Cardy & Kerr

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Einstein

$$S = \frac{1}{16\pi G_N} \int \mathrm{d}^d x \, \sqrt{-g} \, [R - 2\Lambda] + \int \mathrm{d}^d x \, \sqrt{-g} \, \mathcal{L}_{\mathrm{m}}$$

$$\Big\downarrow \; \delta S = 0$$

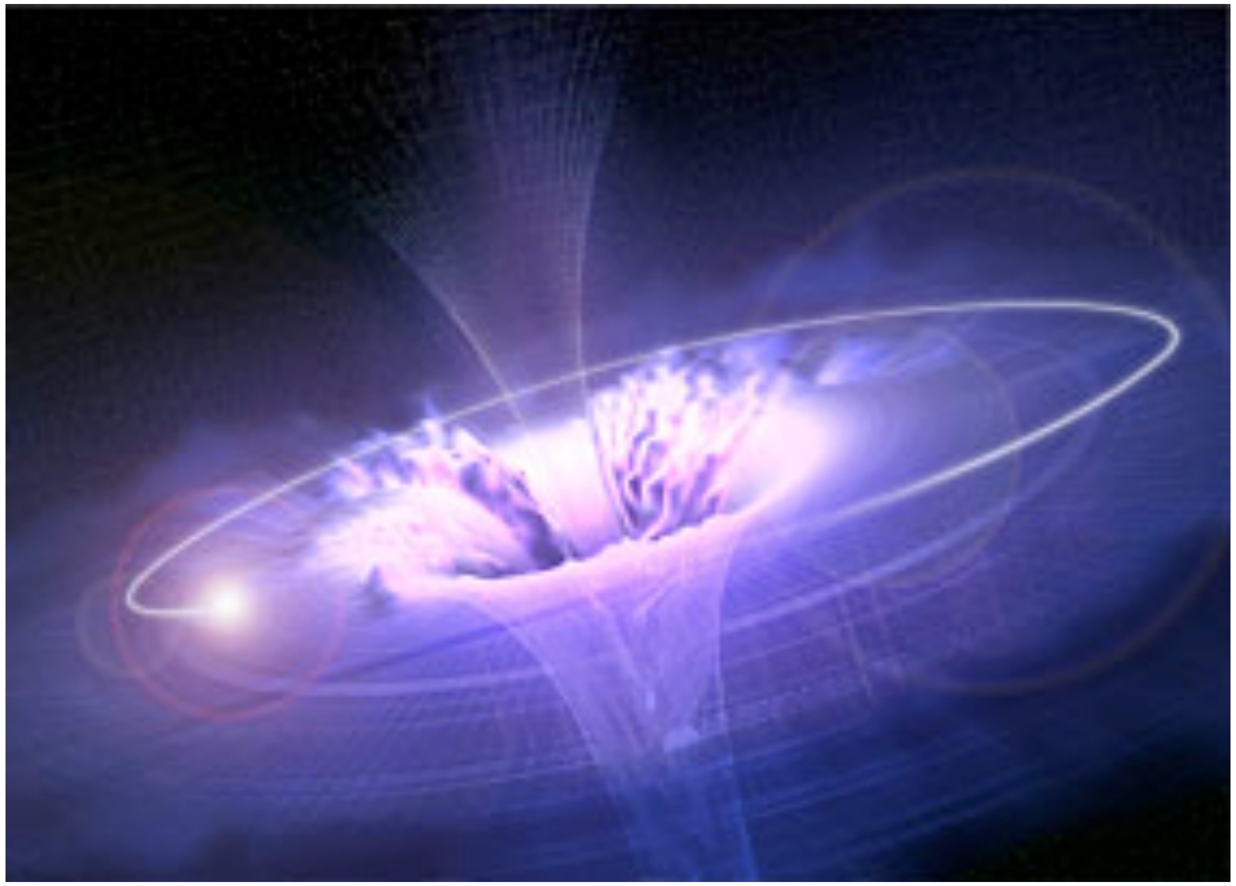
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

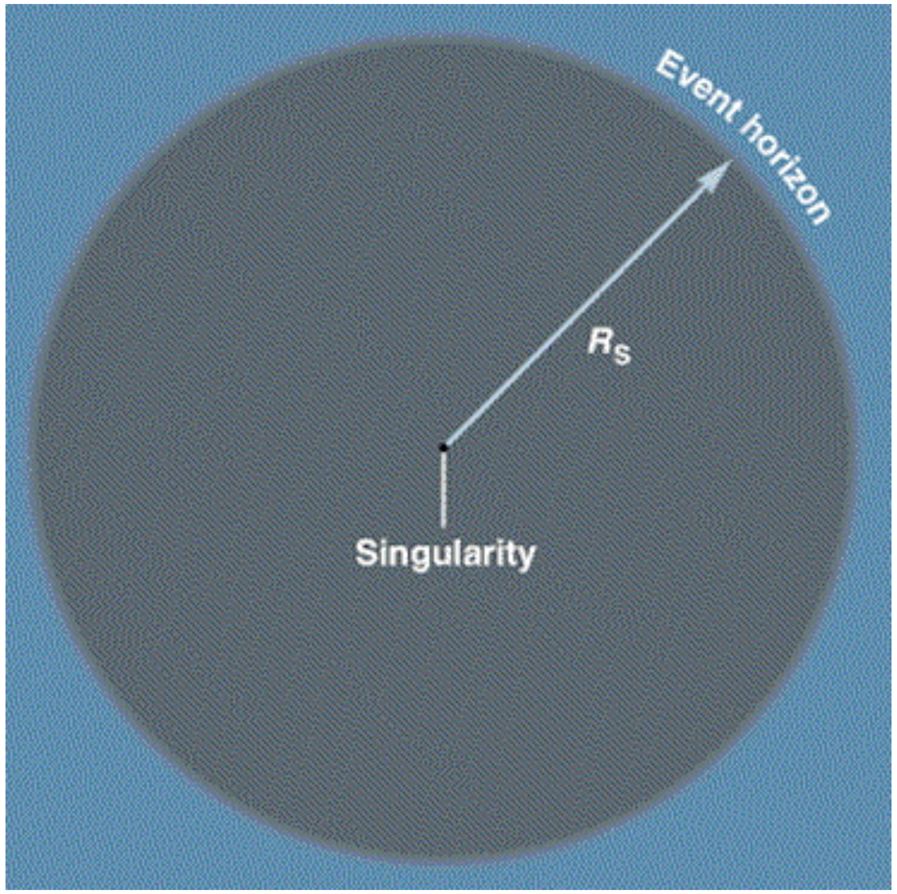
- How matter responds to curvature of spacetime and conversely how geometry responds to presence of energy

- There are singular solutions in General Relativity

- How does string theory resolve spacetime singularities?

Schwarzschild & Friends





- Paradigmatic examples are black holes

- These are thermodynamic objects with entropy

$$\kappa = \text{constant} \neq 0$$

$$dM = \frac{\kappa}{2\pi}\, dA + \Omega\, dJ + \Phi\, dQ$$

$$dS \geq 0$$

Bekenstein & Hawking

- Entropy of a black hole is $\boxed{S_{\rm BH} = \frac{\rm Area}{4\, G_{\rm N}\, \hbar}}$

- String theory enables us to understand the origin of this entropy in very special settings

- Gravitational thermodynamics is then promoted to a theory of statistical mechanics

- Can study physics of $\boxed{e^{S_{\rm BH}} \text{ states}}$

- So far astrophysical black holes are not among these very special settings

Boltzmann

$$S_{\mathrm{BH}} = \frac{A}{4G_N}$$

$$e^{S_{\mathrm{BH}}} \text{ states}$$

Kinetic Theory of Gases

Thermodynamic quantities like T, p, μ describe a gas

We do a statistical averaging

There is an underlying microscopic theory:
$S = k \log \Omega$, where Ω is number of microstates;
this enumerates configurations of gas molecules

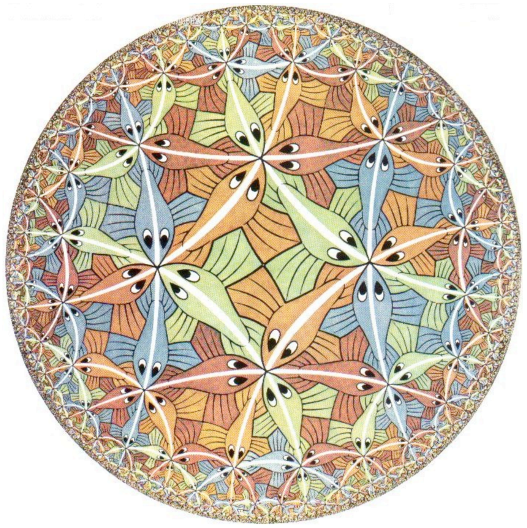
Question: What are the microstates of a black hole?

