Holography & Correlation functions

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Outline

1) Gravity & thermodynamics
2) Emergent IR CFTs
3) Gauge theory expansion parameters

Large degeneracy systems

- Universality of thermodynamics
  (independent of microscopics dynamics)
- Statistical mechanics : bridge between microscopic and macroscopic descriptions

$$\left\{ \begin{array}{l} \text{micro} \sim \text{large \# parameters} \\ \text{macro} \sim \text{few parameters} \end{array} \right\}$$

thermodynamics ~ coarse-grained description

Identification of states

Questions :

1. how do you identify a pure state among the set of degenerate states $\{|\Psi_A\rangle\}$ having the charges of interest (or inside your resolution apparatus scale)?
2. How good is the thermal description ?

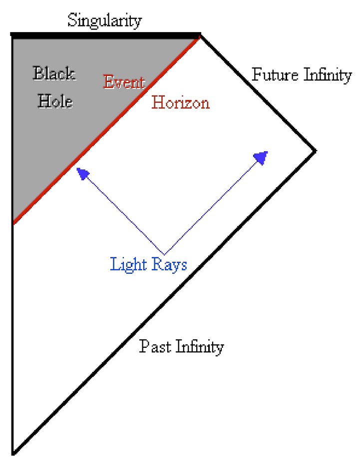
$$\rho_T = \sum_A e^{-\beta E_A} \, |\Psi\rangle_A \langle\Psi|_A$$

Answer : One measures gauge invariant quantities and/or waits for long periods of time

$$_A\langle\Psi|\mathcal{O}|\Psi\rangle_A \qquad , \qquad \mathrm{Tr}\left(\rho_T\, \mathcal{O}\right)$$

Why are BHs so charming ?

- Solutions to Einstein's equations characterised by conserved charges : Mass (M), angular momentum (J), electric/magnetic charge (Q)
- Existence of a null hypersurface (event horizon)



Classical mechanics of BHs

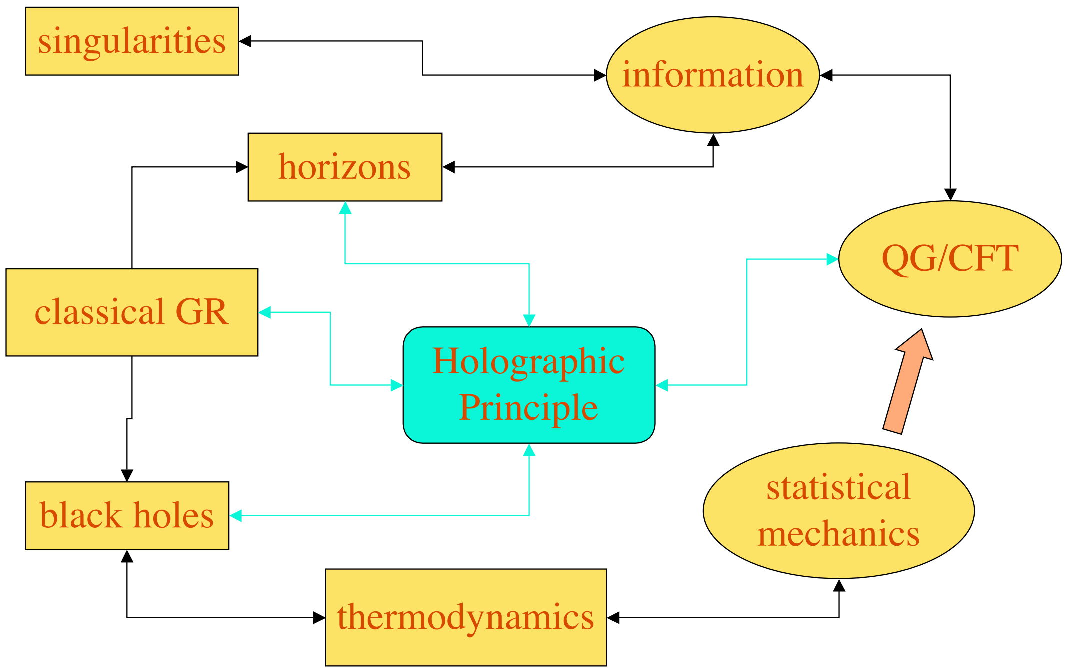
- Variations in a BH with charges (M, J, Q) and event horizon area A satisfy the laws of thermodynamics iff

