SU(2) Yang-Mills Thermodynamics and Radiation in our Universe

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Yang-Mills action

- (thermal) Yang-Mills
  [Pauli, Barker, and Gulmanelli (1953); Yang and Mills (1954)]

$$S = \frac{\text{tr}}{2} \int_0^\beta d\tau \int d^3x \, F_{\mu\nu} F_{\mu\nu} \, ,$$

  where g is (dimensionless) coupling, $\beta \equiv 1/T$,
  $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$, and
  $A_\mu \equiv A_\mu^a t^a \to \Omega A_\mu \Omega^\dagger + i\Omega\partial_\mu\Omega^\dagger$ ($\Omega(x) \in G$) is gauge field
  such that $F_{\mu\nu} \to \Omega F_{\mu\nu} \Omega^\dagger$ and thus S **is gauge invariant**.
- at $T > 0$: admissible changes of gauge respect **periodicity** of A_μ
- in evaluating partition function $Z \equiv \sum_{\{A_\mu\}} e^{-S}$ in **fundamental fields:** Additional **gauge fixing** required $\Rightarrow$
  1) Faddeev-Popov in PT
  2) restriction to Gribov region (or better) otherwise

Propagating modes

- loop expansion of N-point functions in momentum space, propagator $\bar{D}$

$$\bar{D}(\mathbf{p}, \omega_n) \sim \frac{1}{\omega_n^2 + \mathbf{p}^2 + m^2} \,,$$

where $\omega_n \equiv 2\pi\, nT \;\; (n \in \mathbf{Z})$ nth Matsubara frequency.

- re-expressing (but not changing the contour for τ integration in Euclid. action) summation over n by Cauchy's theorem $\Rightarrow$

$$-\frac{1}{\omega_n^2 + \mathbf{p}^2 + m^2} \longrightarrow \frac{i}{p^2 - m^2} + \delta(p^2 - m^2)\frac{2\pi}{e^{\beta|p_0|} - 1} \,,$$

where $\sum_n \int d^3p \longrightarrow \int d^4p$.

Real-time interpretation of loop integrals

Remarks:

- A more elaborate τ integration contour in the action was considered in [Umezawa, Matsumoto, and Tachiki (1982), Niemi and Semenoff (1984)]. This doubles real-time DOEs to avoid **pinch singularities** in PT.

- In Yang-Mills, where selfdual (**nonpropagating**) field configurations contribute to ground-state physics, such a change of contour for physics of propagating excitations is **inconsistent**.

Trivial-holonomy calorons

- in singular gauge (winding number $|k| = 1$ is localized in a point) there is a **superposition principle** of instanton centers in **prepotential** Π ['t Hooft (1976), Jackiw and Rebbi (1976)]:

$$\begin{aligned} \bar{A}_\mu^{+,a}(x) &= -\bar{\eta}^a_{\mu\nu}\,\partial_\nu \log \Pi\,, \\ \bar{A}_\mu^{-,a}(x) &= -\eta^a_{\mu\nu}\,\partial_\nu \log \Pi\,. \end{aligned}$$

- can be used to satisfy at $|k| = 1$ periodic b.c. in strip $(0 \leq \tau \leq \beta) \times \mathbf{R}^3$ [Harrington and Shepard (1978)]:

$$\begin{aligned} \Pi(\tau, \mathbf{x}; \rho, \beta, x_0) &= 1 + \sum_{l=-\infty}^{l=\infty} \frac{\rho^2}{(x - x_l)^2} \\ &= 1 + \frac{\pi\rho^2}{\beta r} \frac{\sinh\left(\frac{2\pi r}{\beta}\right)}{\cosh\left(\frac{2\pi r}{\beta}\right) - \cos\left(\frac{2\pi\tau}{\beta}\right)}\,, \end{aligned}$$

where $r \equiv |\mathbf{x}|$.

Trivial-holonomy calorons, cntd.