A partial answer: Enumerating degenerate supersymmetric vacua in a weakly coupled description in terms of strings and D-branes correctly reproduces Bekenstein-Hawking entropy

Strominger & Vafa, 1996

Maldacena

- Spacetime is an approximation



$AdS_{d+1} \times S^p \times X \quad \Longleftrightarrow \quad \mathrm{CFT}_d$

maximally symmetric solution to Einstein equations with negative c.c.

$$X_0^2 + X_{d+1}^2 - \sum_{i=1}^{d} X_i^2 = L^2 \text{ in } \mathbb{R}^{d,2}$$

$$\mathrm{d}s^2 = L^2\left(-\cosh^2\rho\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + \sinh^2\rho\,\mathrm{d}\Omega_{d-1}^2\right)$$

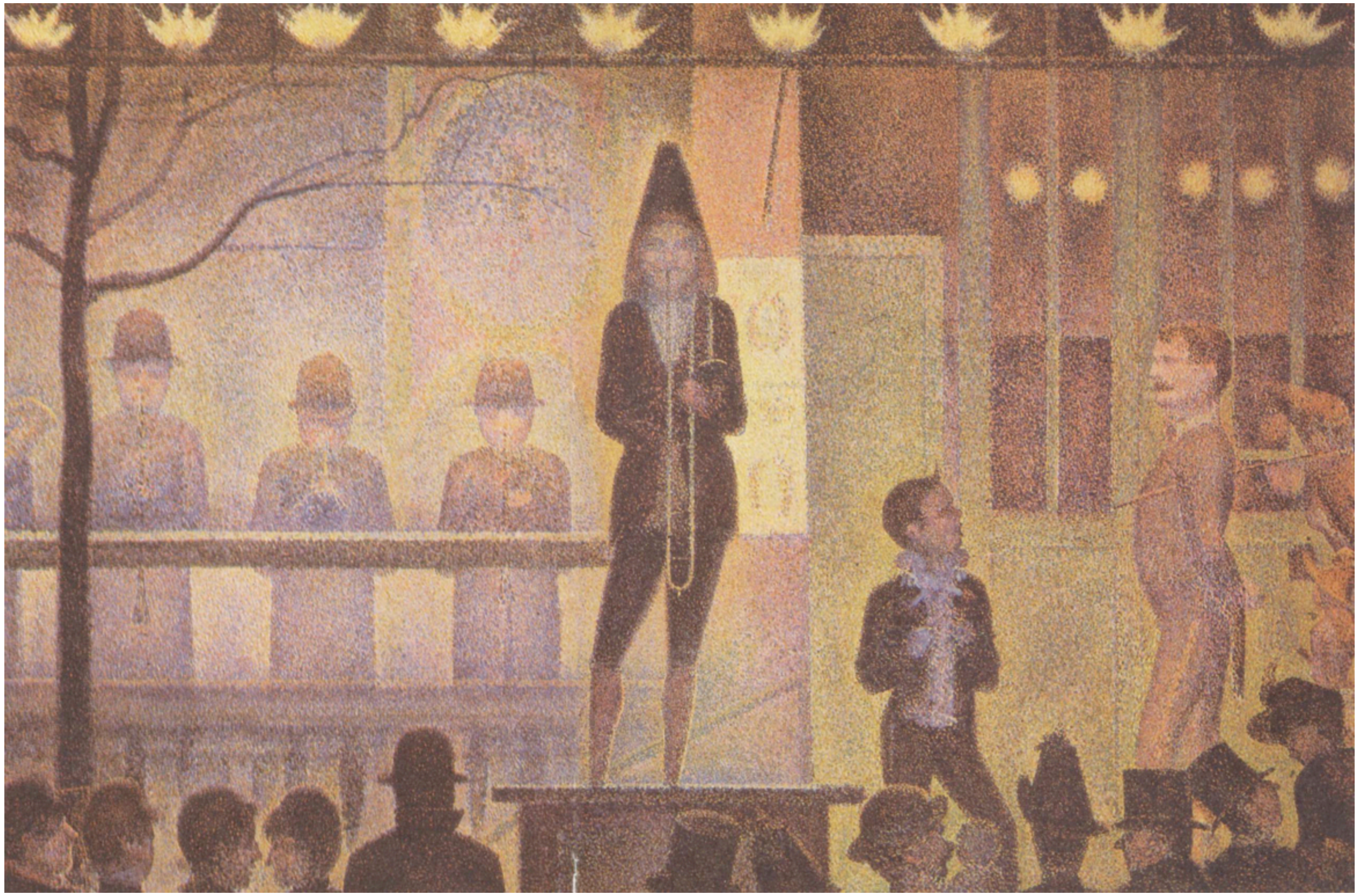
Type IIB string theory on $AdS_5 \times S^5$ $\approx$ $\mathcal{N} = 4$ SYM in $d = 4$

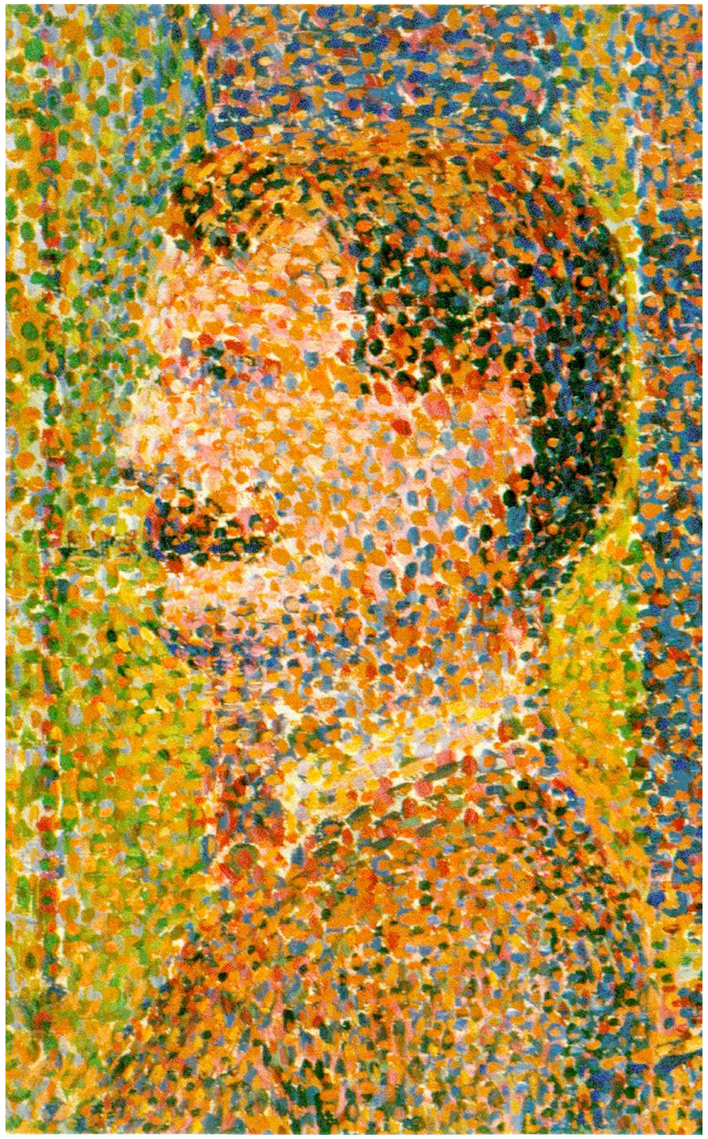
- closed string/open string duality
- strong coupling/weak coupling duality

<u>Question:</u> How do black hole spacetimes emerge?

<u>Question:</u> What are the fundamental degrees of freedom of quantum gravity?

