Charged particle-like branes in ABJM

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Motivation: Construct more general “baryon vertex - like” configurations in ABJM, with potential interest for AdS/CMT applications

In particular: Adding magnetic flux to the particle-like branes in the gravity dual ($AdS_4 \times CP^3$) allows to construct candidates for holographic anyons in ABJM, with the same behavior found by Hartnoll for the anyonic membranes and F- and D-strings in $AdS_7 \times S^4$ and $AdS_5 \times S^5$ (Kawamoto & Lin’09)

Results: More general “baryon vertex - like” configurations, with numbers of quarks depending on the flux, di-baryon with external quarks, etc

Outline:

I. Introduction (3 dim SCFTs, ABJM)

II. The dual gravitational backgrounds of M-theory and Type IIA

III. Add a magnetic flux

IV. Study of the dynamics

1. The 't Hooft monopole with flux
2. The di-baryon with flux
3. The baryon vertex with flux

V. Generalize to non-zero Romans mass

VI. Conclusions

1.Introduction

In the last two years: Important progress in the connection between 3 dimensional SCFTs and multiple M2-branes (Schwarz, Bagger and Lambert, Gustavsson, ABJM)

In the context of AdS/CFT: Construct an explicit Lagrangian for the 3 dim SCFT dual to M-theory on $AdS_4 \times S^7$

Bagger & Lambert, Gustavsson'07: Explicit classical Lagrangian with $\mathcal{N} = 8$ SUSY, scale and parity invariant.
At the expense of a non-associative 3-algebra for the 8 coordinates:

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d \qquad \Rightarrow \qquad \delta X = [\alpha, \beta, X]$$

More generally: $\delta X_d = f^{abc}{}_d \Lambda_{ab} X_c = \tilde{\Lambda}^c{}_d X_c$

$\Rightarrow$ Gauge field with two algebraic indices

CS matter theory:

$$\mathcal{L}_{CS} = \epsilon^{\mu\nu\lambda}(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef})$$

Explicit example: 4 generators and $f^{abcd} = \frac{2\pi}{k}\epsilon^{abcd}$

$\rightarrow$ Only possible choice for unitarity and $\mathcal{N}$ = 8 SUSY

Van Raamsdonk'08: BLG equivalent to an SU(2)xSU(2) CS gauge theory with opposite levels

$$A_\mu = A^{+}_{\mu 4i}\sigma_i\,; \quad \hat{A}_\mu = A^{-}_{\mu 4i}\sigma_i$$

Real vector of SO(4) $\rightarrow$ Bifundamental of SU(2)xSU(2)

2 M2-branes k?, moduli space?

ABJM'08: Generalize to SU(N)xSU(N):

- N M2-branes in $C^4/\mathbb{Z}_k$ described by a supersymmetric $U(N)_k \times U(N)_{-k}$ CS matter theory
- $\mathcal{N}$ = 6 supersymmetric, enhanced to $\mathcal{N}$ = 8 for $k = 1, 2$
- At large N and for $k << N^{1/5}$ the theory is dual to M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ with N units of F_7 flux
- For $N^{1/5} << k << N$ the dual theory is Type IIA on $AdS_4 \times CP^3$ with N units of F_6 flux and k units of F_2 flux
- The string background describes the 't Hooft limit of the theory: $N, k \to \infty$ with $\lambda = N/k$ fixed

