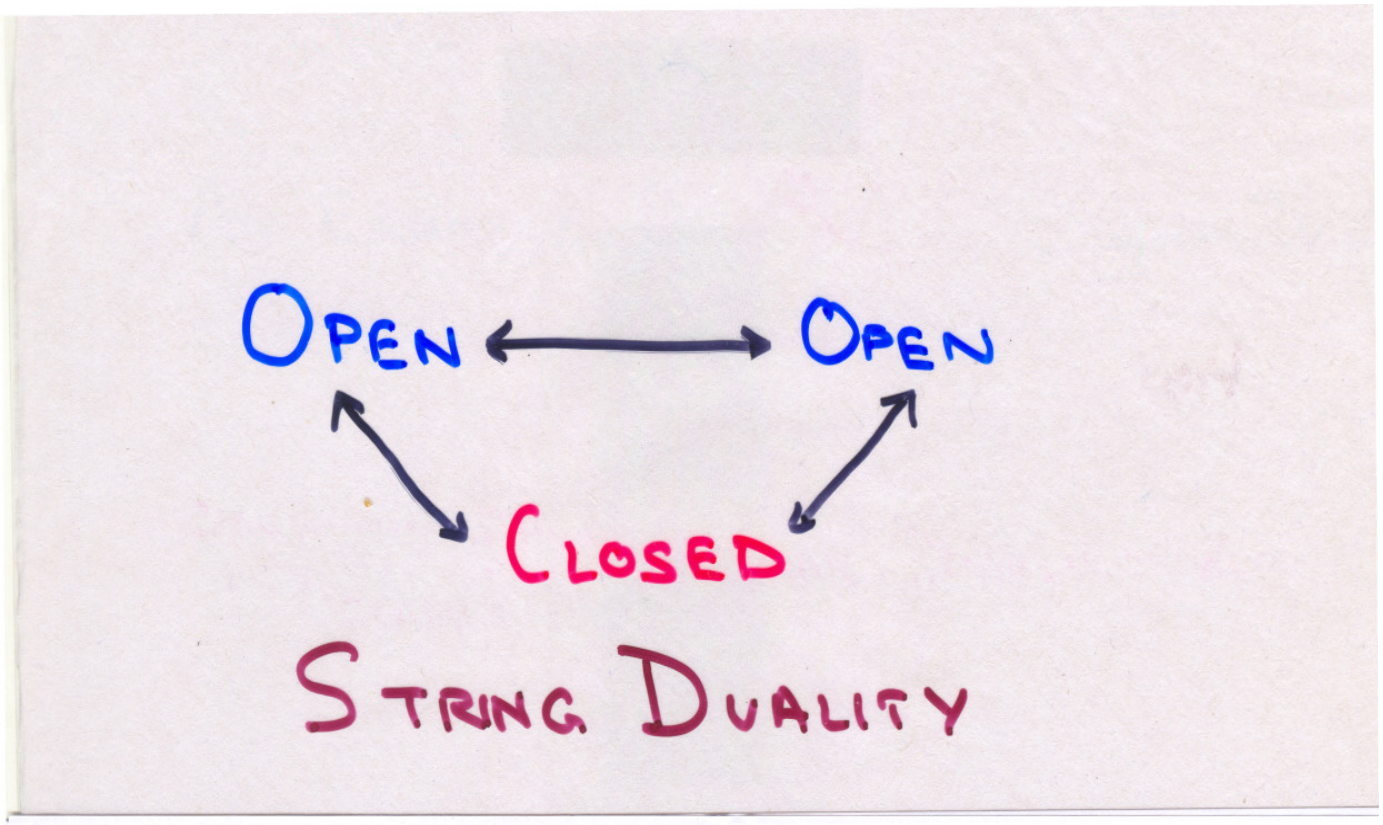

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Jo'burg Workshop

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Outline of the talk

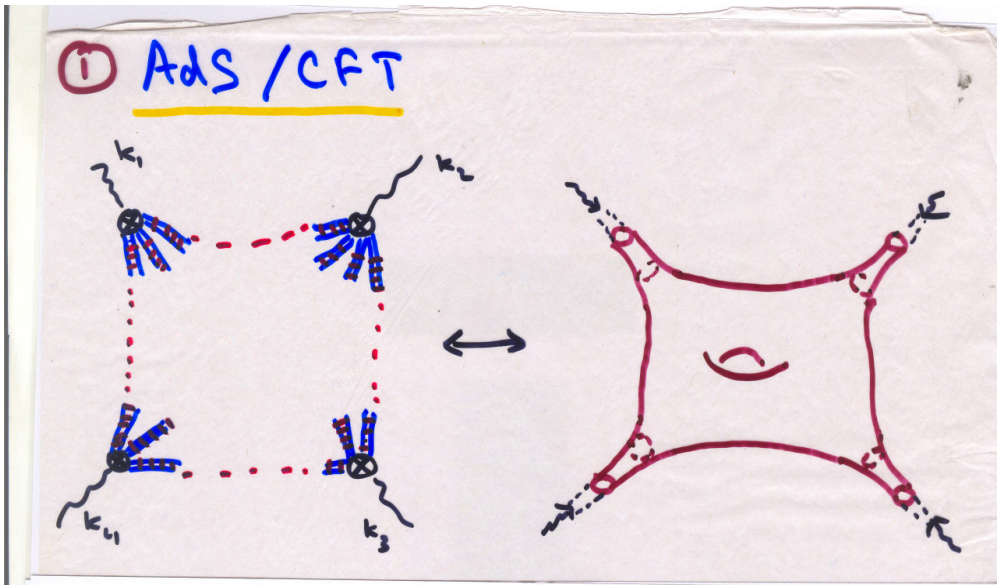
- Introductory Remarks
  - Two kinds of Open-Closed String Duality
- An Example of Open-Closed-Open String Duality
  - "Old" Matrix models
- Open-Open String Duality as Graph Duality
  - From Faces to Vertices
- Reconstructing Closed Strings from Open Strings
  - Horizontal and Vertical Trajectories of Strebel Differentials
- General Lessons?

1 Introductory Remarks

Open-Closed String Dualities

- Open-Closed String Duality underlies Gauge-Gravity Correspondence.
