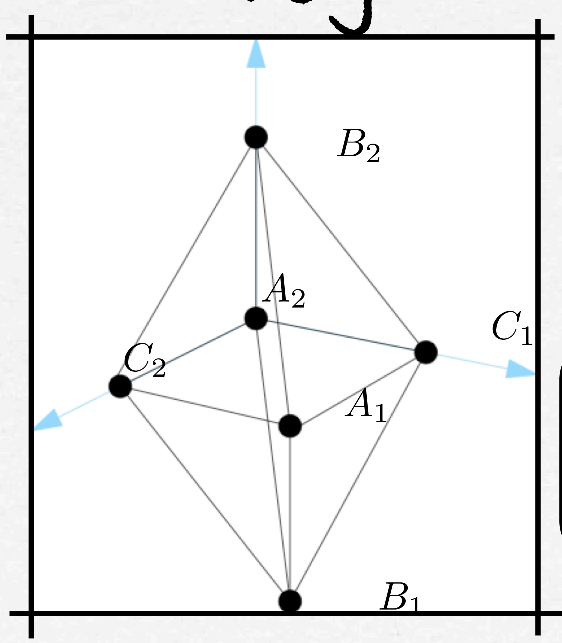
Bdy.		Bulk
$\mathcal{S} : S[A,\, j] \to S[A,\, j] + \int C \wedge dA$	⇔	Bulk E-M duality
$\mathcal{T} : S[A,\, j] \to S[A,\, j] + \int A \wedge dA$		Bulk shift $\theta \to \theta + 2\pi$

(Witten)

- Note that the dual CFT´s are different!

- Suppose we choose b.c. such that the field is dual to a global current in the bdy. Then we are allowed to have M5 branes wrapping holomorphic cycles: DYBARIONS

- These are easy to spot for toric geometries: they are just outer points in the toric diagram



	A_1	A_2	B_1	B_2	C_1	C_2
$U(1)_I$	0	0	1	1	-1	-1
$U(1)_{II}$	-1	-1	0	0	1	1

$$\Rightarrow \mathrm{Ker}Q = \begin{pmatrix} A_1 & A_2 & B_1 & B_2 & C_1 & C_2 \\ \hline 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \end{pmatrix}$$

(Witten's GLSM)

$$\rightsquigarrow \mathcal{C}(Q^{111}) = \frac{SU(2)\times SU(2)\times SU(2)}{U(1)\times U(1)}$$

(Franco, Hanany, Park & D.R-G)

Franco, Klebanov & D.R-G)

- We can also easily identify the corresp. U(1)!

- The metric is known:

$$ds^2 = dr^2 + r^2 \left[\frac{1}{16}\Big(d\psi + \sum \cos\theta_i d\phi_i\Big)^2 + \frac{1}{8}\sum d\theta_i^2 + \sin^2\theta_i d\phi_i^2\right]$$

(D' Auria, Fre & Van Niewenhuizen)

- The divisors where to wrap M5 branes are

$$\Sigma_5 \sim \theta_i = 0,\, \theta_i = \pi\,, \qquad i = 1,2,3$$

Not all independent in cohomology, since $b_2(Q^{111}) = 2$

- Note that

$$\Delta[\Sigma_5] = \frac{N\pi}{6}\,\frac{\mathrm{Vol}(\Sigma_5)}{\mathrm{Vol}(Q^{111})} = \frac{N}{3} \rightsquigarrow \mathcal{O} \sim X^N \,/\, \Delta[X] = \frac{1}{3}$$

Towards a QFT interpretation

- Following ABJM, U(N) CS-matter theories are natural candidates for dual SCFT´s.
- Since we are interested in CY4, we have N=2 SUSY, which is the dimensional reduction of N=1 in 4d. In particular, the vector multiplet is

$$V = -2\,i\,\theta\bar{\theta}\,\sigma + 2\,\theta\gamma^{\mu}\bar{\theta}\,A_{\mu} + \theta^2\,\bar{\theta}^2\,D + \cdots$$

- Crucially, for CS theories, both $\{D,\,\sigma\}$ are auxiliary fields!!!