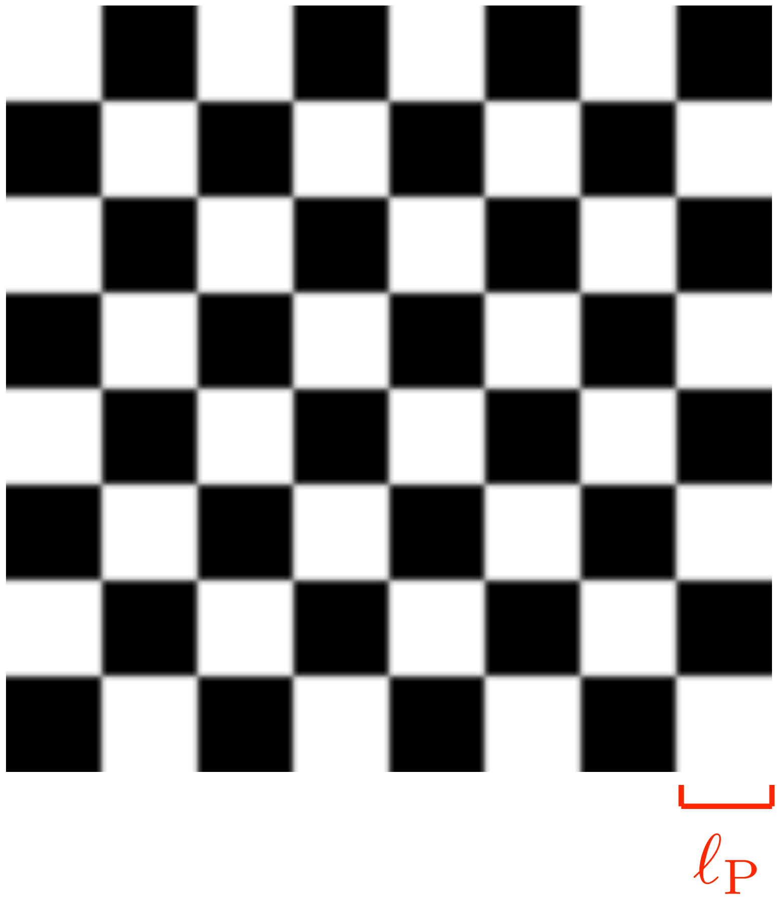
Seurat





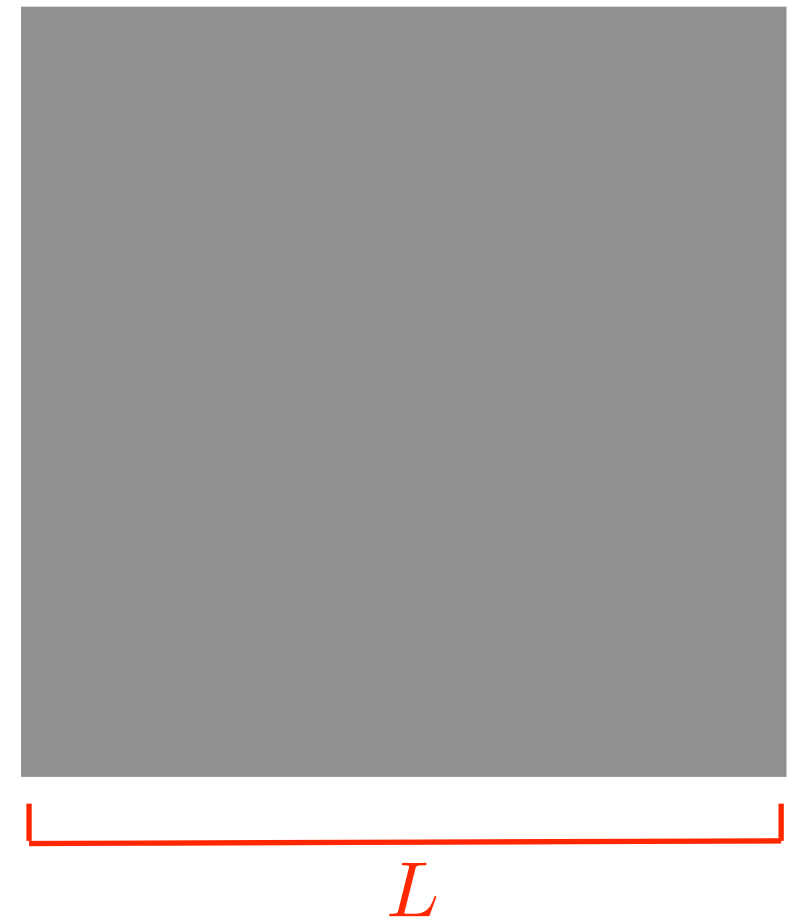
La Parade de Cirque (1888)

Wilson



Quantum data
is washed out

(cf. RG)



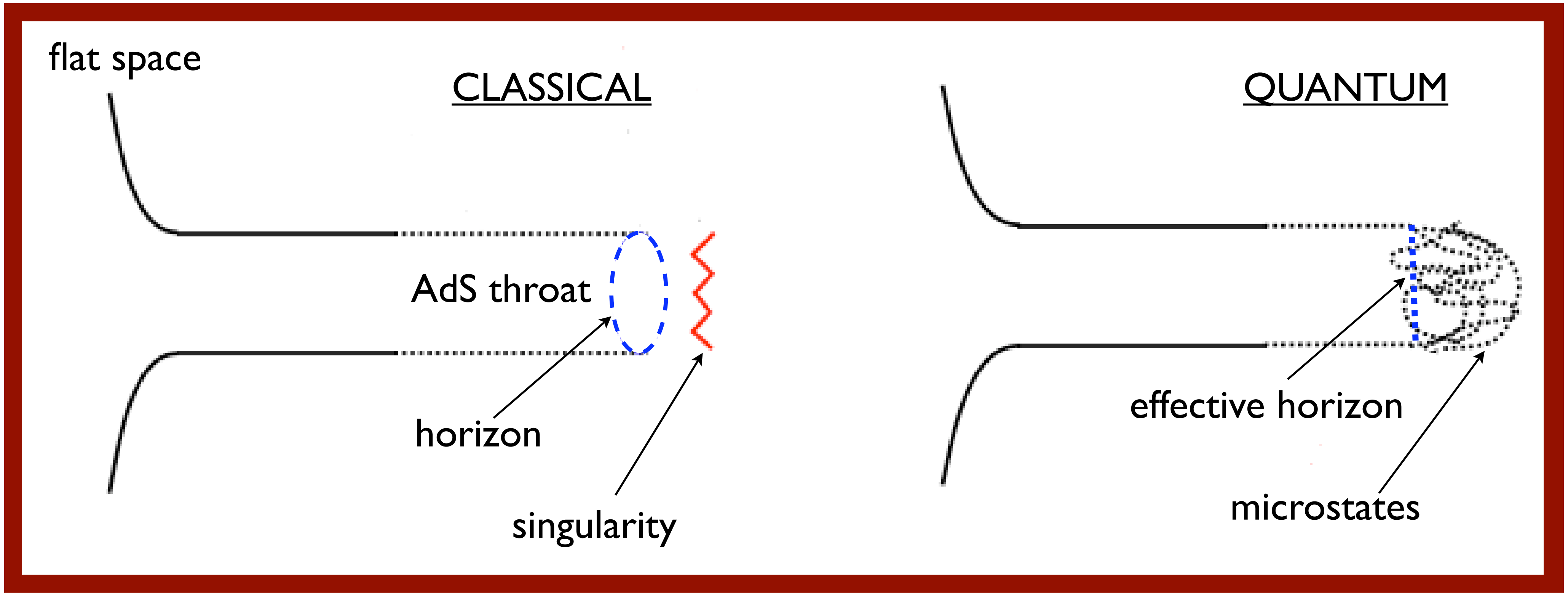
Quantum state
wavefunction

Classical geometry
origin of entropy

- Scale L; on smaller scales all microstates look the same

- For black holes, this new length scale is classical horizon

Mathur



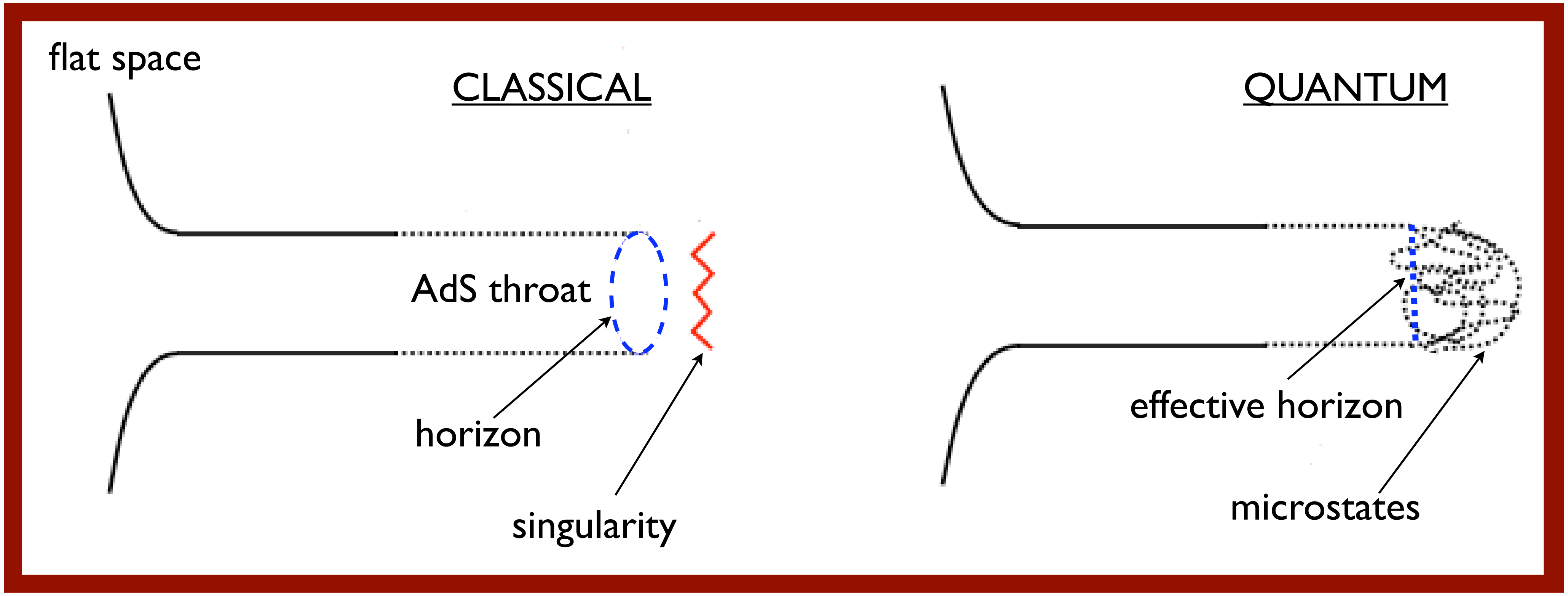
- Many horizonless configurations with the same global charges (M, J, Q) as black hole