- Back reaction of D-branes leads to a modified closed string background – "Holes close up".
- Therefore crucial to understanding the mechanism of open-closed duality. How exactly do holes close up?.
- Actually two different ways in which open string worldsheets close up to form closed strings.
- Known examples realise one or other of these ways.

Open-Closed String Dualities *continued*



✣

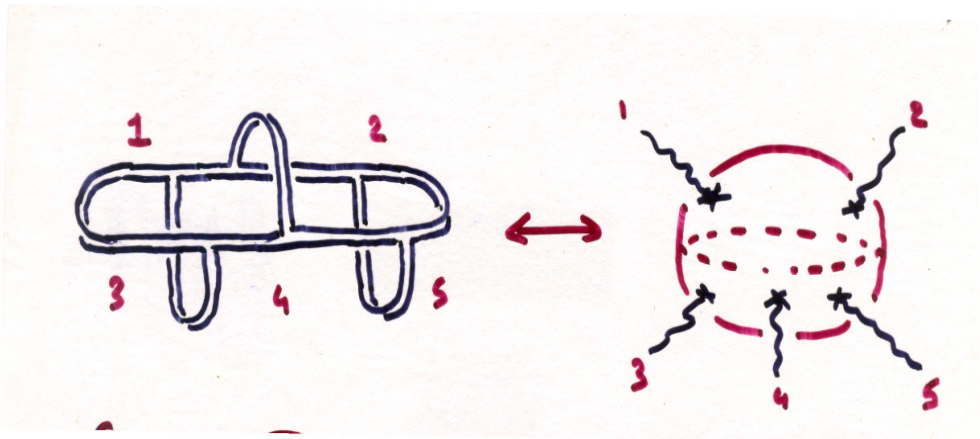
$$\langle \mathcal{O}_1(k_1) \dots \mathcal{O}_n(k_n) \rangle_{sp-time} = \int_{\mathcal{M}_{g,n}} \langle \mathcal{V}_1(k_1, \xi_1) \dots \mathcal{V}_n(k_n, \xi_n) \rangle_{w-sheet}$$

✣ Double lines glued together to form closed string surface. Holes in Feynman diagrams close up.