- holonomy of $\bar{A}_\mu^{\pm,a}(x)$ at $r\to\infty$ trivial:

$$\Pi \stackrel{r\to\infty}{=} 1+\frac{\pi\rho^2}{\beta r} \Rightarrow \lim_{r\to\infty}\bar{A}_4^{\pm} \propto \lim_{r\to\infty}\frac{1}{r^2}=0 \Rightarrow$$
$$\mathcal{P}\,\exp\left[i\int_0^\beta d\tau\,\bar{A}_4^{\pm}\right]=\mathbf{1}_2\,.$$

- Gaussian quantum weight [Gross, Pisarski, and Yaffe (1981)]:

$$S_{\rm eff}=\frac{8\pi^2}{\bar{g}^2}+\frac{4}{3}\sigma^2+16\,A(\sigma)\quad(\sigma\equiv\pi\frac{\rho}{\beta})\,,$$

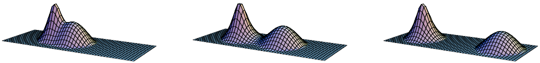
$$A(\sigma)\to-\frac{1}{6}\log\sigma\quad(\sigma\to\infty)\qquad A(\sigma)\to-\frac{\sigma^2}{36}\quad(\sigma\to 0)\,.$$

Conclusion of **semiclassical approx.**:
Trivial-holonomy-caloron weight exponentially suppressed at high T.

Nontrivial holonomy: Magnetic dipoles

- construction based on [Ward 1977, Atiyah and Ward 1977, ADHM 1978, Drinfeld and Manin 1978, Manton 1978, Adler 1978, Rossi 1979, Nahm 1980-1983]

- explicitly carried out in [Lee and Lu 1998, Kraan and Van Baal 1998]: $A_4(\tau, r \to \infty) = -iut^3 (0 \le u \le \frac{2\pi}{\beta})$.



action density of nontrivial-holonomy caloron with $k = 1$ plotted on 2D spatial slice

exact cancellation between A_4-mediated repulsion and A_i-mediated attraction; caloron radius ρ and thus monopole-core separation $D = \frac{\pi}{\beta}\,\rho^2$ increase from left to right (T and holonomy fixed)

Nontrivial holonomy, cntd.

computation of functional determinant about nontrivial holonomy carried out in [Gross, Pisarski, and Yaffe (1981), Diakonov et al. 2004], in latter paper for (relevant) limit $\frac{D}{\beta} = \pi \left(\frac{\rho}{\beta}\right)^2 \gg 1$

conclusions:

- **total suppression** for nontrivial static holonomy in limit $V \to \infty$
- **attraction** of monop. and antimonop. for **small holonomy** $(0 \leq u \leq \frac{\pi}{\beta}(1 - \frac{1}{\sqrt{3}});\ \frac{\pi}{\beta}(1 + \frac{1}{\sqrt{3}}) \leq u \leq 2\,\frac{\pi}{\beta})$
- **repulsion** of monop. and antimonop. for **large holonomy** $(\frac{\pi}{\beta}(1 - \frac{1}{\sqrt{3}}) \leq u \leq \frac{\pi}{\beta}(1 + \frac{1}{\sqrt{3}}))$
- **Instability** of classical configuration under quantum noise $\Rightarrow$ **Nontrivial holonomy does not enter a priori estimate of thermal ground state!**

Inert field ϕ: A priori estimate of thermal ground state

Observations and principles constraining construction of ϕ:

- $F_{\mu\nu} = \pm\tilde{F}_{\mu\nu} \Rightarrow$ vanishing energy-momentum:

$$\begin{aligned}\Theta_{\mu\nu} &= -2\,\mathrm{tr}\Big\{\delta_{\mu\nu}\left(\mp\mathbf{E}\cdot\mathbf{B} \pm \frac{1}{4}(2\mathbf{E}\cdot\mathbf{B} + 2\mathbf{B}\cdot\mathbf{E})\right)\\ &\quad \mp(\delta_{\mu 4}\delta_{\nu i} + \delta_{\mu i}\delta_{\nu 4})\,(\mathbf{E}\times\mathbf{E})_i\\ &\quad \pm\delta_{\mu i}\delta_{\nu(j\neq i)}\,(E_i B_j - E_i B_j) \pm \delta_{\mu(j\neq i)}\delta_{\nu i}\,(E_j B_i - E_j B_i)\Big\} \equiv 0\,.\end{aligned}$$

- spatial isotropy and homogeneity of *effective* local field *not* associated with propagation of energy-momentum by coarse-grained (anti)calorons $\Rightarrow$ **inert scalar** ϕ
- modulo admissible gauge transformations ϕ does not depend on time
- relevance of ϕ (BPS) by gauge-invariant coupling to coarse-grained $k=0$ sector (perturbative renormalizability) $\Rightarrow$ ϕ **adjoint** scalar