- Individual microstates have no entropy (so horizonless)

- The generic state is intrinsically quantum

- A typical state is characteristically stringy all the way to the effective horizon, and well approximated by the black hole metric in the exterior

Mathur



- Horizons and singularities are effective notions in gravity

- They arise only as a consequence of a thermodynamic averaging over microstates, or coarse-graining

- Origin of entropy lies in the inability of a semiclassial observer to distinguish the different quantum microstates

- Quantise moduli space to find $e^{S_{\rm BH}}$ solutions

Cardy & Kerr

- The aim of this talk is suitably modest

- Guica, Hartman, Song, Strominger have proposed a way of computing the entropy of four dimensional Kerr black hole from CFT

- This is analogue of Strominger-Vafa entropy counting, except this time it is a genuine astrophysical setting

- The computation uses the Cardy formula without justification (i.e. outside its regime of validity)

- Our work motivates the Cardy computation of entropy

Brown & Henneaux

- Example: AdS_3

- With suitable b.c., large diffeos generate $\mathrm{Vir} \times \mathrm{Vir}$

$$[L_m, L_n] = (m-n)L_{m+n} + \tfrac{c_L}{12}(m^3 - m)\delta_{m+n,0}$$

$$[\overline{L}_m, \overline{L}_n] = (m-n)\overline{L}_{m+n} + \tfrac{c_R}{12}(m^3 - m)\delta_{m+n,0}$$

$$[L_m, \bar{L}_n] = 0$$

- Central charge is $c_L = c_R = \dfrac{3}{2G_N\sqrt{-\Lambda}} \equiv \dfrac{3\ell}{2G_N}$

- Precursor to $\mathrm{AdS}_3/\mathrm{CFT}_2$

BTZ

- Gravity in 2+1 dimensions is trivial

- Nevertheless there is a black hole in AdS_3

Bañados, Teitelboim, Zanelli, 1992

- We understand the entropy of BTZ in light of the AdS/CFT correspondence

- In fact, this is not really string theory: just use Brown-Henneaux and look at asymptotic growth of states

Strominger, 1997

- This is the blueprint for the Kerr analysis

Kerr

- Four dimensional rotating black hole

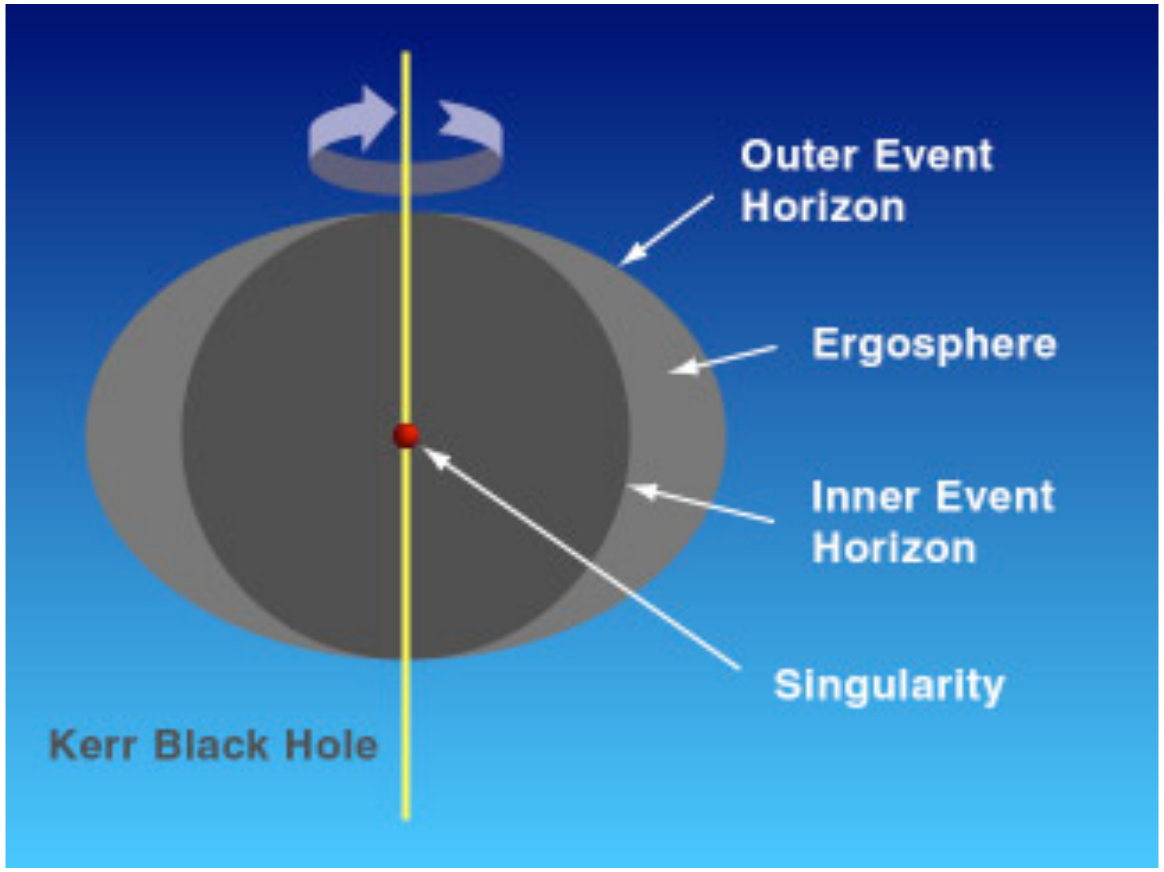
  Theorem: This is unique time-independent vacuum black hole in four dimensional General Relativity.

- Two parameters: $J \leq G_N M_{\rm ADM}^2$ (extremal at equality)

- $\mathbb{R} \times U(1)$ isometry

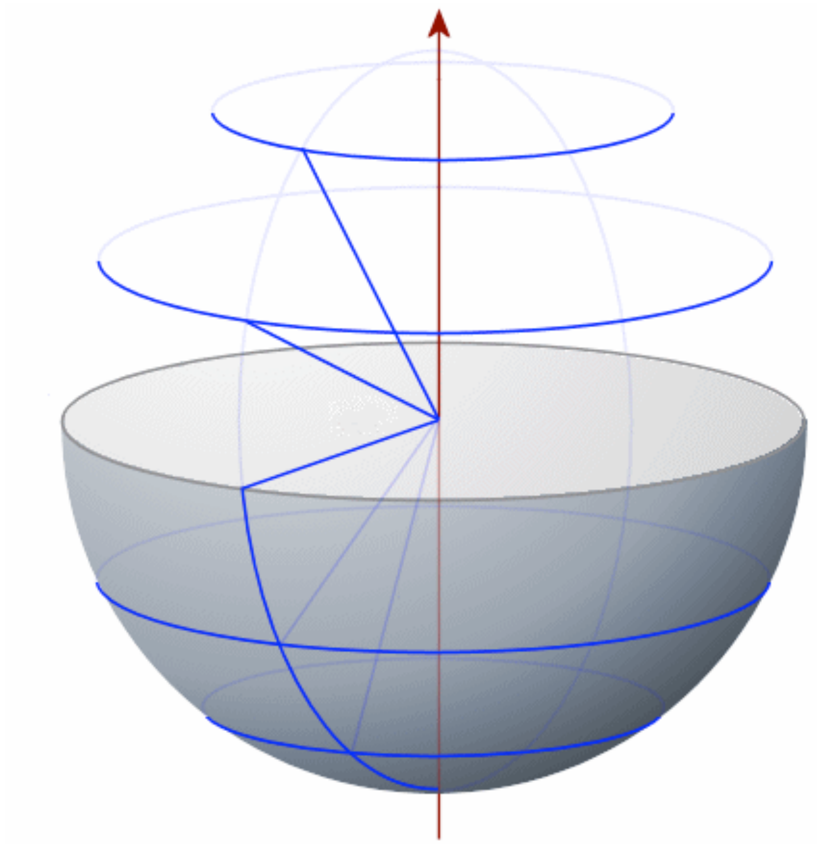
$$\boxed{S_{\rm BH} = \frac{A}{4G_N} = 2\pi J}$$

- What are its microstates?