✣ Each external vertex of gauge theory diagrams ↔ External closed string insertion.

Open-Closed String Dualities *continued*

✦ Chern-Simons/Conifold Duality (R. G. and C. Vafa)



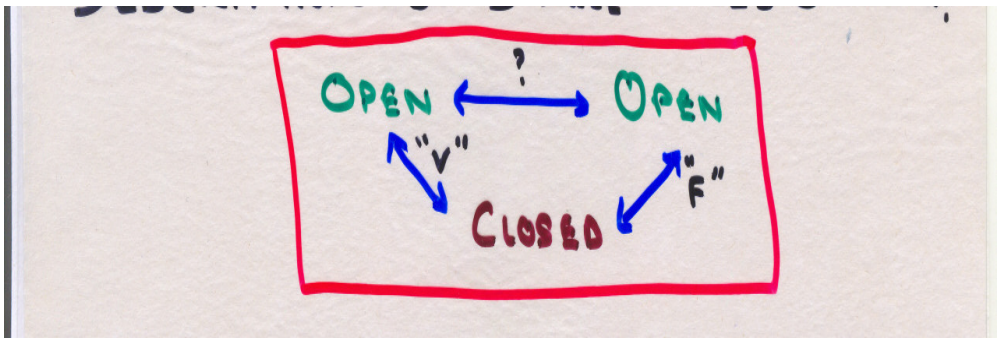
✣ Gauge theory arises directly from an open string field theory. Holes in open string worldsheet close up.

✣ Each external face of gauge theory diagram ↔ External closed string insertion.

✣ Equivalent to replacing D-brane boundary state by local vertex operator insertions.

Open-Closed String Dualities *continued*

- Thus two *different* ways in which closed strings are known to emerge from large N gauge theory diagrams.
  - ★ By replacing **vertices** with closed string punctures.
  - ★ By replacing **faces/boundaries** by closed string punctures.
- Raises the possibility of having two *different* open string descriptions of single closed string.



- Realised in a simple explicit example.
- $(2, 2k+1)$ minimal models coupled to 2-d gravity.

2 An Example

Example of Open-Closed-Open String Duality

✦ Closed String: 2d gravity+Liouville+$(2, 2k+1)$ minimal model.

★ Has physical closed string vertex operators $\mathcal{V}_m$ $(m = 0, 1 \ldots)$.

★ Generating functional of closed string correlators

$$F[t] = \sum_{g=0} \langle \exp(\sum_m t_m \mathcal{V}_m) \rangle_g .$$

★ All $t_m = 0 \leftrightarrow (2, 1)$ minimal model = Topological 2d gravity.

★ Other $(2, 2k+1)$ models correspond to switching on specific value t_k.

★ $F[t]$ can be computed using techniques of topological field theory (Witten).

★ Generating function for certain intersection numbers on moduli space $\mathcal{M}_{g,n}$.

Example of Open-Closed-Open String Duality *Contd.*

✦ Open String - "V-Type": Double scaling limit of 1-matrix model.

★

$$e^{F}[t] \equiv Z[t] = \int [DM]_{N\times N} e^{-N\mathrm{Tr}M^2 + \sum_m \tilde{t}_m \mathrm{Tr}W_m(M)}.$$

★ Double scaling: $N \sim \frac{1}{\epsilon^2} \to \infty, \quad \tilde{t}_m \sim \frac{t_m}{\epsilon^{2m+1}}$.