New AdS/CFT pair in which to test the gauge/gravity correspondence

3 dimensions $\rightarrow$ Applications in condensed matter

Dual string theory is Type IIA

II. The dual gravitational backgrounds of M-theory and Type IIA

A stack of N M2-branes creates the extremal geometry:

$$ds^2 = h^{-2/3} dx_{1,2}^2 + h^{1/3}(dr^2 + r^2 d\Omega_7^2)$$

$$h = 1 + \frac{R^6}{r^6}\ ; \qquad R^6 = 32\pi^2 N l_p^6$$

$$F_4 = d^3x \wedge dh^{-1}$$

In the near horizon limit, $r << R$, the metric becomes that of $AdS_4 \times S^7$:

$$ds^2 = R^2\Big(\frac{1}{4} ds^2_{AdS_4} + ds^2_{S^7}\Big);$$

$$F_4 = 24\frac{r^2}{R^3} d^3x \wedge dr$$

$$\int_{S^7} F_7 = (2\pi)^6 N$$

Use the description of S^7 as an S^1 fibration over CP^3:

$$ds^2_{S^7} = (d\varphi + \omega)^2 + ds^2_{CP^3}$$

Perform the $\mathbb{Z}_k$ quotient:

$$\varphi \equiv \frac{\varphi}{k}\,; \qquad \varphi \sim \varphi + 2\pi\,; \qquad N \equiv Nk$$

Then: $$ds^2_{S^7/\mathbb{Z}_k} = \frac{1}{k^2}(d\varphi + k\omega)^2 + ds^2_{CP^3}$$

Reduce along φ :

$$ds^2 = L^2\Big(\frac{1}{4}ds^2_{AdS_4} + ds^2_{CP^3}\Big); \qquad L = (\frac{32\pi^2 N}{k})^{1/4}$$

$$e^\phi = g_s = \frac{L}{k}$$

$$F_2 = dC_1 = \frac{2L}{g_s} J\ ; \qquad F_4 = dC_3 = \frac{6}{g_s L} dVol(AdS_4)$$

$$F_6 = dC_5 = \frac{6L^5}{g_s} dVol(CP^3)$$

with $dVol(CP^3) = \frac{1}{6} J \wedge J \wedge J$ and $J = \frac{1}{2} d\omega$ the Kähler form on CP^3

M-theory description valid when $\frac{R}{k} >> l_p \Leftrightarrow N >> k^5$

IIA description valid when $\frac{R}{k} << l_p \Leftrightarrow N << k^5$ and $L >> l_s \Leftrightarrow N >> k$

Flux integrals:

$$\int_{CP^1} F_2 = 2\pi k\,; \qquad \int_{CP^3} F_6 = (2\pi)^5 N$$

Induce tadpoles on the D2 and D6 -branes that need to be cancelled adding F-strings:

D6-brane:

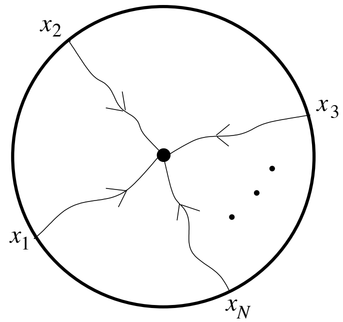
$$S_{CS} = 2\pi\, T_6 \int_{\mathbb{R}\times CP^3} P[F_6] \wedge A = N\, T_{F1} \int dt A_t$$

Cancel this charge with the charge induced by the endpoints of N open F-strings stretching between the D6 and the boundary of AdS (baryon vertex)