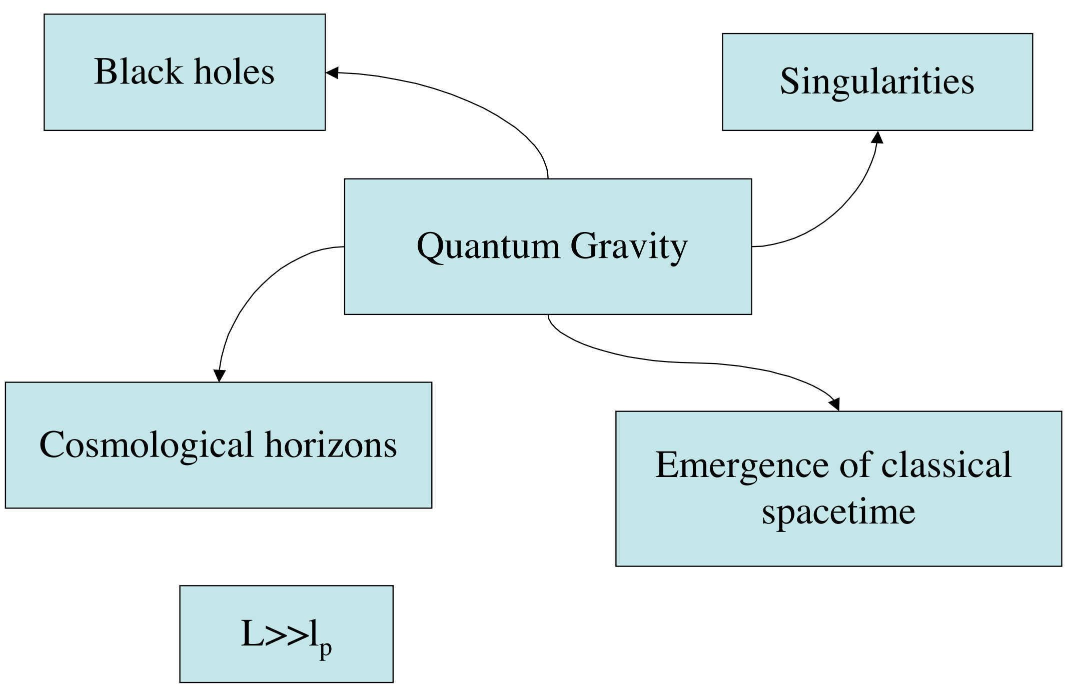
  S ~ A (entropy)

  T = T(κ) (temperature)

- Coefficient computed semiclassically by Hawking

$$S = \frac{A}{4G}$$


singularities
information
horizons
QG/CFT
classical GR
Holographic Principle
statistical mechanics
black holes
thermodynamics


Black holes
Singularities
Quantum Gravity
Cosmological horizons
Emergence of classical spacetime
L>>l_p

Coarse-graining

Conjecture (origin of grav.thermodynamics):