★ Note similarity to AdS/CFT with **vertices** $TrW_m(M) \sim \mathrm{Tr}M^{m+1} + \ldots$ being dual to closed string insertions.

★ The couplings t_m are sources which act as closed string backgrounds.

★ The matrix d.o.f. of M re interpreted as open string modes of ZZ branes.

Example of Open-Closed-Open String Duality *Contd.*

- Open String - "F-Type": Kontsevich matrix model.
  - $$e^{F}[t] \equiv Z[t] = \int [DX]_{n\times n} e^{-\frac{1}{2}\mathrm{Tr}(\Lambda X^2)+\frac{i}{3}\mathrm{Tr}X^3}.$$
  - Here closed string backgrounds encoded in $n \times n$ matrix Λ.
  - $t_m = \frac{1}{(2m-1)!!}\mathrm{Tr}\Lambda^{-(2m+1)}$.
  - Note that t_m do not appear linearly as sources in this model.
  - Cubic matrix model believed to arise from Open String field theory of FZZT branes. (Gaiotto-Rastelli)
  - Like in Chern-Simons theory, **boundaries/faces** of worldsheet are dual to closed string insertions.
  - Can be seen explicitly in Kontsevich's derivation.

Example of Open-Closed-Open String Duality *Contd.*

- Thus the two different open string (Matrix Model) descriptions correspond to two different branes - FZZT and ZZ.
- Are the two matrix models directly related? **YES** (Maldacena, Moore, Seiberg, Shih)
- Identify macroscopic loop operators

$$\mathrm{Tr}\ln(\alpha - M) = \int \frac{dl}{l} e^{-l\alpha} \mathrm{Tr} e^{lM}$$

  as wavefunctions (disc partition function) of FZZT brane.

- Consider an insertion of n FZZT branes via $e^{\mathrm{Tr}\ln(\alpha - M)} = \det(\alpha - M)$.
- $$\langle \prod_{i=1}^{n} \det(\alpha_i - M) \rangle_{gaussian} \equiv \langle \det(\Lambda \otimes \mathbf{1}_N - \mathbf{1}_n \otimes M) \rangle.$$

  $\Lambda = diag(\alpha_1 \ldots \alpha_n)$.
- LHS $= \langle e^{\sum_i \mathrm{Tr}\ln(\alpha_i - M)} \rangle_{gaussian} = \int [DM]_{N\times N} e^{-N\mathrm{Tr}M^2 + \sum_m \tilde{t}_m \mathrm{Tr}W_m(M)}$.

Example of Open-Closed-Open String Duality *Contd.*

- Can write the RHS equivalently as

$$\int [DM][D\chi^\dagger][D\chi] e^{-N\mathrm{Tr}M^2+\chi^\dagger(\Lambda\otimes\mathbf{1}_N-\mathbf{1}_n\otimes M)\chi} = \int [D\chi^\dagger][D\chi] e^{\chi^\dagger\Lambda\chi-\frac{1}{4N}(\chi^\dagger\chi)^2}.$$

- Here χ_{aj} are bifundamental fermion fields we have introduced ($a = 1\ldots N, j = 1\ldots n$) and $(\chi^\dagger\chi)^2 \equiv \sum_{a,b}(\chi^\dagger_{aj}\chi_{bj})(\chi^\dagger_{bi}\chi_{ai})$.

- Now integrate in a new $n \times n$ matrix field X

$$\int [DX][D\chi^\dagger][D\chi] e^{-N\mathrm{Tr}X^2+i\chi^\dagger(X\otimes\mathbf{1}_N)\chi+\chi^\dagger(\Lambda\otimes\mathbf{1}_N)\chi} = \int [DX]\det(\Lambda+iX)^N e^{-N\mathrm{Tr}X^2}$$

- Translating the double scaling limit to this integral then gives the Kontsevich matrix model $Z[t] = \int [DX]_{n\times n} e^{-\frac{1}{2}\mathrm{Tr}(\Lambda X^2)+\frac{i}{3}\mathrm{Tr}X^3}$.