The baryon vertex in $AdS_5 \times S^5$

Gauge invariant coupling of N external quarks

Through AdS/CFT external quarks are regarded as endpoints of F-strings in AdS



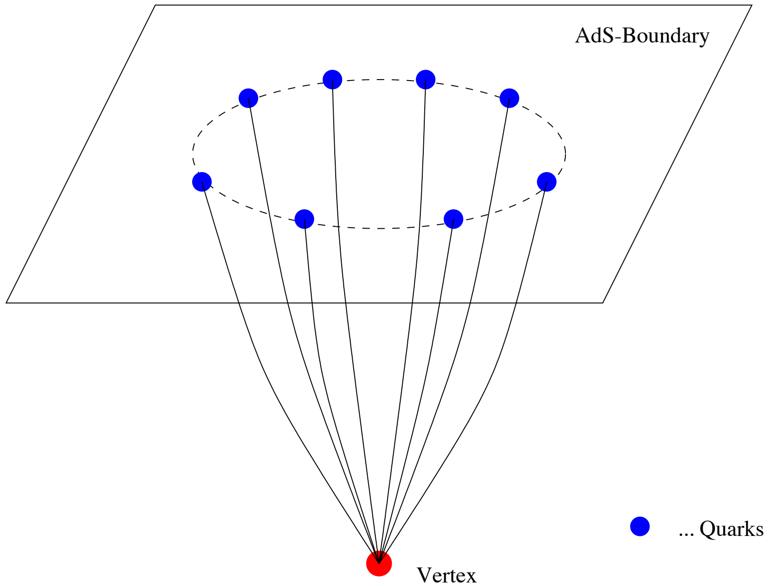
Baryon vertex in the SUGRA dual: D5-brane wrapped on the 5-sphere (Witten'98):

$$S_{CS} = 2\pi\, T_5 \int_{\mathbb{R}\times S^5} P[F_5] \wedge A = N\, T_{F1} \int_{\mathbb{R}} dt A_t$$

N charge cancelled by N F-strings ending on the 5-brane

N F-strings connecting the D5-brane to the boundary of AdS behave as fermions

Dual configuration on the CFT side: N Wilson lines ending on an epsilon tensor $\rightarrow$ Bound state of N quarks:



D2-brane:

$$S_{CS} = 2\pi\, T_2 \int_{\mathbb{R}\times CP^1} P[F_2] \wedge A = k\, T_{F1} \int dt A_t$$

$\Rightarrow k$ open F-strings

k Wilson lines cannot end on an epsilon tensor

If one forms the symmetric product only the endpoint of the Wilson lines is observable and the product behaves like a 't Hooft operator creating one unit of magnetic flux at a point (ABJM) $\rightarrow$ 't Hooft monopole

Other particle-like branes:

D4-brane wrapped on the $CP^2 \subset CP^3$:

Baryon charge N, $m_{D4}L = N \Rightarrow \Delta = \frac{m_{D4}L}{2} = \frac{N}{2}$

$\Rightarrow$ Dual configuration composed of N chirals

No tadpoles

$\Rightarrow$ Dual to the di-baryon $O^{D4} = \epsilon_{i_1 \ldots i_N} \epsilon^{j_1 \ldots j_N} A^{i_1}_{j_1} \ldots A^{i_N}_{j_N}$

D0-brane:

Baryon charge k, $m_{D0}L = k \Rightarrow \Delta = \frac{m_{D0}L}{2} = \frac{k}{2}$

$\Rightarrow$ Dual configuration composed of k chirals

No tadpoles

$\Rightarrow$ Dual to the di-monopole

$$O^{D0} = (Sym_k W)_{i_1 \dots i_k} (Sym_k W)^{j_1 \dots j_k} A^{i_1}_{j_1} \dots A^{i_k}_{j_k}$$

Remark: We will assume the right boundary conditions that allow each brane to exist

III. Add a magnetic flux

Take $F = \mathcal{N} J$:

- D2-brane: D0-brane charge induced:

$$S_{CS} = 2\pi\, T_2 \int_{\mathbb{R}\times CP^1} C_1 \wedge F = \frac{\mathcal{N}}{2}\, T_0 \int_{\mathbb{R}} C_1$$

- D4-brane: $\displaystyle\int_{CP^2} F \wedge F = \mathcal{N}^2 \pi^2 \quad \Rightarrow$ D0-brane charge

It captures the F_2 flux and develops a tadpole $\Rightarrow$ F-strings ending on it :

$$S_{CS} = \frac{1}{2}(2\pi)^2\, T_4 \int_{\mathbb{R}\times CP^2} P[F_2] \wedge F \wedge A = \frac{k\mathcal{N}}{4}\, T_{F1} \int dt A_t$$