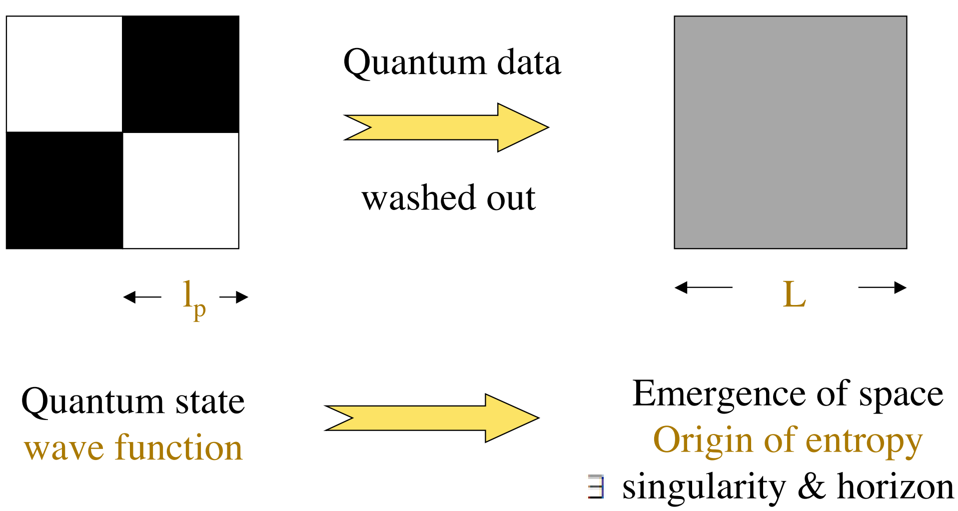
∃ scale L, such that by coarse-graining quantum data at smaller scales (non-accessible to a semiclassical observer), all microstates look alike

Consequence : BH ~ coarse-grained object

Emergence of spacetime & singularities

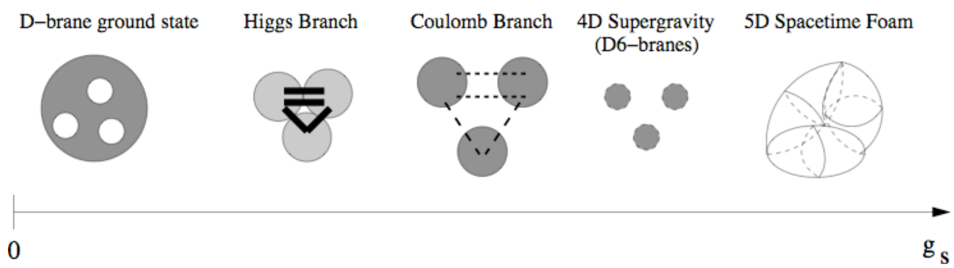
Loss of information : consequences for information paradox

Heuristic RG picture



String Theory Perspective

- Degeneracy of D-branes compatible with (M, J, Q) is a partition problem (Ramanujan,…)
- Counting done at g_s N « 1, but d_{micro} does NOT change, because of susy, and is valid at g_s N » 1



AdS/CFT perspective

- Its lorentzian formulation is essential to carry the information on the microstate, i.e. boundary conditions of bulk fields
- Euclidean partition function evaluated by saddle point approximation knows about the number of states, but not about the specific microstates
- A state is characterised by an infinite set of of one-point functions !! whereas a black hole geometry requires a few parameters !!

Speculations

- The scale L corresponds to the scale at which the transition between closed and open strings occurs
- Quantum gravity effects are not confined to the singularity of the black hole

Distinction of states

- Correlation functions

$$c^k_\psi(x^1,\ldots,x^k) = \langle\psi|\mathcal{O}(x^1)\cdots\mathcal{O}(x^k)|\psi\rangle$$

$$\begin{aligned}\langle c^k(x^1,\ldots,x^k)\rangle_{\mathcal{M}_{sup}} &= \int D\psi\, c^k_\psi(x^1,\ldots,x^k)\\ \mathrm{var}[c^k(x^1,\ldots,x^k)]_{\mathcal{M}_{sup}} &= \int D\psi\,(c^k_\psi(x^1,\ldots,x^k))^2 - \langle c^k(x^1,\ldots,x^k)\rangle^2_{\mathcal{M}_{sup}}\end{aligned}$$

$$\frac{\sigma[c^k(x^1,\ldots,x^k)]_{\mathcal{M}_{sup}}}{\langle c^k(x^1,\ldots,x^k)\rangle_{\mathcal{M}_{sup}}} = \frac{\sqrt{\mathrm{var}[c^k(x^1,\ldots,x^k)]_{\mathcal{M}_{sup}}}}{\langle c^k(x^1,\ldots,x^k)\rangle_{\mathcal{M}_{sup}}}$$

- Comparison with thermal answer
- At which spacetime scale do these deviations show up ?