3 Open-Open Duality as Graph Duality

Open-Closed-Open Duality and Graph Duality

- Relation between Kontsevich model and double scaled 1-Matrix model has a nice geometric interpretation.

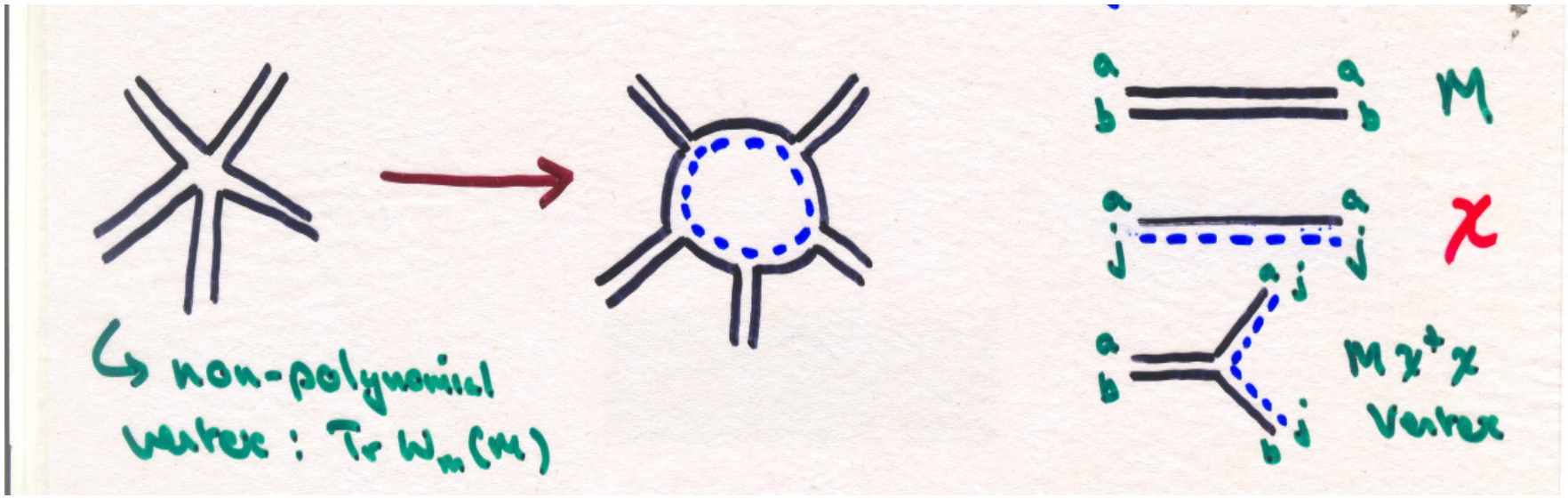
- Essentially, holes of "V-type" open string worldsheet close up while vertices open up into holes of "F-type" theory.

- In this process graphs go into *dual* graphs.

- We can follow the individual Feynman diagrams through the steps of integrating in and out of fields.