In fact, Freed-Witten anomaly (see later)

- D6-brane: $\int_{CP^3} F \wedge F \wedge F = \mathcal{N}^3 \pi^3 \quad \Rightarrow$ D0-brane charge

It captures the F_2 flux $\Rightarrow$ Additional F-strings ending on it:

$$S_{CS} = \frac{1}{6}(2\pi)^3 T_6 \int_{\mathbb{R}\times CP^3} P[F_2] \wedge F \wedge F \wedge A = \frac{k\mathcal{N}^2}{24} T_{F1} \int dt A_t$$

$\Rightarrow N + \dfrac{\mathcal{N}^2}{24}$ F-strings

All branes topologically stable

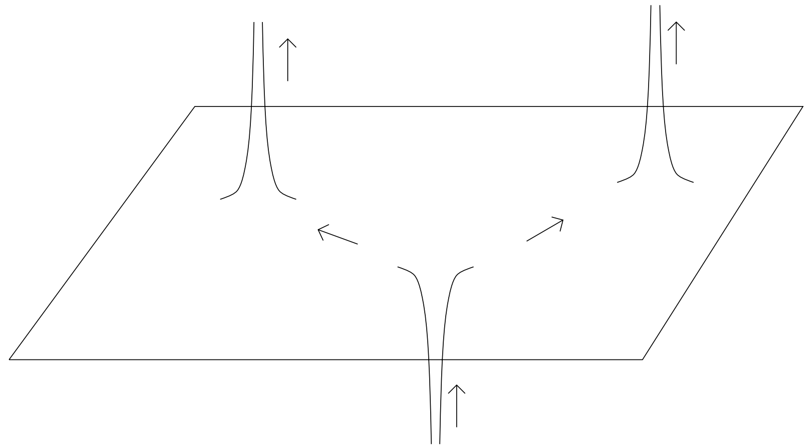
Stability in the AdS direction: Look at the equations of motion

Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98: Study the stability of the baryon vertex in $AdS_5 \times S^5$ in the probe brane approximation

However, in order to be able to ignore the deformation caused by the string tension and the electric field the strings should be uniformly distributed on the brane, and then all SUSYs are broken

In order to preserve SUSY: All strings end on a point

But then the branes are deformed by the string tension and electric field to form spiky solutions, and the probe brane approximation is not valid



In $AdS_5 \times S^5$: The energy obtained in the non-susy probe brane approximation is corrected, with the bound state becoming marginal as one would expect from supersymmetry (Imamura'98)

Partial studies of spiky solutions in $AdS_4 \times CP^3$ in Kawamoto, Lin'09

Are the Dp-branes with magnetic flux supersymmetric?

- Checking explicitly the kappa symmetry conditions in $AdS_4 \times CP^3$ is complicated

- The D2-D0 bound state is probably non-supersymmetric, as in flat space

- For the D4-brane:

$$S_{DBI} = -\frac{T_4}{g_s} \int d^5\xi \sqrt{g_{CP^2}} \left(L^4 + 2\,(2\pi)^2 F_{\alpha\beta} F^{\alpha\beta} \right)$$

and:

$$E_{D4_{\mathcal{N}}} = E_{D4} + E_{\mathcal{N}^2/8\,D0}$$

$\Rightarrow$ Threshold BPS intersection of D4 and D0-branes

- The D6-brane with charge is probably non-supersymmetric

Effect of the F-strings: Supersymmetry preserved if all strings end on a point

IV. Study of the dynamics

Consider: $S = S_{Dp} + S_{qF1}$

$$S_{Dp} = -Q_p \int dt \, \frac{2\rho}{L} \, , \qquad Q_p = \frac{\pi^{p/2}\, T_p}{(\frac{p}{2})!\, g_s} \Big(L^4 + (2\pi\mathcal{N})^2 \Big)^{p/4}$$

$$S_{qF_1} = -q\, T_{F1} \int dt dx \sqrt{\frac{16\rho^4}{L^4} + \rho'^2}$$

$\rho = \rho(x)$: Position in AdS

Bulk equation of motion: $$\frac{\rho^4}{\sqrt{\frac{16\rho^4}{L^4} + \rho'^2}} = c$$