Quantum Gravity Scales

Lagrangian vs Holographic perspectives

l_p vs L

information requires space !!

Black hole requires r_h to describe degeneracy

$r_h \sim L \sim N^a \, l_p \quad (a > 0)$

$N \sim$ number of constituents

Classical fuzzball

Set-up : look for classical configurations satisfying

1. Same asymptotics and same charges
2. Horizonless
3. Globally smooth (includes absence of CTCs)

classical moduli space

Remark : not all microstates will allow a reliable classical description

“Quantum” fuzzball

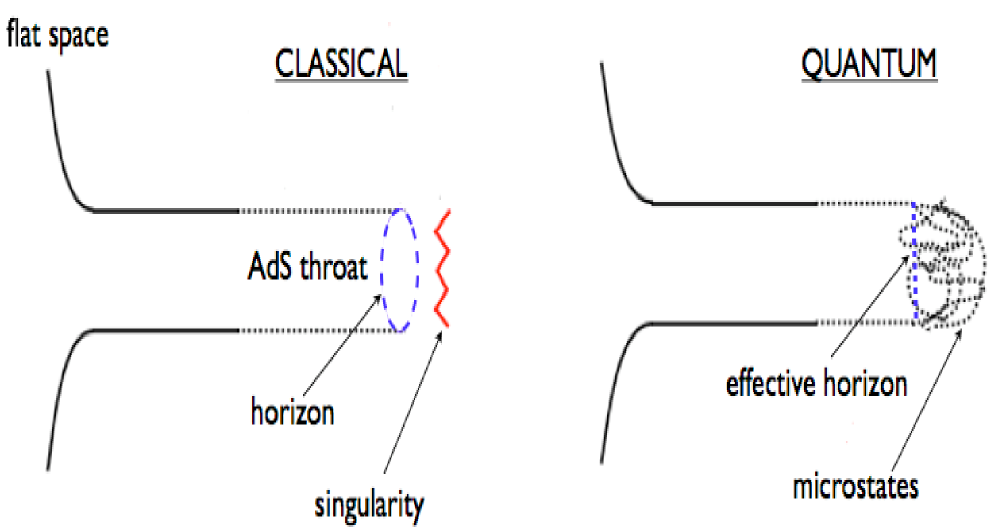
Classical moduli space → Quantum moduli space

geometric quantization

This procedure generates a Hilbert space by working in a subspace of field configurations

- Counting of states (macroscopic)
- Validity of classical description (size of fluctuations)

An intuitive picture for the fuzzball



Status report

- ✓ Small BPS black holes (in certain U-duality frames)
- Existence of scaling solutions for large BPS black holes

  not enough states, quantum corrections ?

- Preliminary work on extremal non-BPS and non-extremal black holes

Large Black Holes

Given a **large black hole**

- It defines a singular lorentzian geometry (in the deep interior), are there any kind of smooth configurations whose quantization reproduces the entropy of the BH ?
- It defines a smooth euclidean configuration, where is the information regarding pure states in the euclidean path integral?

Euclidean Black Holes

- Regular euclidean geometries with suitable boundary conditions
- Saddle points in euclidean path integrals
- Existence of a hole in the geometry

  (interior of the lorentzian black hole !!)

Remarks

- Importance of α' and g_s corrections
- Relation between different formulations :

  microcanonical vs canonical ensembles

$$\Updownarrow$$

lorentzian vs euclidean geometries

Ensemble comparison

$\exists$ scaling solutions, with not enough entropy

α' and g_s corrections ?

$\exists$ saddle points smooth, but interior removed

where is information on microstates stored in the euclidean path integral ?

Questions

- Rules for euclidean path integrals
- Role of complex metric configurations
- $\exists$ euclidean microcanonical path integral or exact path integral vs saddle point approximations and choice of boundary conditions/euclidean topology

$$\begin{aligned}
\int_{f_a} D[g_a] D[\Phi_a]\, e^{-I_{f_a}} &= e^{-I_{f_a}+S} = e^{-I_{BH}} \\
I_{f_a} &= \beta F = \beta\,(E - \mu_i Q_i) \\
I_{BH} &= \beta F = \beta\,(E - \mu_i Q_i) - S
\end{aligned}$$

Information Paradox

- If black hole ~ thermal state , how can unitary evolution evolve a pure state into a thermal density matrix ?