- The bosonic potential is

$$V_b = \mathrm{Tr}\Big\{ -4k_g\,\sigma_g\,D_g + D_g\,\mu_g - \sum_{X_{g_1g_2}} |\sigma_{g_1}\,X_{g_1g_2} - X_{g_1g_2}\sigma_{g_2}|^2 - |\partial_{X_{g_1g_2}} W|^2 \Big\}$$

$$\mu_g = X_{gg_2}\,X^\dagger_{gg_2} - X^\dagger_{gg_2}\,X_{gg_2}$$

(Tomasiello & Jafferis, Martelli & Sparks, Hanany & Zaffaroni)

- In order to find the moduli space, we look at the "geometric branch"

$$X = \mathrm{diag}(x_1, \cdots, x_N)\,, \quad \sigma = \sigma\ \forall g\ \Rightarrow \mu_g = 4\,k_g\,\sigma$$

- This leaves G-2 effective D-terms. For toric W the master space has dim=G+2. Therefore we have a CY four-fold.

- We are not yet done. We have to be more careful with the gauge symm: the overall gauge field is decoupled from bifunds. It only appears as

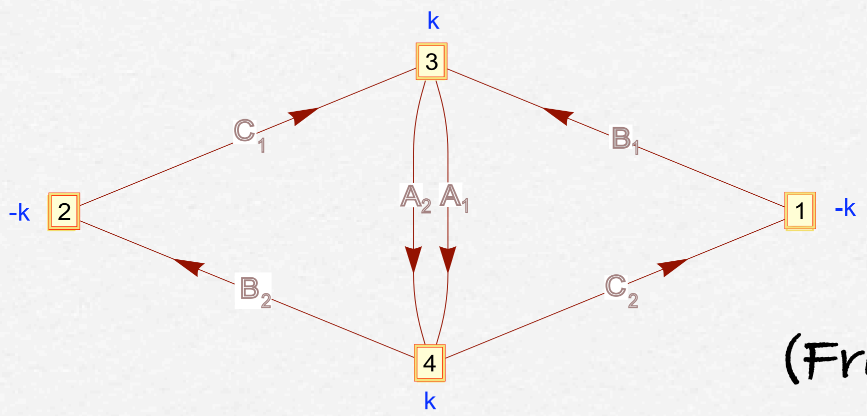
$$\frac{k}{2\pi G}\int \mathcal{B}_{G-1}\wedge d\mathcal{B}_G\,, \qquad \mathcal{B}_{G-1}=\frac{1}{k}\sum k_g\,\mathcal{A}_g\,, \quad \mathcal{B}_G=\sum_g \mathcal{A}_i\,, \quad k=\mathrm{gdc}(k_g)$$

- Dualizing this

$$\Delta S=\frac{1}{2\pi}\int \tau\, d\mathcal{F}_{\mathcal{B}_G} \quad \Rightarrow \mathcal{B}_{G-1}=\frac{G}{k}\,d\tau\,, \quad S=\int d\Big(\frac{\tau}{2\pi}\,\mathcal{F}_{\mathcal{B}_G}\Big)$$

- In turn, since the scalar is compact, the discrete gauge transformations get quantized: further orbifold quotient of the moduli space (basically the M-theory circle)!!!

- For our example



$$W = \epsilon^{ij}\,\mathrm{Tr}\Big(C_2\,B_1\,A_i\,B_2\,C_1\,A_j\Big)$$

(Franco, Hanany, Park & D.R-G)

- At k=1, for U(1), the smallest GIO's are

$$\begin{array}{llll} w_1 = A_1\,B_2\,C_1\ , & w_2 = A_2\,B_1\,C_2\ , & w_3 = A_1\,B_1\,C_2\ , & w_4 = A_2\,B_2\,C_1\ , \\ w_5 = A_1\,B_1\,C_1\ , & w_6 = A_2\,B_1\,C_1\ , & w_7 = A_1\,B_2\,C_2\ , & w_8 = A_2\,B_2\,C_2\ . \end{array}$$

- ...and satisfy

$$\left\{\begin{array}{ccccccc} w_1\,w_2 - w_3\,w_4 & = & w_1\,w_2 - w_5\,w_8 & = & w_1\,w_2 - w_6\,w_7 & = & 0\ , \\ w_1\,w_3 - w_5\,w_7 & = & w_1\,w_6 - w_4\,w_5 & = & w_1\,w_8 - w_4\,w_7 & = & 0\ , \\ w_2\,w_4 - w_6\,w_8 & = & w_2\,w_5 - w_3\,w_6 & = & w_2\,w_7 - w_3\,w_8 & = & 0\ . \end{array}\right\} \sim \mathcal{C}(Q^{111})$$