Kerr, 1963

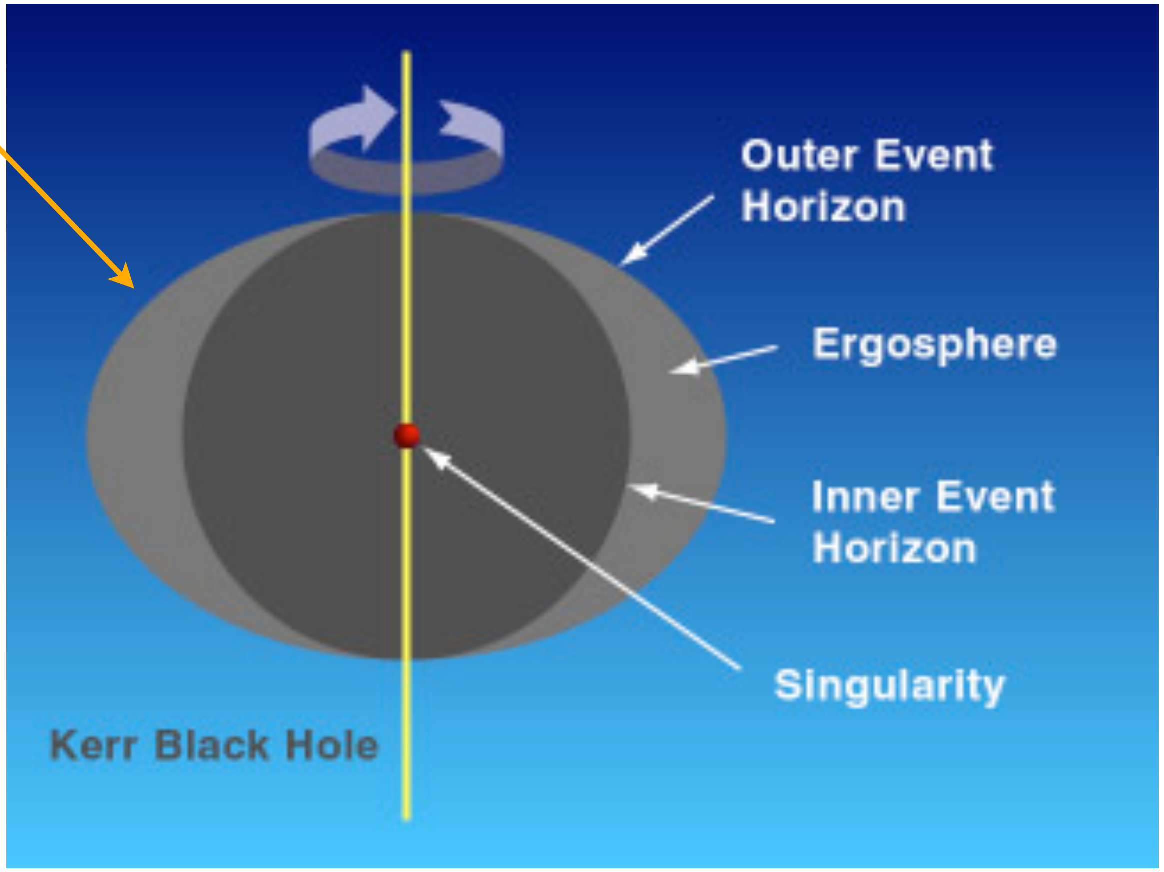
Kerr



- divide into slices of latitude

- each slice contains BTZ in warped AdS_3

$$\boxed{S_{\mathrm{BH}} = \frac{A}{4G_N} = 2\pi J}$$



- What are its microstates?

Kerr, 1963

Bardeen & Horowitz

- Near-horizon extremal Kerr has metric

$$\boxed{\mathrm{d}s^2 = 2G_4 J \Omega(\theta)^2 \left(-(1+r^2)\, d\tau^2 + \frac{dr^2}{1+r^2} + d\theta^2 + \Lambda(\theta)^2 (d\varphi + r\, d\tau)^2 \right)}$$

$$\Omega(\theta)^2 = \frac{1+\cos^2\theta}{2}\,, \qquad \Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta}\,, \qquad \theta \in [0,\pi]\,, \qquad \varphi \sim \varphi + 2\pi$$

- τ, r combine to give AdS_2

- φ is a circle fibered on top

- constant θ : $\quad$ warped AdS_3

Zaslavsky, 1997;
Bardeen, Horowitz, 1999

Bardeen & Horowitz

- Near-horizon extremal Kerr has metric

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- Geodesically complete with timelike infinity at $r = \pm\infty$

- Isometry group is enhanced to $SL(2,\mathbb{R}) \times U(1)$

- Can we apply Brown-Henneaux here?

Zaslavsky, 1997;
Bardeen, Horowitz, 1999

GHSS

- Devise b.c. on metric falloff and then find all the allowed diffeos that preserve them

- Find the generators and show they are well-preserved, finite, and conserved

$$\boxed{\zeta_\epsilon = \epsilon(\varphi)\partial_\varphi - r\epsilon'(\varphi)\partial_r} \Longrightarrow \boxed{\mathbb{R} \times \mathrm{Vir}}$$

- Obtain chiral CFT with $c_L = 12J$

Guica, Hartman, Song, Strominger, 2008

GHSS

- Frolov-Thorne vacuum with $T_L = \dfrac{1}{2\pi}\ ,\ T_R = 0$

- Extremality = No excitations on right side: $c_R = 0$

- Entropy from Cardy formula

$$\boxed{S_{\rm CFT} = \frac{\pi^2}{3} c_L T_L = 2\pi J = S_{\rm BH}}$$

- So we are done...

Guica, Hartman, Song, Strominger, 2008

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- So we are done...

- Except we are not...

AHMR / DRS / BDSS

- No dynamics in near-horizon extremal Kerr

  Theorem: Any solution to vacuum GR that is asymptotic to NHEK is as well diffeomorphic to NHEK.

Amsel, Horowitz, Marolf, Roberts, 2009;
Dias, Reall, Santos, 2009

- Look at near-horizon BTZ; DLCQ of a non-chiral CFT dual to AdS_3 is a chiral CFT

  Question: The CFT dual of Kerr is a DLCQ limit of what theory?

Balasubramanian, de Boer, Sheikh-Jabbari, Simón, 2009

Cardy

$$c_L = 12J$$

$$T_L = \frac{1}{2\pi}$$

$$S_{\mathrm{CFT}} = \frac{\pi^2}{3} c_L T_L = 2\pi J = S_{\mathrm{BH}}$$

Cardy

$$\boxed{\begin{array}{c} c_L = 12J \\ \\ T_L = \frac{1}{2\pi} \end{array}} \Rightarrow \boxed{S_{\rm CFT} = \frac{\pi^2}{3} c_L T_L = 2\pi J = S_{\rm BH}}$$

- Cardy formula extracts the leading term in the high-temperature expansion of the partition function

Hardy, Ramanujan, 1918;
Rademacher, 1937;
Cardy, 1986

Cardy

$c_L = 12J$

$T_L = \frac{1}{2\pi}$

$$\boxed{S_{\mathrm{CFT}} = \frac{\pi^2}{3} c_L T_L = 2\pi J = S_{\mathrm{BH}}}$$

- Cardy formula extracts the leading term in the high-temperature expansion of the partition function

- Here, $T \approx \mathcal{O}(1)$!

- So why does Cardy work?

On Cardy Off Cardy

- Similar situation exists for Strominger-Vafa: Cardy formula works outside its regime of validity

- In fact, this is generic in string theory when there is a CFT dual to gravitational system

- No general argument or explanation, but we often know how!

- Mathur gives us a paradigm for how to count

Lunin & Mathur

- $$\left.\begin{array}{r} n_1 \ \mathrm{D1-branes} \\ \\ n_5 \ \mathrm{D5-branes} \end{array}\right\} \text{wrapping } \mathcal{M} = \mathrm{T}^4 \text{ or K3}$$

- CFT is a sigma model with target $\mathcal{M}^N/S_N$

$$c = 6N = 6n_1 n_5$$

de Boer, 1998;
Seiberg, Witten, 1999;
Larsen, Martinec, 1999

Lunin & Mathur

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- System is dual to an effective string

n_1 units of winding

n_5 units of momentum

$8 \text{ bosonic}, 8 \text{ fermionic oscillators} \Longrightarrow c_L = 12$

$N_L = n_1 n_5$ is oscillator level

- $T_L \approx \dfrac{\sqrt{L_0}}{c_L} \gg 1$ ⟶ Cardy works!