(general thermalization problem in physics)

Resolution : coarse-graining

Technically, what does go wrong ?

Breaking down of EFT ?

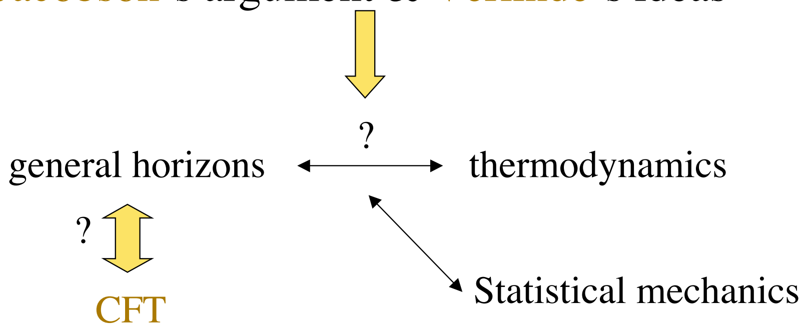
- The use of EFT and thermal state lead to loss of unitarity ⟹ EFT breaks down
- Conjecture :

$$\langle \phi_1 \dots \phi_N \rangle_{\mathrm{QG}} = \langle \phi_1 \dots \phi_N \rangle_{\mathrm{EFT}} + \mathcal{O}\left(e^{-(S-N)}\right)$$

when we probe a “BH state” with an operator as “heavy” as the state, the EFT breaks down because information about the microscopics of the “thermal state” becomes relevant and not subleading

GR is thermodynamical in nature

- Physics of black holes (event horizons)
- De Sitter physics
- FRW spacetimes and apparent horizons
- Jacobson's argument & Verlinde's ideas



General Holography

- Brown-Henneaux requires a boundary
- Entropy may require a horizon

Solodukhin/Carlip : horizon as a boundary

$\exists$ chiral Virasoro algebra for any Killing horizon

$$c = \frac{3A}{2\pi G \kappa}$$

Cardy vs Bekenstein-Hawking

Using Cardy's formula : $S = 2\pi\sqrt{\frac{c}{6}\left(\Delta - \frac{c}{24}\right)}$

we can reproduce Bekenstein-Hawking :

$$S = \frac{\pi^2}{3} c T = \frac{A}{4G}$$

also some equipartition (relation to Verlinde's assumption ?)

$$E = ST/2$$

Extremal BHs/CFT ??

- A Brown-Henneaux analysis at the boundary of the near horizon geometry + boundary conditions

$\exists$ asymptotic symmetry group compatible with a chiral Virasoro algebra

What is Brown-Henneaux ?

Semiclassical identification of normalizable modes and symmetry generators acting on them

$$g_{\mu\nu} = g^0_{\mu\nu} + \delta g_{\mu\nu}$$ boundary condition

$$(\mathcal{L}_\xi g)_{\mu\nu} \sim \delta g_{\mu\nu}$$ symmetry generator

Near horizon geometries :

$$\begin{aligned} ds^2 &= \Gamma(\theta)\left[-r^2dt^2 + \frac{dr^2}{r^2} + \alpha(\theta)d\theta^2\right] + \gamma(\theta)(d\phi + krdt)^2 \\ A &= f(\theta)(d\phi + krdt) \end{aligned}$$

Boundary conditions :

$(t,\ \phi,\ \theta,\ r)$

$$\begin{aligned} \delta g_{\mu\nu} &\sim \mathcal{O}\begin{pmatrix} r^2 & 1 & 1/r & 1/r^2 \\ & 1 & 1/r & 1/r \\ & & 1/r & 1/r^2 \\ & & & 1/r^3 \end{pmatrix} \\ \delta A_\mu &\sim \mathcal{O}(r, 1/r, 1, 1/r^2) \end{aligned}$$

Symmetry generators :

$$\begin{aligned} \xi_\epsilon &= \epsilon(\phi)\partial_\phi - r\epsilon'(\phi)\partial_r \\ \bar{\xi} &= \partial_t \end{aligned}$$

Central charge & Temperature :

$$c = 3k\int_0^\pi d\theta\,\sqrt{\Gamma(\theta)\,\alpha(\theta)\,\gamma(\theta)} \qquad T = \frac{1}{2\pi k}$$

Cardy vs Bekenstein-Hawking :

$$S = \frac{\pi^2}{3}cT = \frac{A}{4}$$

Lessons ?