- At larger k

(Franco, Klebanov & D.R-G)

$$(w_1,\,w_2,\,w_3,\,w_4,\,w_5,\,w_6,\,w_7,\,w_8) \to (w_1,\,w_2,\,w_3,\,w_4,\,e^{i\frac{2\pi}{k}}w_5,\,e^{i\frac{2\pi}{k}}w_6,\,e^{-i\frac{2\pi}{k}}w_7,\,e^{i\frac{2\pi}{k}}w_8) \Rightarrow \frac{\mathcal{C}(Q^{111})}{\mathbb{Z}_k}$$

- At large k monopoles decouple. One can then study the non-abelian theory.
- The chiral ring precisely matches the coordinate ring of the variety (Franco, Klebanov & D.R-G)
- Crucially, in order to have such achievement, F-terms must be used (cf. the conifold in IIB)

  ⇨ Non-trivial check of W!

- However, in the U(N) theory...where are the GIO dual to the M5 branes wrapping toric divisors???

- Consider k=1. The action roughly looks like

$$S_U \sim \int \mathcal{A}_I \wedge d\mathcal{A}_{II} + \int \mathcal{B}_3 \wedge d\mathcal{B}_4 + \int \mathcal{L}_R \quad \dashrightarrow \quad S_{SU} \sim \int \mathcal{B}_3 \wedge d\mathcal{B}_4 + \int \mathcal{L}_R$$

- We can just consider S_{SU} by itself. We can act on it with the S operation on one global symm.

$$S_{SU} \to S_{SU}[\mathcal{A}_I] + \int \mathcal{C}_I \wedge d\mathcal{A}_I$$

- We can act again with S, but now on $C_I - \mathcal{A}_{II}$

$$S_{SU}[\mathcal{A}_I] + \int \mathcal{C}_I \wedge d\mathcal{A}_I \to S_{SU}[\mathcal{A}_I, \mathcal{A}_{II}] + \int \mathcal{C}_I \wedge d\mathcal{A}_I + \int \mathcal{C}_{II} \wedge d\big(C_I - \mathcal{A}_{II}\big)$$

- We can perform the path integral over the C-fields $\rightsquigarrow S_U$

Relation between $U(1)^2 \times SU(N)^4 \leftrightarrow U(N)^4$

- From the bulk perspective the S-operation is electric-magnetic duality
- In the SU(N) th. we have B=0, and thus we CAN have electric M5 brane sources
- This provides a way to identify the ungauged U(1)´s: they are precisely the Betti multiplets!!! (c.f. the GLSM description)
- In the SU(N) theory we can form baryonic operators

$$\mathfrak{B}_{A_{I_1}\cdots A_{I_N}} = \frac{1}{N!}\,\epsilon^{i_1\cdots i_N}\,\epsilon_{j_1\cdots j_N}\,(A_{I_1})^{j_1}_{i_1}\cdots(A_{I_N})^{j_N}_{i_N}$$

$$\mathfrak{B}_{B_i} = \frac{1}{N!}\det B_i e^{i\,(-1)^{i-1}\,N\tau}$$

$$\mathfrak{B}_{C_i} = \frac{1}{N!}\det C_i e^{i\,(-1)^{i-1}\,N\tau}$$

Gravity suggests R=1/3

Just the right one to put the theory at a SCFT point!!!