Kerr

- Charged black holes in string theory have a lower bound on mass set by the charges

- Consider extremal solutions: $T_{\rm H} = 0 \; , \; \; S_{\rm BH} \neq 0$

near-horizon is AdS

- Global charges encode central charge of CFT

- Goal: Start with neutral Kerr in four dimensions and find a duality frame such that we have a black hole in string theory whose CFT dual is in the Cardy regime

Kerr

- If we lift four dimensional Kerr black hole to five dimensions and add a magnetic charge, we get a rotating Kaluza-Klein black hole with the same entropy

$$\boxed{S_{\mathrm{BH}} = \frac{A}{4G_N} = 2\pi J}$$

- If we take limit of large radius, this is a Myers-Perry black hole at the tip of a Taub-NUT cigar

- We shall use and develop these observations

Emparan, Maccarrone, 2007;
Horowitz, Roberts, 2007

Kaluza & Klein

metric of M-theory lifted four-dimensional extremal, dyonically charged rotating black hole

$$\boxed{\mathrm{d}s^2 = \frac{H_q}{H_p}(dy + \mathbf{A})^2 - \frac{\Delta_\theta}{H_q}(dt + \mathbf{B})^2 + H_p\left(\frac{dr^2}{\Delta} + d\theta^2 + \frac{\Delta}{\Delta_\theta}\sin^2\theta\, d\phi^2\right)}$$

$$H_p = r^2 + r\,p + \frac{p^2\,q}{2(p+q)} + \frac{q\,p^2\cos\theta}{2(p+q)}\,\frac{\alpha}{m}\;, \quad H_q = r^2 + r\,q + \frac{q^2\,p}{2(p+q)} - \frac{p\,q^2\cos\theta}{2(p+q)}\,\frac{\alpha}{m}\;, \text{ etc.}$$

$$2G_4 M = \frac{p+q}{2}\;, \qquad G_4 J = \frac{(pq)^{\frac{3}{2}}}{4(p+q)}\frac{\alpha}{m}\;, \qquad Q^2 = \frac{q^3}{4(p+q)}\;, \qquad P^2 = \frac{p^3}{4(p+q)}$$

D6−brane charge (monopole)

D0−brane charge

(momentum on M-theory circle)

Emparan, Maccarrone, 2007;
Horowitz, Roberts, 2007

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D6$-$brane charge (monopole)

D0$-$brane charge

(momentum on M-theory circle)

- In sugra: we have a Taub-NUT/ALE space with momentum flowing along the circle

$$\boxed{Q = \frac{2G_4N_0}{R}\ ,\ P = \frac{RN_6}{4}\ ,\ R \gg 1}$$

Emparan, Maccarrone, 2007;
Horowitz, Roberts, 2007

Myers & Perry

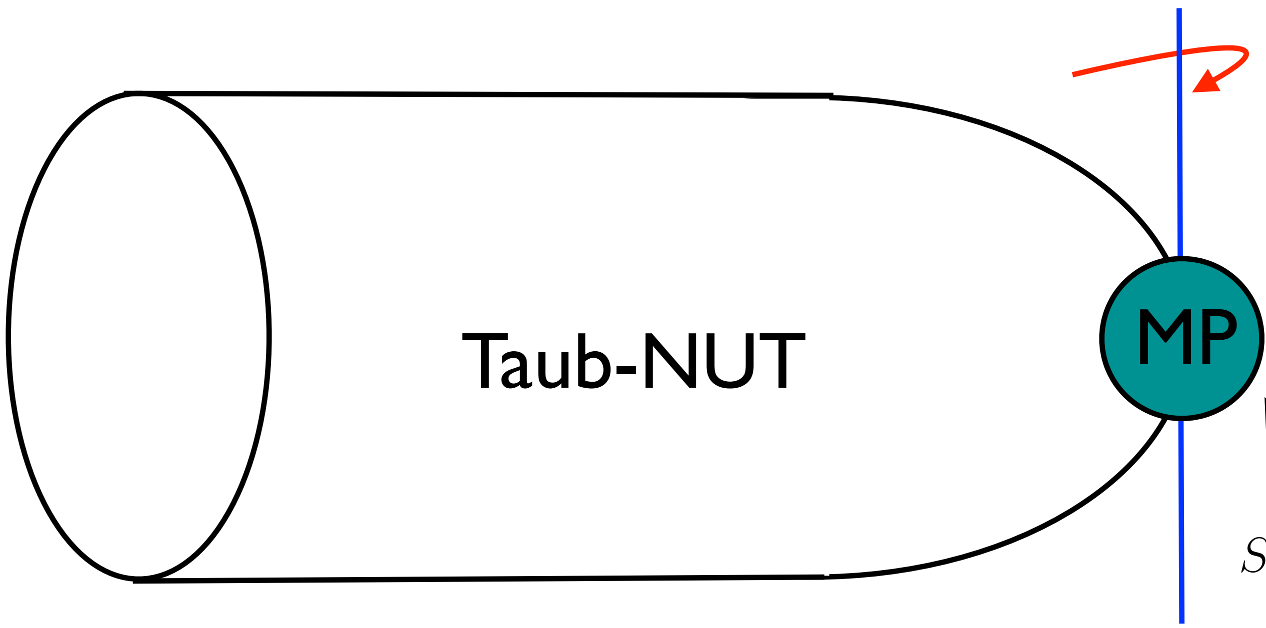
Take limits

$$\boxed{\begin{array}{lcr} p \to \infty & y \to \infty & \psi = \frac{y}{p} \\ q \to 0 & pq = \frac{\mu}{4} & \\ \alpha \to 0 & p\alpha = \frac{1}{8}(\mu-(a+b)^2)^{\frac{1}{2}}(a-b) & \\ m \to 0 & pm = \frac{1}{8}[\mu(\mu-(a+b)^2)]^{\frac{1}{2}} & \\ r \to 0 & pr = \frac{1}{4}[\rho^2 - \frac{1}{2}(\mu - a^2 - b^2 - 8pm)] & \end{array}}$$

metric of five dimensional Myers-Perry black hole placed at tip of Taub-NUT cigar

$$\boxed{ds^2 = -dt^2 + \frac{\mu}{\Sigma}(dt - a\,\sin^2\theta\, d\psi - b\,\cos^2\theta\, d\phi)^2 + \Sigma\left(\frac{d\rho^2}{\Delta} + d\theta^2\right) + (\rho^2+a^2)\sin^2\theta\, d\psi^2 + (\rho^2+b^2)\cos^2\theta\, d\phi^2}$$

$$\Sigma = \rho^2 + a^2\cos^2\theta + b^2\sin^2\theta\ , \ \ \Delta = \frac{(\rho^2+a^2)(\rho^2+b^2) - \mu\rho^2}{\rho^2}$$



$$J_1 = \frac{\pi\,\mu\,a}{4G_5N_6}\ ,\ \ J_2 = \frac{\pi\,\mu\,b}{4G_5N_6}$$

$$J_{1,2} = \frac{N_0N_6}{2} \pm J$$

$$S = 2\pi\sqrt{J^2 - \frac{N_0^2N_6^2}{4}}$$

Emparan, Maccarrone, 2007;
Horowitz, Roberts, 2007

Myers & Perry

$\phi \leftrightarrow -\phi \ , \ \ b \leftrightarrow -b$ is a symmetry

$\therefore J_2 \leftrightarrow -J_2 \ , \ $ Cartans flipped

This is just a reflection of Myers-Perry

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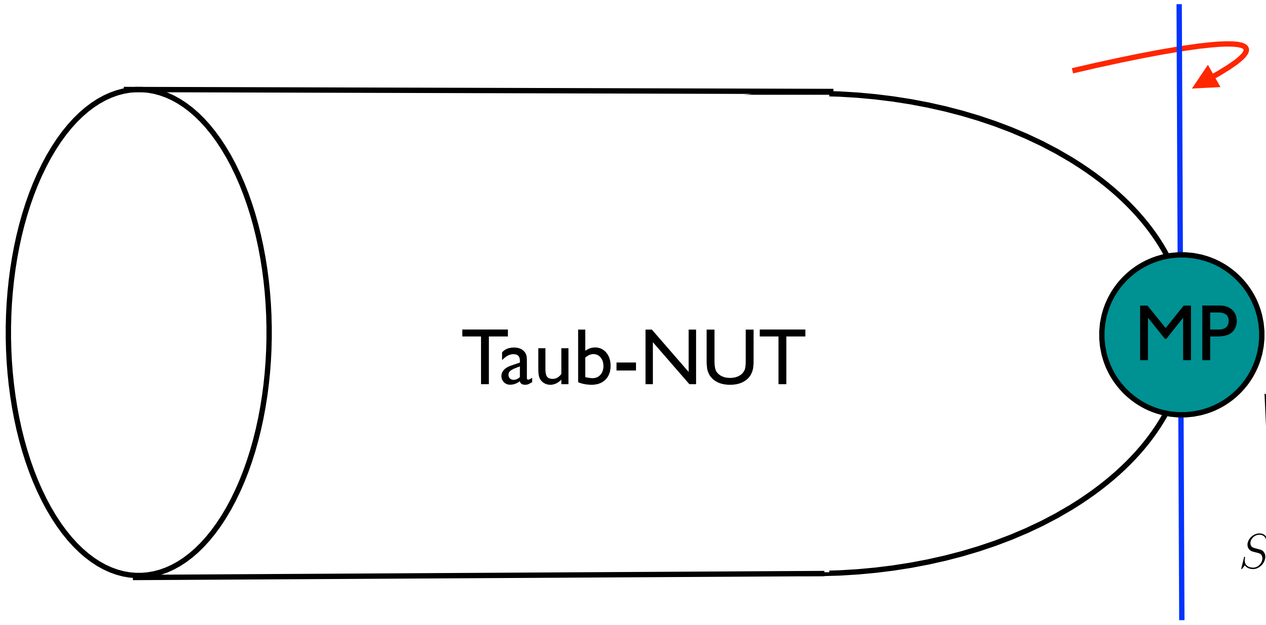