Inert field ϕ

Observations and principles constraining construction of ϕ, cntd:

- $F_{\mu\nu} \equiv \pm\tilde{F}_{\mu\nu} \Rightarrow$ any *local* "power" of $F_{\mu\nu}$ with an insertion of t^a **vanishes**
- **only trivial holonomy** in $F_{\mu\nu} \equiv \pm\tilde{F}_{\mu\nu}$ allowed
- $|\phi|$ is spacetime homogeneous $\Rightarrow$ information on ϕ's EOM is encoded in phase $\hat{\phi} \equiv \frac{\phi}{|\phi|}$
- definition of possible phases $\{\hat{\phi}\}$: due to BPS of $A_\mu^\pm$ **no explicit** T **dependence**, **flat measure** for admissible **integration over moduli** (excluding temporal shifts and global gauge rotations), Wilson lines between spatial points along **straight lines**

Inert field ϕ

Unique definition of $\{\hat{\phi}\}$ [Herbst and RH 2004]:

$$\{\hat{\phi}^a\} \equiv \sum_{\pm} \mathrm{tr} \int d^3x \int d\rho \, t^a \, F_{\mu\nu}(\tau, \mathbf{0}) \, \{(\tau, \mathbf{0}), (\tau, \mathbf{x})\}$$

$$\times F_{\mu\nu}(\tau, \mathbf{x}) \, \{(\tau, \mathbf{x}), (\tau, \mathbf{0})\} \ ,$$

where

$$\{(\tau, \mathbf{0}), (\tau, \mathbf{x})\} \equiv \mathcal{P} \exp \left[i \int_{(\tau,\mathbf{0})}^{(\tau,\mathbf{x})} dz_\mu \, A_\mu(z) \right] \ ,$$

$$\{(\tau, \mathbf{x}), (\tau, \mathbf{0})\} \equiv \{(\tau, \mathbf{0}), (\tau, \mathbf{x})\}^\dagger \ ,$$

and sum is over **Harrington-Shepard** (trivial-holonomy) caloron and anticaloron of scale ρ.

Higher n-point functions, higher topol. charge k? **No.**

(Would introduce mass dimension $d = 3 - n - m$ of object, $m > 1$ number of dimension-length caloron moduli at $k > 1$, but d needs to vanish.)

Inert field ϕ

Some observations, conventions:

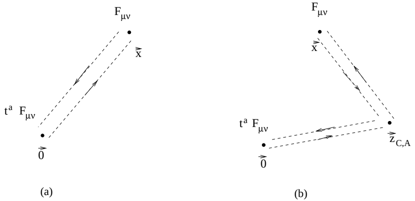
- $\hat{\phi}$ indeed transforms as an adjoint scalar:

$$\hat{\phi}^a(\tau) \to R^{ab}(\tau)\hat{\phi}^b(\tau)\,,$$

where R^{ab} is τ dependent matrix of adjoint rep.

$$R^{ab}(\tau)t^b = \Omega^\dagger(\tau,\mathbf{0})t^a\Omega(\tau,\mathbf{0})\,.$$

- What about shift of spatial center $\mathbf{0} \to \mathbf{z}_\pm$?



(a) graphical representation of **definition**
(b) only possible generalization to $\mathbf{z}_\pm \neq 0$

Shift of center amounts to spatially *global* gauge rotation induced by the group element $\Omega^\pm_z = \{(\tau,\mathbf{0}),(\tau,\mathbf{z}_\pm)\}$.

Inert field ϕ

Some observations, conventions, cntd:

- one has

$$\int_{(\tau,\mathbf{0})}^{(\tau,\mathbf{x})} dz_\mu A_\mu(z)|_\pm = \pm \int_0^1 ds\, x_i A_i(\tau, s\mathbf{x})$$

$$= \pm t_b x_b\, \partial_\tau \int_0^1 ds\, \log \Pi(\tau, sr, \rho) \ \Rightarrow$$

  integrand in the exponent of $\{(\tau, \mathbf{0}), (\tau, \mathbf{x})\}_\pm$ varies along a fixed direction in su(2) (a hedge hog); **Path-ordering can be ignored.**
- temporal shift freedom in $A_\mu^\pm$: set $\tau_\pm = 0$ and re-instate later
- parity: $F_{\mu\nu}(\tau, \mathbf{x})_+ = F_{\mu\nu}(\tau, -\mathbf{x})_-$ and