Resolving the cone

- The CY cone admits a crepant resolution

$$
\begin{aligned}
ds^2(X) &= \kappa(r)^{-1}dr^2 + \kappa(r)\frac{r^2}{16}\Big(d\psi + \sum_{i=1}^{3}\cos\theta_i d\phi_i\Big)^2 + \frac{(2a+r^2)}{8}\Big(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2\Big) \\
&\quad + \frac{(2b+r^2)}{8}\Big(d\theta_3^2 + \sin^2\theta_3 d\phi_3^2\Big) + \frac{r^2}{8}\Big(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2\Big)\ , \\
\kappa(r) &= \frac{(2A_- + r^2)(2A_+ + r^2)}{(2a+r^2)(2b+r^2)}\,, \quad A_\pm = \frac{1}{3}\Big(2a + 2b \pm \sqrt{4a^2 - 10ab + 4b^2}\Big)
\end{aligned}
$$

- Consider partial resolution. Locating the branes at, say, the N of the blown-up sphere

$$
\Delta_x h[y] = \frac{(2\pi\ell_p)^6 N}{\sqrt{\det g_X}}\delta^8(x-y) \ \Rightarrow \ h(r,\theta_3) = \sum_{l=0}^{\infty} H_l(r)\, P_l(\cos\theta_3)\ ,
$$

$$
H_l(r) = \mathcal{C}_l\left(\frac{8b}{3r^2}\right)^{3(1+\beta)/2} {}_2F_1\left(-\tfrac{1}{2} + \tfrac{3}{2}\beta, \tfrac{3}{2} + \tfrac{3}{2}\beta, 1 + 3\beta, -\tfrac{8b}{3r^2}\right)\ ,
$$

$$
\beta = \beta(l) = \sqrt{1 + \frac{8}{9}l(l+1)}\ , \quad \mathcal{C}_l = \frac{3\Gamma(\frac{3}{2} + \frac{3}{2}\beta)^2}{2\Gamma(1+3\beta)}\left(\frac{3}{8b}\right)^3 (2l+1)\, R^6\ , \quad R^6 = \frac{(2\pi\ell_p)^6 N}{6\mathrm{vol}(Q^{111})} = \frac{256}{3}\pi^2 N \ell_p^6
$$

- Looking at the assymptotic form of the warp factor

$$h(r,\theta_3) \sim \frac{R^6}{r^6}\left(1+\frac{18b\,\cos\theta_3}{5r^2}+\cdots\right) \;\to \langle\mathcal{U}\rangle \sim b\,\cos\theta_3\,/\,\Delta(\mathcal{U})=1$$

- This is suggestive of SSB of a global symmetry. Indeed the natural dim. 1 op. is

$$\mathcal{U} \sim \sum_{X_g} q_X\, X_g\, X_g^\dagger \sim \mu_g$$

(c.f. with the conifold example. Klebanov & Murugan)

- Since this would break a baryonic symmetry, it is natural to guess that a baryon gets a VEV: look for baryonic condensates=euclidean M5 branes!

- The baryonic condensate is an euclidean brane wrapping the cone over the dual 5-cycle. By the AdS/CFT duality, its action is the desired VEV

$$S(r_c) = T_5 \int_{r \le r_c} h \sqrt{\det g_6}\, d^6x \ ,$$

- After some computations we get

$$e^{-S(r_c)} = e^{7N/18} \left(\frac{8b}{3\, r_c^2}\right)^{\frac{N}{3}} \left(\sin \frac{\theta_3}{2}\right)^N \quad .$$

- This is expected to be

$$\langle \mathfrak{B}_X \rangle \sim b^{\Delta_X\, N} \Rightarrow \Delta_X = \frac{1}{3}$$

- Non-trivial check of the R-charge assignation

- An M2 brane wrapped on the blown-up cycle is SUSY. In analogy with the IIB case, it is the string of the SSB

(c.f. the conifold in IIB. Klebanov, Murugan, D.R-G, Ward)

- It sources a 3-form fluctuation

$$\delta C_3 = \tilde{A} \wedge \beta \ , \ d\beta = 0 \, , \ d(h \star_8 \beta) = 0 \Rightarrow d \star_3 d\tilde{A}$$