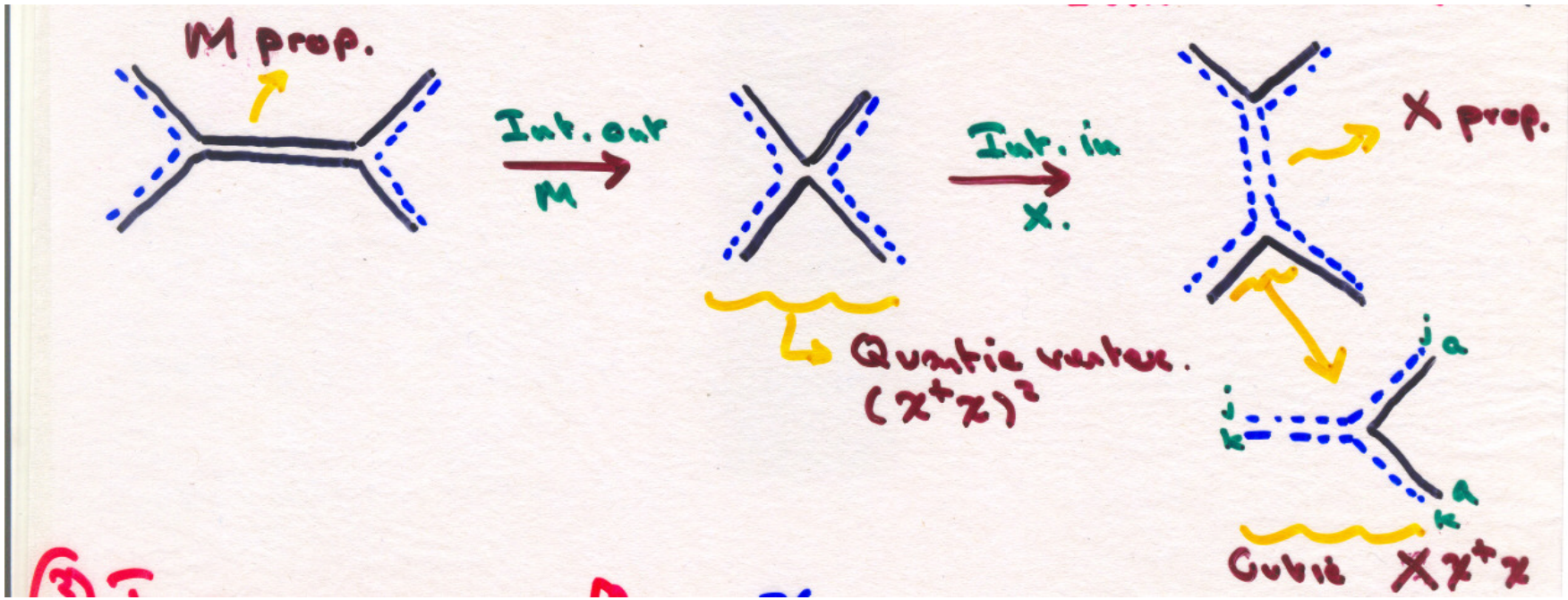
Open-Closed-Open Duality and Graph Duality *Contd.*

- Introducing fermions χ_{aj}



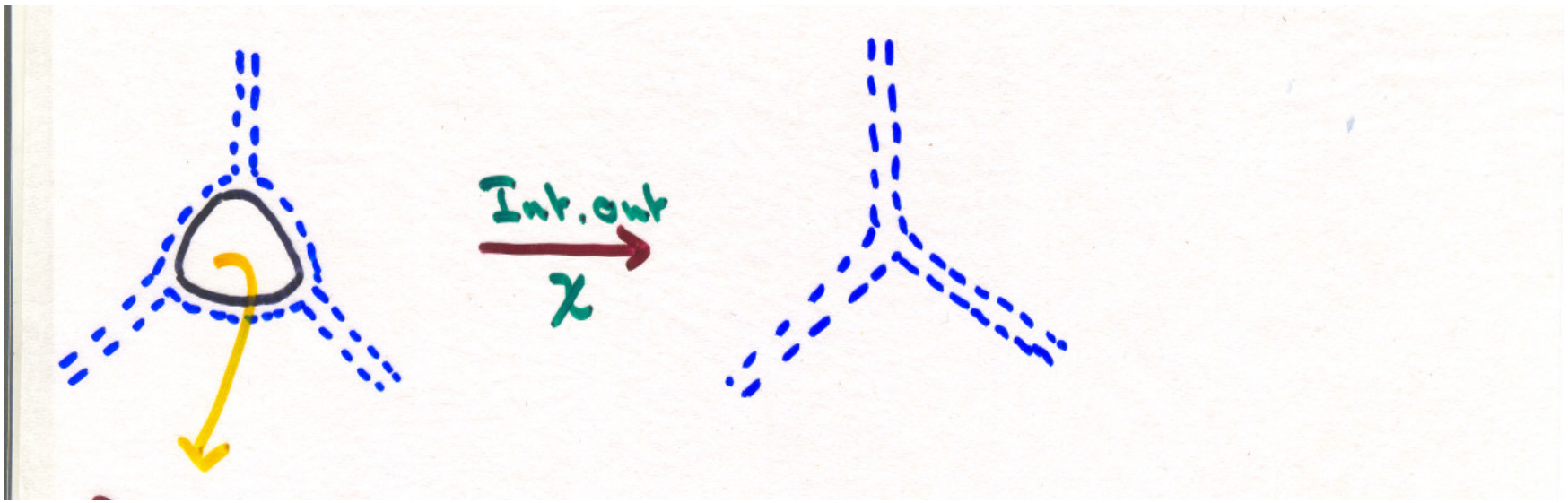
Open-Closed-Open Duality and Graph Duality *Contd.*