$$\begin{aligned}\{(\tau, \mathbf{0}), (\tau, \mathbf{x})\}_+ &= \left(\{(\tau, \mathbf{x}), (\tau, \mathbf{0})\}_+\right)^\dagger = \{(\tau, \mathbf{0}), (\tau, -\mathbf{x})\}_- \\ &= \left(\{(\tau, -\mathbf{x}), (\tau, \mathbf{0})\}_-\right)^\dagger \ \Rightarrow\end{aligned}$$

  $-$ contribution to the integrand in **definition** obtained by $\mathbf{x} \to -\mathbf{x}$ in $+$ contribution

Inert field ϕ

Some observations, conventions, cntd:

after tedious computation [Herbst and RH 2004]
$+$ contribution to integrand in **definition** reads:

$$-\,i\,\beta^{-2}\frac{32\pi^4}{3}\frac{x^a}{r}\frac{\pi^2\hat{\rho}^4+\hat{\rho}^2(2+\cos(2\pi\hat{\tau}))}{(2\pi^2\hat{\rho}^2+1-\cos(2\pi\hat{\tau}))^2}\times F[\hat{g},\Pi]\,,$$

where $\hat{\rho}\equiv\frac{\rho}{\beta}$, $\hat{r}\equiv\frac{r}{\beta}$, $\hat{\tau}\equiv\frac{\tau}{\beta}$, and functional F is

$$F[\hat{g},\Pi]=2\cos(2\hat{g})\left(2\frac{[\partial_\tau\Pi][\partial_r\Pi]}{\Pi^2}-\frac{\partial_\tau\partial_r\Pi}{\Pi}\right)$$
$$+\sin(2\hat{g})\left(2\frac{[\partial_r\Pi]^2}{\Pi^2}-2\frac{[\partial_\tau\Pi]^2}{\Pi^2}+\frac{\partial_\tau^2\Pi}{\Pi}-\frac{\partial_r^2\Pi}{\Pi}\right)\,,$$

and

$$\{(\tau,\mathbf{0}),(\tau,\mathbf{x})\}_\pm\equiv\cos\hat{g}\pm 2it_b\frac{x^b}{r}\,\sin\hat{g}\,.$$

One shows that $\hat{g}$ saturates exponentially fast for $\hat{r}>1$.

Inert field ϕ

discussion:

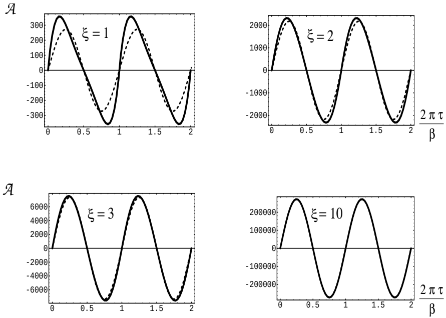
- angular integration would yield **zero** if radial integration was regular
- **but:** radial integration diverges logarithmically due to term $\frac{\partial_r^2 \Pi}{\Pi}$; this term arises from the **magnetic-magnetic** correlation (no convergence in PT due to weakly screened magnetic sector!)
- zero$\times$infinity yields undetermined, multiplicative, and real constants $\Xi_\pm$
- without restriction of generality (global choice of gauge), angular integration regularized by defect azimuthal angle in 1-2 plane of su(2) for both $+$ and $-$ contributions $\Rightarrow$ **Members of $\{\hat{\phi}\}$ all move in hyperplane of su(2)!**
- re-instate $\tau \to \tau + \tau_\pm \Rightarrow$

Inert field ϕ

discussion, cntd:

result:

$$\{\hat{\phi}^a\} = \{\Xi_+(\delta^{a1}\cos\alpha_+ + \delta^{a2}\sin\alpha_+)\,\mathcal{A}\,(2\pi(\hat{\tau}+\hat{\tau}_+)) + \Xi_-(\delta^{a1}\cos\alpha_- + \delta^{a2}\sin\alpha_-)\,\mathcal{A}\,(2\pi(\hat{\tau}+\hat{\tau}_-))\}\,, \quad \text{where}$$



τ dependence of function $\mathcal{A}(\frac{2\pi\tau}{\beta})$;
saturation property (cutoff independence) for $\hat{\rho}$ integration.