- $\exists$ emergent IR CFTs

  connection to condensed matter systems ?

- Emergence of IR CFTs in large charge sectors of N=4 SYM (excitations on top of bound states of giant gravitons)

  CFT circle is emergent in the gauge theory !!

Further evidence

- Computation of greybody factors compatible with a CFT result
- Wave equations ~ conformal close to horizon (Rindler)
- Assuming AdS/CFT like duality

$$Z_{CFT}[\phi_0(\vec{x})] = \langle e^{-\int \phi_0(\vec{x})\,\mathcal{O}(\vec{x})} \rangle_{CFT} = Z_{\text{string}}[\phi_0(\vec{x})] \sim e^{-S_{\text{SUGRA}}[\phi_0(\vec{x})]}$$

match of extremal 3 point functions

Emergent IR CFTs : CM view

- Consider d-dimensional CFT with a global U(1) symmetry at finite charge density
- Dual description : black hole in d+1 AdS with U(1) current mapped to a U(1) gauge field in AdS
- Probe the black hole with scalar/fermionic operators, compute their retarded Green's function. Study their behaviour in the extremal limit and for small bulk field frequencies

Quantum criticality, Fermi surfaces and AdS_2

- The $T = 0, \quad \omega \to 0$ limit of the bulk retarded Green's function

$$G_R(\omega, k) = K \frac{b_+^{(0)} + \omega\, b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega\, b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega\, a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega\, a_-^{(1)} + O(\omega^2)\right)}$$

where $\mathcal{G}_k(\omega)$ is the retarded Green's function of the IR CFT

$$\mathcal{G}_k(\omega) = e^{-i\pi\nu_k} \frac{\Gamma(-2\nu_k)\Gamma(1+\nu_k - iqe_d)}{\Gamma(2\nu_k)\Gamma(1-\nu_k - iqe_d)} \frac{\left(m + \frac{ikR}{r_*}\right) R_2 - iqe_d - \nu_k}{\left(m + \frac{ikR}{r_*}\right) R_2 - iqe_d + \nu_k} (2\omega)^{2\nu_k}$$

Brief comments

- Non-analytic part controlled by IR CFT
- Low energy behaviour depends on the conformal dimensions in the IR CFT and $G_R(\omega = 0, \vec{k})$

- CM physics $\Longleftrightarrow$ AdS_2x R^{d-1}

   absence of fragmentation

String theory embedding

- Black hole backgrounds are R-charged black holes in AdS
- Their BPS limits ~ distribution of giants gravitons, whose dual operators are known
- Can we get any gravity insight by studying the near horizon limits of these backgrounds in near extremal limits ? Can we borrow any of the ideas developed in BMN in these limits ?

The AdS backgrounds

$$ds_{10}^2 = \sqrt{\Delta}\, ds_5^2 + \frac{1}{\sqrt{\Delta}}\, d\Sigma_5^2$$

$$ds_5^2 = -\frac{f}{H_1 H_2 H_3} dt^2 + \frac{dr^2}{f} + r^2\, d\Omega_3^2$$

$$d\Sigma_5^2 = \sum_{i=1}^{3} L^2 H_i \left(d\mu_i^2 + \mu_i^2\, [d\phi_i + a_i\, dt]^2\right)$$

$$B_4 = -\frac{r^4}{L} \Delta\, dt \wedge d^3\Omega - L \sum_{i=1}^{3} \tilde{q}_i\, \mu_i^2 \left(L\, d\phi_i - \frac{q_i}{\tilde{q}_i} dt\right) \wedge d^3\Omega$$