✦ Integrating out M and integrating in X



Open-Closed-Open Duality and Graph Duality *Contd.*

- Integrating out χ



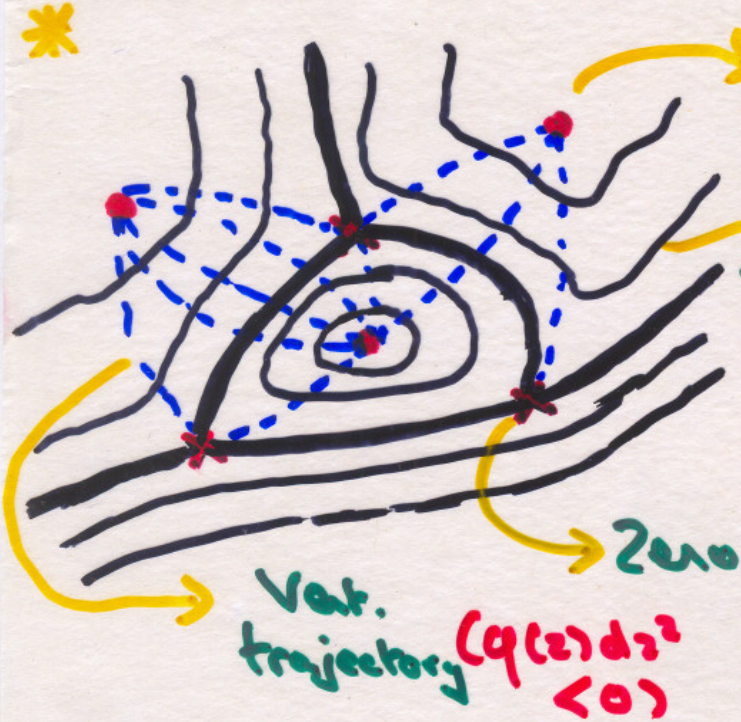
- We see that the original vertices (of M) open up into faces of X while the original faces (with solid lines) close up - dynamic graph duality.

4 Reconstructing Closed String Worldsheets

Reconstructing Closed String Worldsheets

- The relation between the two different open string descriptions being related by graph duality is probably general.

- It is also consistent with the approach to obtain closed string worldsheets from Feynman graphs via the Strebel parametrisation of $\mathcal{M}_{g,n}$.



RELIES ON EXISTENCE OF UNIQUE QUAD. DIFFERENTIALS $\varphi(z)dz^2$ WHICH FOLIATE $\Sigma_{g,n}$ VIA HORIZONTAL TRAJECTORIES.

THERE ARE n POLES AT THE LOCATION OF n PUNCTURES. GIVES RIEMANN SURFACE CONFORMALLY EQUIV. TO n SEMI-INFINITE CYLINDERS (CENTRED ABOUT EACH POLE).