Boundary equation of motion: $$\frac{\rho'_0}{\sqrt{\frac{16\rho_0^4}{L^4} + \rho_0'^2}} = \frac{2Q_p}{L\, q\, T_{F_1}}$$

Define $\sqrt{1-\beta^2} = \frac{2Q_p}{L\,q\,T_{F1}}$ with $\beta \in [0,1]$

The two equations can be combined into:

$$\frac{\rho^4}{\sqrt{\frac{16\rho^4}{L^4} + \rho'^2}} = \frac{1}{4}\,\beta\,\rho_0^2\,L^2$$

Integrating: Size of the configuration:

$$\ell = \frac{L^2}{4\rho_0}\int_1^\infty dz\,\frac{\beta}{z^2\sqrt{z^4-\beta^2}}$$

Same form for the baryon vertex in $AdS_5 \times S^5$

Same dependence on L^2 in $AdS_5 \times S^5$: Prediction of AdS/CFT for the strong coupling behavior of the gauge theory

On-shell energy:

$$E = E_{Dp} + E_{qF1} = qT_{F_1}\rho_0\Big(\sqrt{1-\beta^2} + \int_1^\infty dz \frac{z^2}{\sqrt{z^4-\beta^2}}\Big)$$

Binding energy:

$$E_{\rm bin} = q\,T_{F_1}\,\rho_0\Big(\sqrt{1-\beta^2} + \int_1^\infty dz \Big[\frac{z^2}{\sqrt{z^4-\beta^2}} - 1\Big] - 1\Big)$$

where we have substracted the energy of the constituents (when the brane is located in $\rho_0 = 0$ the strings become radial and correspond to free quarks)

- $E_{\rm bin}$ negative and decreases monotonically with β
- $E_{\rm bin} = 0$ for $\beta = 0$ (q free radial strings stretching from ρ_0 to ∞ plus a Dp-brane at ρ_0) (Only for non-zero magnetic flux)

Configuration of free quarks degenerate. The location of the Dp has become a moduli of the system

As a function of ℓ :

$$E_{\rm bin} = -f(\beta)\frac{(g_s N)^{2/5}}{\ell} \quad \text{with} \quad f(\beta) \geq 0$$

$\Rightarrow$ - The configuration is stable

- $E_{\rm bin} \sim 1/\ell$ dictated by conformal invariance

- As a function of the 't Hooft coupling, $\lambda = N/k$,

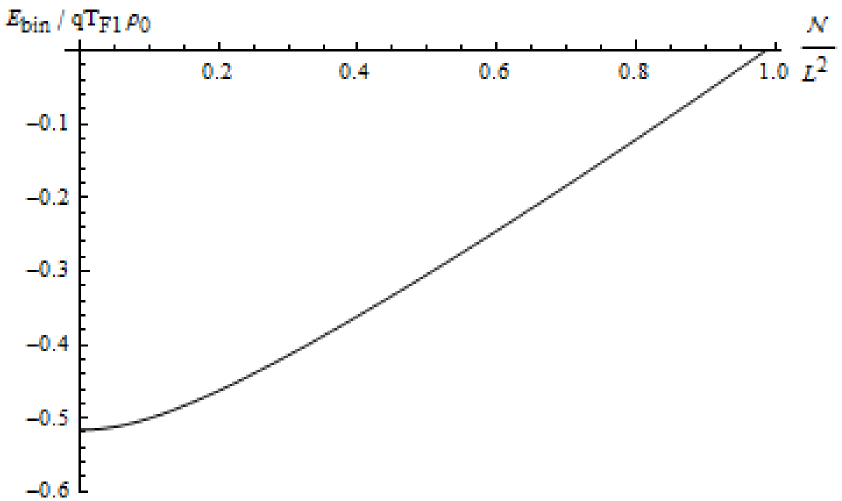
  $E_{\rm bin} \sim \sqrt{\lambda}$, as in $AdS_5 \Rightarrow$ Non-trivial prediction for the non-perturbative regime of the gauge theory.