- In the unwarped case this can be explicitly solved. In the warped case we can argue an appropriate solution exists

$$I = \int_X \beta \wedge \star_8 h \, \beta = \int_0^\pi d\theta_2 \int_0^\pi d\theta_3 \int_0^\infty d\varrho \, h \, \sqrt{\det g} \sin\theta_2 \sin\theta_3 \Big[(\partial_\varrho f_0)^2 + \\ + \frac{(\partial_{\theta_3} f_0)^2}{U\,(l_3^2+\varrho)} + \frac{(\partial_{\theta_2} f_0)^2}{U\,(l_2^2+\varrho)} + \frac{(f_0 - \partial_{\theta_3} f_3 - \cot\theta_3 f_3)^2}{(l_3^2+\varrho)^2} + \frac{(\partial_{\theta_3} f_2)^2 + (\partial_{\theta_2} f_3)^2}{(l_2^2+\varrho)(l_3^2+\varrho)} + \\ + \frac{(-1 + f_0 - \cot\theta_2 f_2 - \partial_{\theta_2} f_2)^2}{(l_2^2+\varrho)^2} + \frac{U}{(l_2^2+\varrho)}(\partial_\varrho f_2)^2 + \frac{U}{(l_3^2+\varrho)}(\partial_\varrho f_3)^2 + \frac{f_0^2}{\varrho^2} \Big] \, .$$

- The Hodge dual is

$$\delta G_7 = \star_3 dA \wedge h \star_8 \beta \ .$$

- This can be re-written as

$$\star_3 dA = dp \Rightarrow \delta G_7 = dp \wedge h \star_8 \beta = d\left(p\, h \star_8 \beta\right) \ ,$$

- Together with the boundary behavior of the 2-form

$$\beta \sim \frac{2}{3}\, e_{\theta_2} \wedge e_{\phi_2} - \frac{1}{3}\, e_{\theta_1} \wedge e_{\phi_1} - \frac{1}{3}\, e_{\theta_3} \wedge e_{\phi_3} \ ,$$

- This suggests

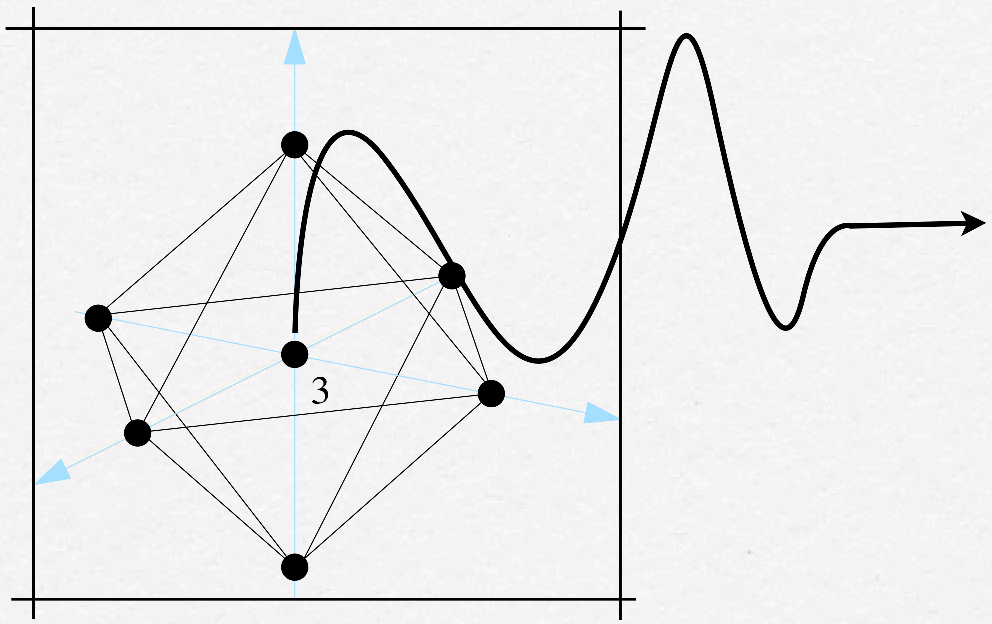
$$\langle J_\mu \rangle = f_\pi^{-1}\, \partial_\mu p \,, \quad f_\pi \sim b^{-1}$$

(excpetional) 6-cycles

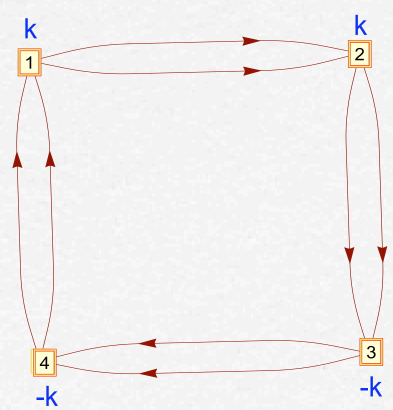
- For isolated singularities with no 6-cycles we expect to be able to do the same analysis as for Q111. This is basically because when a lagrangian description exists, it is basically the minimal GLSM.
- What about (exceptional) 6-cycles???
- We use as toy model an orbifold of Q111 along the U(1) R-symmetry known as Q222