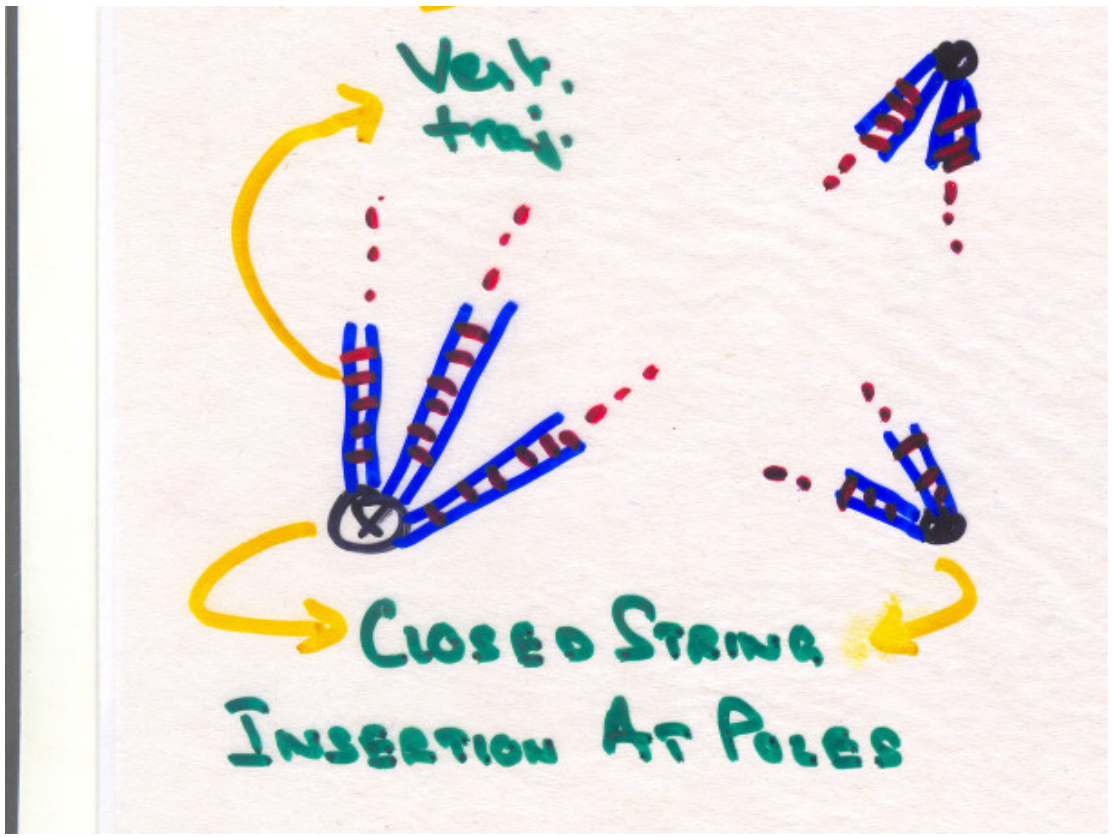
Reconstructing Closed String Worldsheets *Contd.*

- The Horizontal trajectories joining zeroes of $\phi(z)$ give a graph with cubic vertices.

- The length of the edges $\leftrightarrow$ Schwinger lengths of propagators.
- This is the proposal for how closed string moduli space emerges in F-type duality.

Reconstructing Closed String Worldsheets *Contd.*

- Alternatively, consider Vertical trajectories joining poles of $\phi(z)$ gives the **dual** graph with arbitrary valent vertices.



- The widths of the edges $\leftrightarrow$ *inverse* Schwinger lengths of propagators.
- This is the proposal for how closed string moduli space emerges in V-type duality.

To Conclude

- Need to refine our notion of open-closed string duality - distinguish V-type and F-type dualities.
- Possibility of both descriptions existing for a single theory -from two different brane descriptions.
- Relation between the two descriptions a dynamic graph duality - one set of holes close up and another set open up.
- Graph duality fits nicely with emergence of closed string geometry from Strebel parametrisation of $\mathcal{M}_{g,n} \times R_+^n$ - horizontal vs. vertical trajectories.
- Explore possibility of dual open string ("F-type") description in AdS/CFT - from giant gravitons?

The end