$$H_i = 1 + \frac{q_i}{r^2}, \qquad a_i = \frac{\tilde{q}_i}{q_i} \frac{1}{L} \left(\frac{1}{H_i} - 1\right),$$

$$f = 1 - \frac{\mu}{r^2} + \frac{r^2}{L^2} H_1\, H_2\, H_3, \qquad \Delta = H_1\, H_2\, H_3 \left[\frac{\mu_1^2}{H_1} + \frac{\mu_2^2}{H_2} + \frac{\mu_3^2}{H_3}\right],$$

$$\mu_1 = \cos\theta_1, \qquad \mu_2 = \sin\theta_1 \cos\theta_2, \qquad \mu_3 = \sin\theta_1 \sin\theta_2.$$

Charges & giants

- Mass and R-charge

$$M = \frac{\pi}{4G_N^{(5)}} \left(\frac{3}{2}\mu + q_1 + q_2 + q_3 + \frac{3L^2}{8}\right)$$

$$J_i = \frac{\pi L}{4G_N^{(5)}} \tilde{q}_i\,, \qquad \tilde{q}_i = \sqrt{q_i(\mu + q_i)}$$

- Number of giants

$$N_i = \frac{2J_i}{N} = \frac{\pi^4}{2N} \cdot \frac{L^8}{G_N^{(10)}} \cdot \frac{\tilde{q}_i}{L^2}$$

Getting some intuition in AdS$_5$

- One R-charge : $\forall \mu \neq 0 \quad \Rightarrow \quad S \sim \frac{N^2}{L^2}\,\mu$
- Two R-charges :

$$0 < \mu < \mu_c = \frac{q_1 q_2}{L^2} \quad \Rightarrow \quad \nexists \text{ horizon}$$

$$\mu \geq \mu_c \quad \Rightarrow \quad S \sim \frac{N\sqrt{\mu - \mu_c}}{L}\sqrt{N_1 N_2}$$

- Three R-charges :

$$\mu < \mu_c \quad \Rightarrow \quad \nexists \text{ horizon}$$

$$\mu \geq \mu_c \quad \Rightarrow \quad S \sim \sqrt{N}\sqrt{N_1 N_2 N_3}$$

Two R-charge near-extremal limits

- Near-BPS limit

$$\mu \to 0\,, \qquad \mu_c \to 0\,, \qquad \mu - \mu_c \to 0 \qquad \Rightarrow \qquad q_i \to 0 \;\; \text{(dilute approx)}$$

- Near-extremal non-BPS limit

$$\mu - \mu_c \to 0 \qquad \{\mu,\, \mu_c\} \text{ fixed}$$

Near-BPS limit

- The limit :

$$r = \epsilon\tilde{\rho}, \qquad \theta_i = \theta_i^0 - \epsilon^{1/2}\hat{\theta}_i,\ 0 \le \theta_i^0 \le \pi/2,$$
$$\mu - \mu_c = \epsilon^2 M, \qquad q_i = \epsilon\hat{q}_i, \qquad \psi_i = \frac{1}{\epsilon^{1/2}}\left(\phi_i - \frac{t}{L}\right),\ i = 2,3,$$

- The resulting metric :

$$ds^2 = \epsilon\left[R_S^2\left(ds_{AdS}^2 + d\Omega_3^2\right) + \frac{L^2}{R_S^2}ds_{C_4}^2\right]$$
$$ds_{AdS}^2 = -(\rho^2-\gamma^2)d\tau^2 + \frac{d\rho^2}{\rho^2-\gamma^2} + \rho^2 d\phi_1^2$$

$$\gamma^2 \equiv \frac{\mu - \mu_c}{\mu_c}$$

Remarks

- Using connection to AdS_3

$$\gamma = -1\ \ (\mu = 0) \quad \Rightarrow \quad \mathrm{AdS}_3$$
$$-1 < \gamma < 0\ \ (\mu_c > \mu > 0) \quad \Rightarrow \quad \text{conical defects}$$
$$\gamma \ge 0\ \ (\mu \ge \mu_c) \quad \Rightarrow \quad \text{BTZ black holes}$$