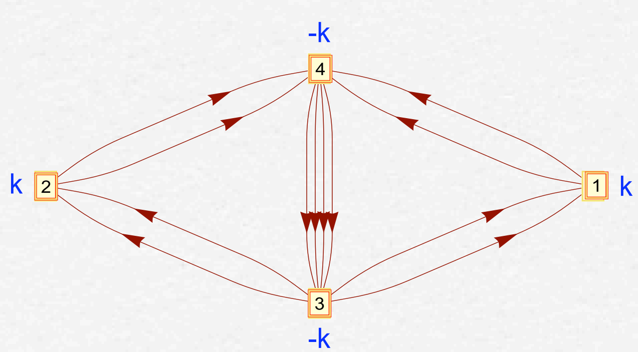
(D' Auria, Fre & Van Niewenhuizen)

- The metric is the same (cutting in half the U(1)-R angle period). The toric diagram is



- A U(N) theory has been proposed





$$W_I = \mathrm{Tr}\, \epsilon_{ij}\, \epsilon_{mn}\, X^i_{12}\, X^m_{23}\, X^j_{34}\, X^n_{41}\ ,$$
$$W_{II} = \mathrm{Tr}\left(\epsilon_{ij}\, \epsilon_{mn}\, X^i_{32}\, X^m_{24}\, X^{jn}_{43} - \epsilon_{ij}\, \epsilon_{mn}\, X^m_{31}\, X^i_{14}\, X^{jn}_{43} \right) .$$

(Franco, Klebanov & D.R-G)

- The abelian moduli space gives the expected variety. The (large k) chiral ring matches the expectation (again with crucial use of F-terms)
- However, the GLSM is NOT the minimal one. The baryonic symmetry story would be the same as in its Q111 cousin. Thus, it seems there is not enough room in the master space to accomodate the exceptional 6-cycle...
- On the other hand, 6-cycles should have a different interpretation...

- Once blown-up, we can wrap an E5 brane on it
- As opposed to the IIB case, this brane sees the same SUSY´s as if there was no "sourcing" M2's
- Therefore, naively the counting of fermionic zero modes suggests this E5 to generate a non-perturbative W
- The warped volume should give (the real) part of the W

$$W = e^{-S_{DBI} - iS_{WZ}}\,, \quad S_{DBI} = T_5 \int_D \sqrt{\det g_D}\, h\, d^6x$$

(Witten, Baumann, Dymarsky, Klebanov, Maldacena & Murugan)

- In the case at hand

$$W = e^{-S} = \exp\left[\frac{2\,N}{3}\,\frac{r_\star^6}{r_0^6}\,{}_2F_1\Big(\frac{3}{4},\,1,\,\frac{7}{4},\Big(\frac{r_\star}{r_0}\Big)^8\Big)\right]\ ,$$

There are very nice mathematical results one can prove which give warped volumes in generic CY´s

- Thus, either the cycle is not blown-up or the M2 branes are constrained to live on it (reduced mesonic moduli space)
- The complex part is missing, as well as how to write this in terms of field theory quantities
- Besides, the field theory interpretation is not clear

Conclusions

- AdS_4/CFT_3 is much richer than its 4d cousin
- We set out for the study of (Betti) vector multiplets in AdS_4. Suitable choice of b.c. allow to consider a theory with dybarions
- These, being dual to M5 branes, allow for a number of non-trivial checks (VEV´s, SSB...)
- However there are many aspects to be better understood: resolution sequences, explicit action of T/S, generic cases, relation to flavored th... (Benini, Closset & Cremonesi, Jafferis)

- Also one would like to understand the role of the M-th circle. For example, it should give a (Killing vector-like) gauge field in AdS. What is its role???
- Non-perturbative W's and 6-cycles also require more understanding. Do these cycles really contribute a W? If so, what is the FT interpretation? How to holomorphically write the warped volumes?...

Many thanks!!!!