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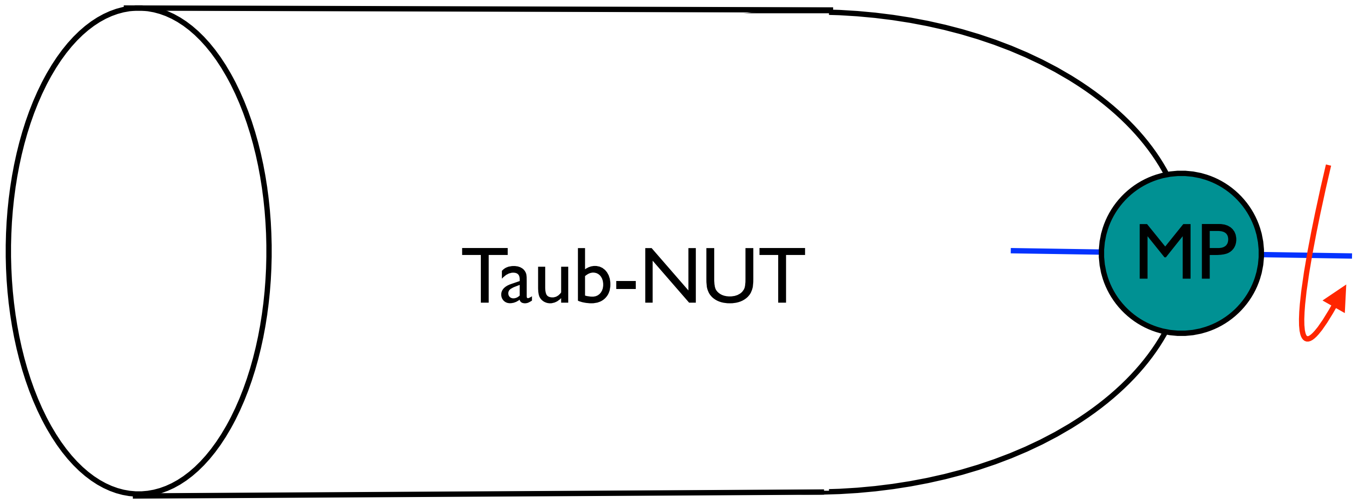
$$J_1 = J_2 \Longrightarrow J = 0$$

This exchanges angular momentum and charge



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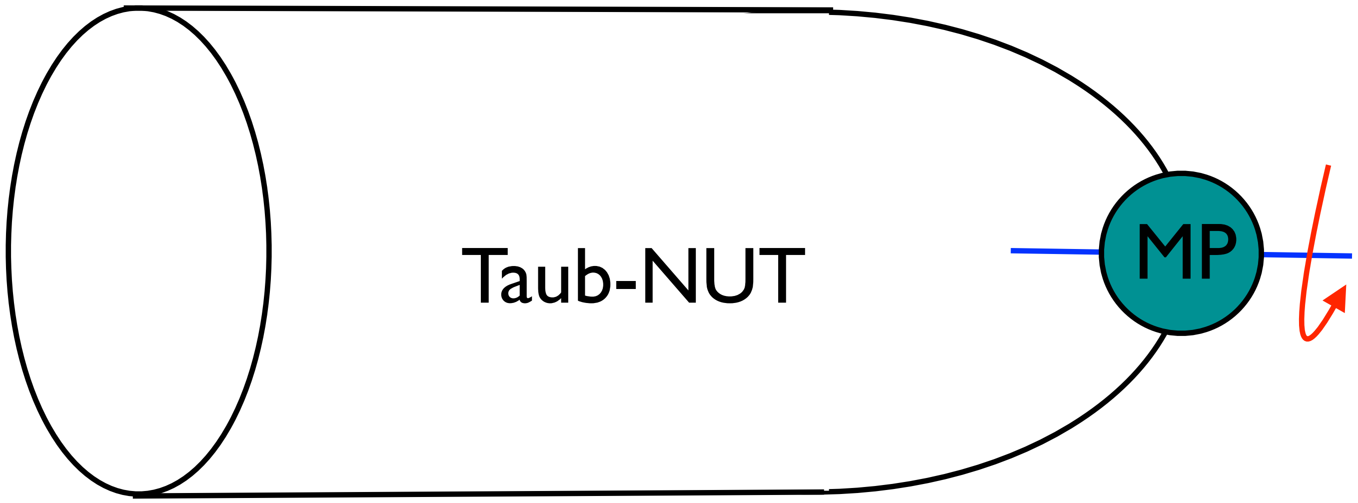
This exchanges angular momentum and charge

Trade J for N_0

Entropy stays the same



$$J_1 = \frac{\pi \, \mu \, a}{4 G_5 N_6} \ , \ J_2 = \frac{\pi \, \mu \, b}{4 G_5 N_6}$$

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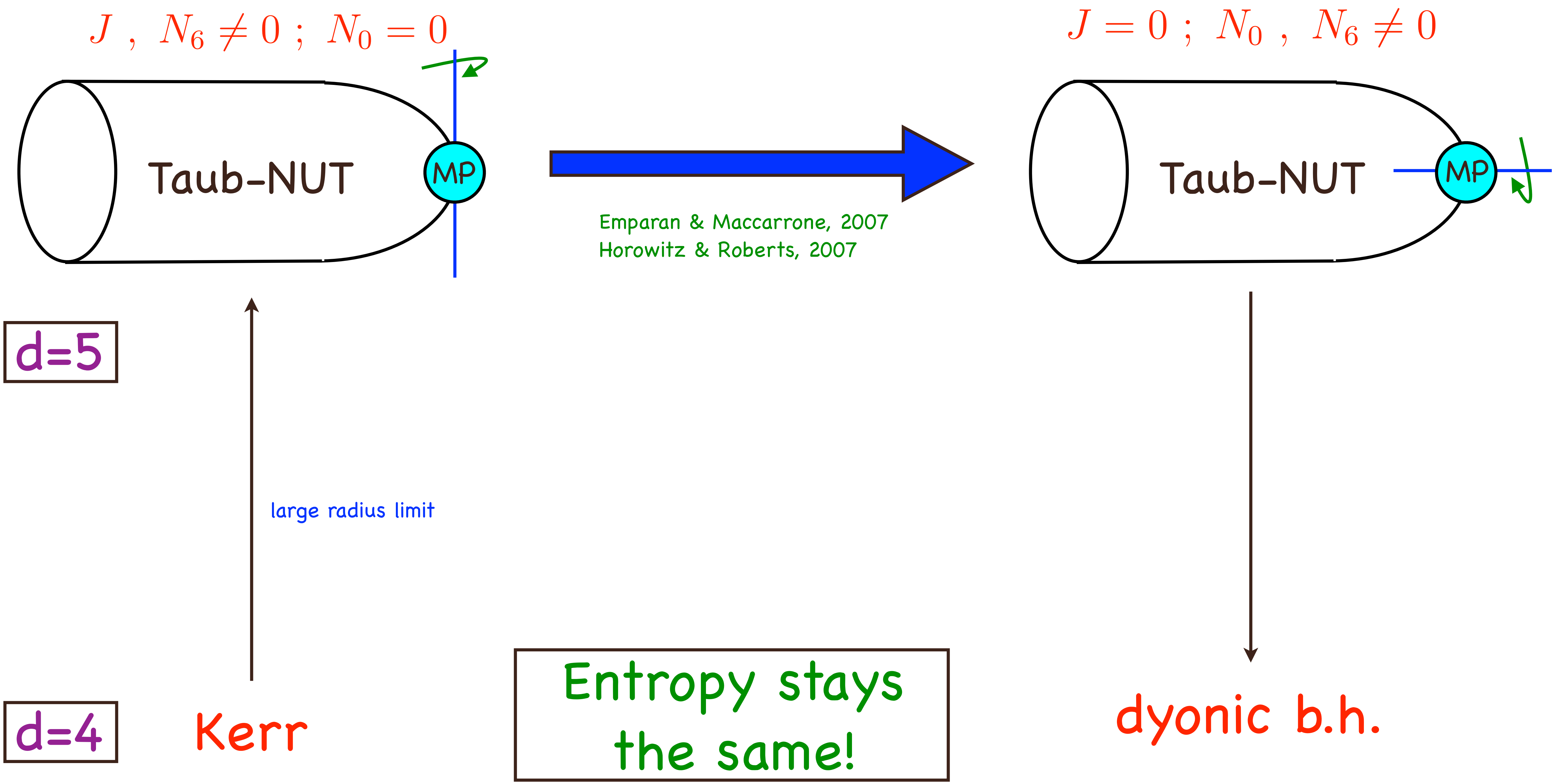

Myers & Perry

- We have a system of D0- and D6-branes

N_0 D0−brane charge

$N_6 = 1$ D6−brane charge

- Use four dimensional U-duality

$m = \frac{J}{n}$ D0−brane charge

$n \gg 1$ D6−brane charge

Entropy stays the same!