1. The 't Hooft monopole with flux

In this case:

$$\beta = \sqrt{1 - \frac{1}{4\pi^2}\left(1 + \frac{4\pi^2\mathcal{N}^2}{L^4}\right)}$$

Allowed values for the magnetic flux: $\frac{\mathcal{N}}{L^2} \leq \sqrt{1 - \frac{1}{4\pi^2}}$



Binding energy per string

At the bound the strings become radial and the configuration ceases to be stable

Similar to the baryon vertex with flux in $AdS_5 \times S^5$

2. The di-baryon with flux

In this case:

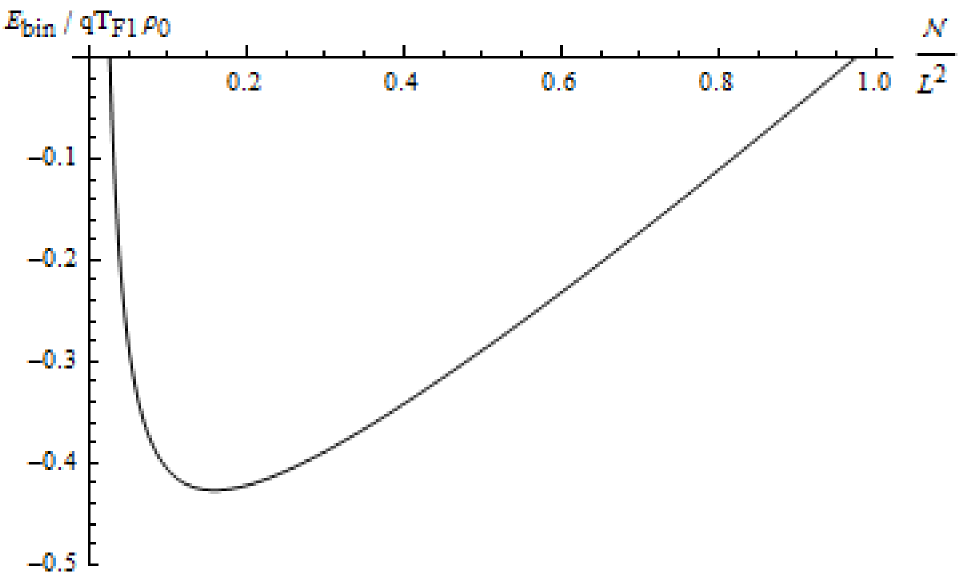
$$\beta = \sqrt{1 - \frac{L^4}{16\pi^4\mathcal{N}^2}\Big(1 + \frac{4\pi^2\mathcal{N}^2}{L^4}\Big)^2}$$

Allowed values for the magnetic flux:

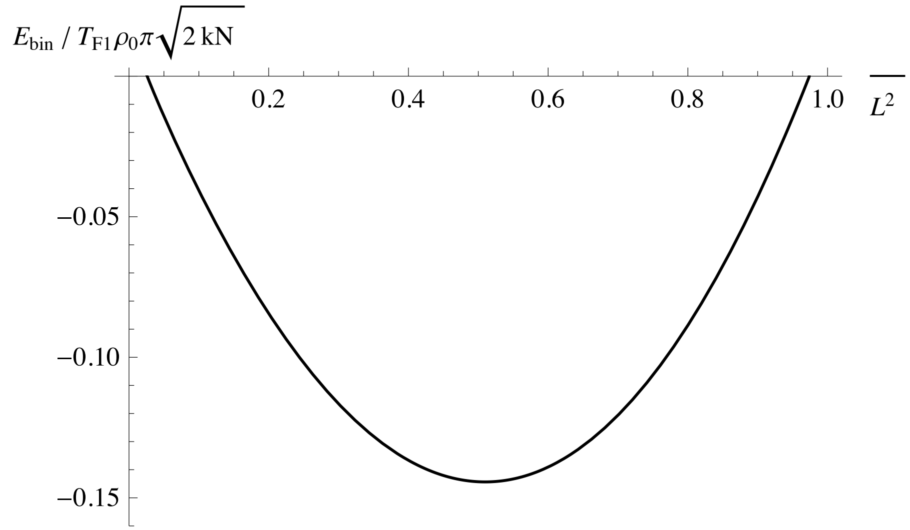
$$\frac{1}{2}\Big(1 - \sqrt{1 - \frac{1}{\pi^2}}\Big) \leq \frac{\mathcal{N}}{L^2} \leq \frac{1}{2}\Big(1 + \sqrt{1 - \frac{1}{\pi^2}}\Big)$$

$\Rightarrow$ Allowed values for the F-strings ending on it $\left(q = \frac{k\mathcal{N}}{4}\right)$

At the bounds the strings become radial and the configuration ceases to be stable



Binding energy per string



Total binding energy

Configuration maximally stable for $\dfrac{\mathcal{N}}{L^2} = \dfrac{1}{2\pi}$

Dual operator:

$$\mathcal{O}^{D4_{\mathcal{N}}} = \epsilon_{i_1 \ldots i_N} \epsilon^{j_1 \ldots j_N} A^{i_1}_{j_1} \ldots A^{i_N}_{j_N} (Antisym_k W)^{\mathcal{N}/4} (\mathcal{O}^{D0})^{\mathcal{N}^2/8}$$

3. The baryon vertex with flux

In this case:

$$\beta = \sqrt{1 - \frac{1}{36\pi^2(1+\frac{4\pi^2\mathcal{N}^2}{3L^4})^2}\Big(1+\frac{4\pi^2\mathcal{N}^2}{L^4}\Big)^3}$$

Allowed values for the magnetic flux: $\frac{\mathcal{N}}{L^2} \lesssim 1.034$

$\Rightarrow$ More general baryon vertex configurations in which the number of quarks can be increased to $\lesssim 15N$

Also, more general configurations with less number of quarks (some strings stretch between ρ_0 and 0) (BISY):

$$\frac{1}{2}(N + \frac{k\mathcal{N}^2}{24})(1+\sqrt{1-\beta^2}) \leq q \leq N + \frac{k\mathcal{N}^2}{24}$$

V. Generalize to non-zero Romans mass

Gaiotto & Tomasiello'09: A perturbation of $AdS_4 \times CP^3$ by a mass term should be dual to a perturbation of ABJM with levels $k_1 + k_2 = F_0$:

In the D0: $S_{CS} = T_0 \int dt\, F_0 A_t \quad \Rightarrow$ F0 strings ending on it

Di-monopole dual operator:

$$O^{D0} = (\mathrm{Sym_k})_{i_1 \ldots i_{k+F_0}} (\overline{\mathrm{Sym}_k})^{j_1 \ldots j_k} A^{i_1}_{j_1} \ldots A^{i_k}_{j_k}$$