- We can add the third R-charge perturbatively, giving rise to rotating BTZ black holes

N=4 SYM interpretation

- Requiring gravity to be weakly curved determines

$$\epsilon \sim \frac{1}{\sqrt{N}}$$

- BMN-like interpretation

$$\Delta \sim J_i \sim N^{3/2}, \qquad \lambda = g_{YM}^2 N \sim N \to \infty$$

$$\frac{J_i}{N^{3/2}} \equiv \frac{\hat{q}_i}{L^2} = \text{fixed}, \qquad (\Delta - \sum_{i=2,3} J_i) \cdot \frac{1}{N} = \frac{\hat{\mu}}{L^2} = \text{fixed}$$

Near-extremal non-BPS limit

- Limit

$$r = \epsilon \tilde{\rho}, \qquad t = \frac{\tilde{\tau}}{\epsilon}, \qquad \mu - \mu_c = \epsilon^2 M$$

$$\phi_1 = \frac{\varphi}{\epsilon}, \qquad \phi_i = \psi_i + \frac{\tilde{q}_i}{q_i L} \frac{\tilde{\tau}}{\epsilon}, \quad i = 2, 3$$

- Resulting metric

$$ds_{10}^2 = \mu_1 \left(R_{AdS_3}^2\, ds_3^2 + R_S^2\, d\Omega_3^2\right) + \frac{1}{\mu_1} ds_{\mathcal{M}_4}^2 \qquad R_{AdS_3}^2 = \frac{R_S^2}{f_0},$$

$$ds_3^2 = -(\rho^2 - \rho_0^2) d\tau^2 + \frac{d\rho^2}{\rho^2 - \rho_0^2} + \rho^2 d\varphi^2 \qquad R_S^2 \equiv \sqrt{q_2 q_3} = \sqrt{L^2 \mu_c}\ ,$$

$$ds_{\mathcal{M}_4}^2 = \frac{L^2}{R_S^2} \left[q_2\, (d\mu_2^2 + \mu_2^2\, d\psi_2^2) + q_3\, (d\mu_3^2 + \mu_3^2\, d\psi_3^2)\right]$$

$$\rho_0^2 = \frac{M}{\mu_c}\ , \qquad f = f_0 - \frac{M}{\tilde{\rho}^2}$$

Remarks

- $\exists$ consistent truncation to d=6 dealing with warping
- In d=6, similar comments as before
- Perturbative inclusion of the third charge as before

N=4 SYM interpretation

- Requiring gravity to be weakly curved

$$\epsilon \sim \frac{1}{N}$$

- BMN-like interpretation

$$\Delta \sim J_i \sim N^2, \qquad \lambda \sim N \to \infty,$$

$$\frac{J_i}{N^2} \equiv \frac{\tilde{q}_i}{2L^2} = \text{fixed}, \qquad \frac{\mathcal{E} - \mathcal{E}_0}{N} = \text{fixed}$$

$$\mathcal{E} = \frac{R_{AdS_3}}{R_S} \cdot \frac{N^2}{4\epsilon} \cdot \frac{\mu}{L^2} = \mathcal{E}_0 + \frac{R_{AdS_3}}{R_S} \cdot (2\pi T_s^{(6)} M)$$

$$\mathcal{E}_0 = \frac{R_{AdS_3} R_S^3}{16 L^4} \cdot N^3$$

Remarks

- Some features are reminiscent of the appearance of Rindler as the near horizon geometry for non-extremal BHs
- It is not obvious how to even define the 2d CFTs dual to the 3d local AdS_3 geometries (work in progress)
- Gravity suggests to study the gauge theory in very precise large N scalings of the different charges and impurities, to test integrability in the non-planar regime

General lesson : extremal vanishing horizons

- Existence of points in the space of extremal black holes with vanishing horizon
- Near horizon geometries classically controlled by local AdS_3

$$S, T \to 0 \qquad \frac{S}{T} \sim c \ \text{fixed}$$

- Gravity predicts existence of non-trivial IR physics in large charge sectors of N=4 SYM