- Reverse uplift to four dimensions

Kerr black hole on $\mathbb{R}^4/\mathbb{Z}_N$; with angular momentum J

Cardy

- Azimuthal angle has periodicity $\dfrac{2\pi}{N_6}$

- Temperature is then $T_L = \dfrac{\Delta\varphi}{4\pi^2} = \dfrac{1}{2\pi N_6}$

- Central charge increases by same factor N_6

$$\boxed{S_{\rm CFT} = \frac{\pi^2}{3} c_L T_L = 2\pi J = S_{\rm BH}}$$

- Family of CFTs with correct entropy

- Still not in the Cardy regime; in fact further than where we started as temperature is now even smaller

Cardy

Type IIA on T^6

$$\vec{Q} = (q_0 = N_0, 0, \ldots, 0)$$

$$\vec{P} = (0, p^0 = N_6, 0, \ldots, 0)$$

$$\vec{Q} \cdot \vec{P} = \vec{Q} \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes 1 \right) \vec{P} = q_0 \, p^0 = N_0 N_6 \gg 1$$

Cardy regime

$$S_{\mathrm{BH}} = \pi \sqrt{-I}$$

$$\boxed{\begin{array}{c} p^0 = 0 \\ I = (\vec{Q} \cdot \vec{Q})^2 (\vec{P} \cdot \vec{P})^2 - (\vec{Q} \cdot \vec{P})^2 \gg 6 (p^1)^2 (\vec{P} \cdot \vec{P})^2 \end{array}}$$

Nampuri, Tripathy, Trivedi, 2007

Cardy

Type IIA on T^6

$$\vec{Q} = (q_0 = N_0, 0, \ldots, 0)$$

$$\vec{P} = (0, p^0 = N_6, 0, \ldots, 0)$$

$$\vec{Q}\cdot\vec{P} = \vec{Q}\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes 1\right)\vec{P} = q_0\, p^0 = N_0 N_6 \gg 1$$

U-duality

Cardy regime

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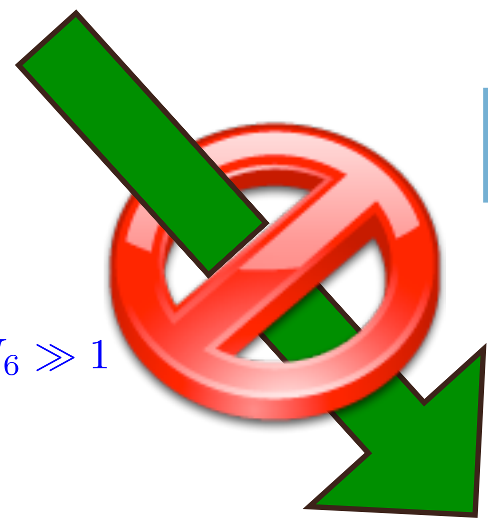

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infinitesimally close to

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D0/D2/D2/D4/D4/D4 system

Cardy

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$$\updownarrow$$

$$|q_0| \gg 1\ ,\ \ |p^0| \gg 1$$

$$S_{\rm BH} = \pi\sqrt{-I}$$

Cardy finally applies!

Cardy

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$$\updownarrow$$

$$|q_0| \gg 1 \;,\; |p^0| \gg 1$$

Cardy finally applies!

- This correctly reproduces entropy of Kerr black hole

Kerr & Cardy

- Assumption: Near-horizon geometry remains invariant under duality transformations

  Thus the leading entropy remains same in sugra

- Assumption: Attractor mechanism operates for non-supersymmetric black holes and the moduli are under control in map from D0/D6 to D0/D2/D2/D4/D4/D4

  Can check this (I will point out how momentarily)

- Note: At a point in moduli space where the four dimensional coupling is strong, D0/D2/D2/D4/D4/D4 gives BTZ

  Thus, this is naturally related to $\mathrm{AdS}_3/\mathrm{CFT}_2$

Kerr & Cardy

5d KK black hole embedded in M-theory $\mathbb{R}^{1,3} \times K3 \times S_M \times S_1 \times S_2$

reduced to type IIA $\mathbb{R}^{1,3} \times K3 \times S_1 \times S_2$

dual to heterotic $\mathbb{R}^{1,3} \times T^3 \times S_M \times S_1 \times S_2$

- The S_M circle is the eleven-dimensional M-theory circle with radius R_{11}.
- The S_1 and S_2 are circles with radii R_1 and R_2, respectively. We can think of these two circles as being T^2 at a special point with volume denoted as V_2.
- Of the 22 two-cycles of $K3$, 16 arise as fixed points of $T^4/\mathbb{Z}_2$ at the orbifold point and are not of interest. Of the remaining six two-cycles, D2-branes in the type IIA frame wrap the cycles corresponding to momentum and winding charges on the S_1 circle in the heterotic frame; these have volumes labelled by $V_{(2)}$ and $V_{(3)}$, respectively.
- The volume of the six-dimensional manifold, $K3 \times T^2$ is denoted by V_6.
- The quantities ℓ_{11} and ℓ_5 denote Planck lengths in eleven dimensions and five dimensions, respectively.
- The four-dimensional string coupling constant is denoted as g_4; unless explicitly stated, this is represented in the heterotic frame.

Kerr & Cardy

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- from comparing masses and tensions of objects, map moduli

$$\frac{V_{K3}}{\ell_{11}^4} = (g_4^2)^{\frac{2}{3}} \left(\frac{R_{11}}{\ell_s}\right)^{\frac{2}{3}} \left(\frac{V_2}{\ell_s^2}\right)^{\frac{2}{3}}$$

$$\frac{R_1}{\ell_{11}}\sqrt{\frac{V_{K3}}{\ell_{11}^4}} = \frac{R_1}{\ell_s}$$

$$\frac{R_2}{\ell_{11}}\sqrt{\frac{V_{K3}}{\ell_{11}^4}} = \frac{R_2}{\ell_s}$$

$$\frac{R_{11}}{\ell_{11}}\sqrt{\frac{V_{K3}}{\ell_{11}^4}} = \frac{R_{11}}{\ell_s}$$

$$\frac{R_{11}}{\ell_5} = \left(\frac{R_{11}}{\ell_s}\right)^{\frac{2}{3}} \left(\frac{1}{g_4^2}\right)^{\frac{1}{3}}$$

M-theory/heterotic moduli

$$\frac{R_1}{\ell_s} = \sqrt{\frac{R_1}{R_2}}\frac{1}{g_4}$$

$$\frac{R_2}{\ell_s} = \sqrt{\frac{R_2}{R_1}}\frac{1}{g_4}$$

$$\frac{V_{(2)}}{\ell_s^2} = \frac{R_{11}}{R_1}$$

$$\frac{V_{(3)}}{\ell_s^2} = \frac{R_{11}R_1}{\ell_s^2}$$

$$\frac{V_2}{\ell_s^2} = \frac{1}{g_4^2}$$

$$\frac{1}{g_{4,A}^2} = \frac{V_2}{\ell_s^2}$$

$$\frac{V_6}{\ell_s^6} = \frac{1}{g_4^2}\frac{R_{11}^2}{\ell_s^2}$$

type IIA/heterotic moduli

Kerr & Cardy

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reduced to type IIA $\mathbb{R}^{1,3} \times K3 \times S_1 \times S_2$

dual to heterotic $\mathbb{R}^{1,3} \times T^3 \times S_M \times S_1 \times S_2$

- from comparing masses and tensions of objects, map moduli

- want gravity to be weak in 5d so sugra is a good approximation

- attractor mechanism fixes the radii of $\frac{R_{11}}{\ell_s} = \frac{R_1}{\ell_s} = \sqrt{q_0}$

- duality maps are under control when $\frac{R_2}{\ell_s} \gg 1\ ,\ \frac{R_2}{\ell_s} > g_4^{-2} q_0^{\frac{1}{3}}$

Kerr

- Extremal D0/D6 uplifted to five dimensions has the near-horizon geometry $\mathrm{AdS}_2 \times S^3$

- Black hole is not a thermal excitation over pure AdS_2 so it is unique solution (modulo diffeos) with given charges

- CFT is uniquely characterised by D0/D6 quantum numbers that fix radius, central charge consistent with NHEK analysis

Amsel, Horowitz, Marolf, Roberts, 2009;
Dias, Reall, Santos, 2009

Kerr

- We have gravitational thermodynamics of extremal Kerr (analogue of Strominger-Vafa), but what are microstates of extremal Kerr (analogue of Mathur's fuzzball programme)?

- Notably the CFT dual of Kerr is chiral — does this arise as a DLCQ of a non-chiral CFT as in BDSS? — if so, what is the parent theory?

Balasubramanian, de Boer, Sheikh-Jabbari, Simón, 2009

- What about non-extremal Kerr?

  Instead of near-horizon with conformal symmetry, look for conformal symmetry in space of solutions of wave equation in Kerr background

  Derive temperature from periodicities, assume central charges are 12J

  Correctly derive entropy from Cardy formula

- Are there hidden conformal symmetries?

Castro, Maloney, Strominger, 2010

Thank You!