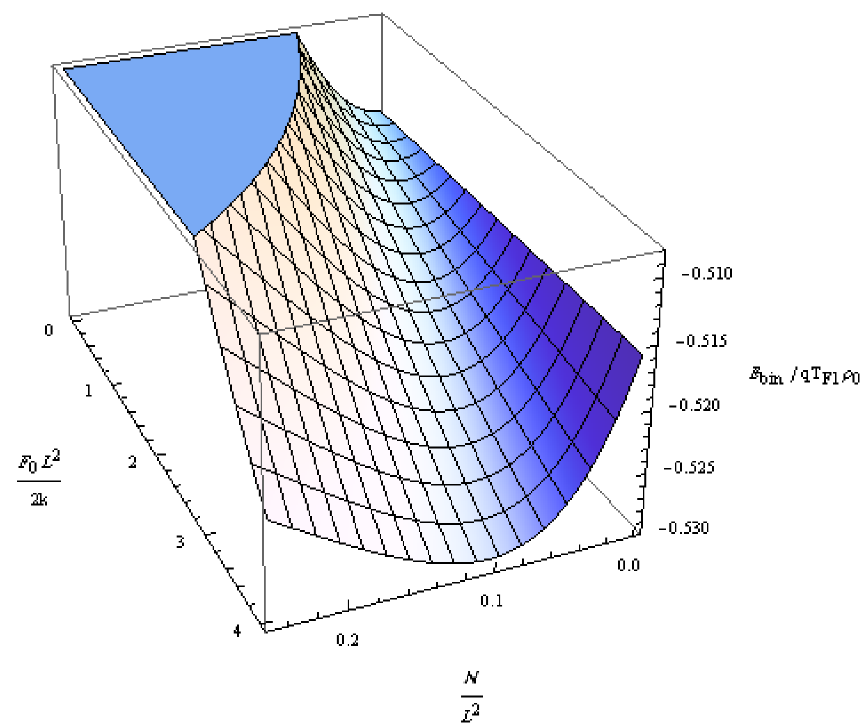
ABJM can be deformed in different ways such that the levels do not sum to zero. In all cases the deformed theory flows to a CFT, with different amounts of symmetries and SUSY

Simplest case: $\mathcal{N}=0$ CFT with SO(6) global symmetry, dual to a perturbation of the $\mathcal{N}=6$ solution by a small mass $F_0 << k, N$

Add $F = \mathcal{N} J$: All branes develop tadpoles proportional to the mass. For instance the D2:

$$S_{CS} = 2\pi T_2 \int F_0 \, A \wedge dA = \frac{F_0 \mathcal{N}}{2} \int dt A_t$$

This changes the stability, like:



Generally configurations more stable for non-zero mass

VI. Conclusions

- Particle-like branes in ABJM with magnetic flux
- New "baryon vertex-like" configurations: 't Hooft monopole , baryon vertex with extra quarks, di-baryon with quarks
- The magnetic flux has to satisfy some bounds $\mathcal{N}_{\rm max} \sim \sqrt{\lambda}$
  Same non-analytic behavior than the binding energy and size
  $\Rightarrow$ Related to the conformal symmetry?
- Find explicit spike solutions (in progress)
- As mentioned, the D4 wraps a CP^2 , which is not spin, so it is subject to the Freed-Witten anomaly (Aharony, Hashimoto, Hirano, Ouyang'09)

The path integral measure is well-defined with a $F_{FW} = \frac{J}{2}$

This flux can be cancelled with a flat B_2: $B_2 = -2\pi \frac{J}{2} \Rightarrow$

The actual dual to ABJM involves this B_2 field. This modifies some quantities in the previous calculations, which are however cancelled with higher curvature terms.

For example, for the D6:

$$S_{CS} = 2\pi\, T_6 \int_{\mathbb{R}\times CP^3} P\Big[F_6 + \frac{1}{6} F_2 \wedge B_2 \wedge B_2\Big] \wedge A$$

But the contribution of $F_2 \wedge B_2 \wedge B_2$ is precisely cancelled with

$$AS = 2\pi^4\, T_6 \int_{\mathbb{R}\times CP^3} C_1 \wedge B_2 \wedge \sqrt{\frac{\hat{\mathcal{A}}(T)}{\hat{\mathcal{A}}(N)}}$$

XVI EUROPEAN WORKSHOP ON STRING THEORY 2010

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