ζ dependence of $\Xi_\pm$

$\rho_{max} \equiv \zeta\beta$:

$$\int d\rho \to \int_0^{\zeta\beta} d\rho\,, \qquad (\zeta > 0)\,.$$

- $\Xi_\pm = 272\,\zeta^3 \times$ unknown, fixed real, $\quad(\zeta > 5)$
- integral over ρ is strongly dominated by contributions just below upper limit
- since upper limit set by $|\phi|^{-1}$ (yet to be determined), only (anti)calorons with $\rho \sim |\phi|^{-1}$ **contribute to effective theory**
- since $\zeta_\phi \equiv (|\phi|\beta)^{-1} \geq 8.22$ (later) semiclassical analysis of nontrivial-holonomy calorons in limit $\frac{D}{\beta} = \pi\left(\frac{\rho}{\beta}\right)^2 \geq (8.22)^2 \times \pi \gg 1$ [Diakonov et al. 2004] **is justified**.

Kernel of a differential operator D and potential for ϕ

- set $\{\hat{\phi}\}$ contains two real parameters for each "polarization": $\Xi_\pm$ and $\tau_\pm$; $\{\hat{\phi}\}$ is annihilated by **linear**, **second-order** differential operator $D = \partial_\tau^2 + \left(\frac{2\pi}{\beta}\right)^2 \Rightarrow$ $\{\hat{\phi}\}$ coincides with **kernel** of D and determines D uniquely
- linearity $\Rightarrow$ also $D\phi = 0$
- **but:** D depends on β explicitly, not allowed (BPS, caloron action given by topolog. charge)
- therefore seek potential $V(|\phi|^2)$ such that (Euclidean) action principle applied to

$$\mathcal{L}_\phi = \mathrm{tr}\left((\partial_\tau \phi)^2 + V(\phi^2)\right) \,.$$

  yields solutions annihilated by D, where $\mathcal{L}_\phi$ does not depend on β explicitly; demand that energy density $\Theta_{44} = 0$ on those solutions

Potential $V(\phi^2)$ and modulus of ϕ

- pick motion in 1-2 plane of su(2) (gauge invariance $\Rightarrow$ V **central** potential $\Rightarrow$ cons. angular momentum); ansatz:

$$\phi = 2\,|\phi|\,t_1\,\exp(\pm\frac{4\pi i}{\beta}t_3\tau)\,.$$

(circular motion in 1-2 plane, $|\phi|$ time independent!)

- apply E-L to $\mathcal{L}_\phi \Rightarrow$

$$\begin{aligned}\partial_\tau^2\phi^a &= \frac{\partial V(|\phi|^2)}{\partial|\phi|^2}\,\phi^a \text{ (in components) } \Leftrightarrow \\ \partial_\tau^2\phi &= \frac{\partial V(\phi^2)}{\partial\phi^2}\,\phi \text{ (in matrix form)}\,.\end{aligned}$$

- $\Theta_{44} = 0$ on ansatz $\phi \Rightarrow |\phi|^2\left(\frac{2\pi}{\beta}\right)^2 - V(|\phi|^2) = 0$ but also: $\partial_\tau^2\phi + \left(\frac{2\pi}{\beta}\right)^2\phi = 0 \Rightarrow$

$$\frac{\partial V(|\phi|^2)}{\partial|\phi|^2} = -\frac{V(|\phi|^2)}{|\phi|^2}\,.$$

Potential $V(\phi^2)$ and modulus of ϕ, cntd

- $\Rightarrow V(|\phi|^2) = \frac{\Lambda^6}{|\phi|^2}$
  where Λ integration constant of mass dim. unity.

- $\Rightarrow |\phi| = \sqrt{\frac{\Lambda^3\beta}{2\pi}}$ (power-like decay of field ϕ with increasing T)

The field ϕ describes coarse-grained effect of **noninteracting** *trivial-holonomy calorons and anticalorons. It does not propagate, and its modulus $|\phi|$ sets the scale of maximal off-shellness of intermediates in effective theory.*

- Indeed: cutting off ρ and r integrations at $|\phi|^{-1}$, τ dependence of $\mathcal{A}(\frac{2\pi\tau}{\beta})$ is perfect sine
  (Error at level smaller than 10^{-22} if knowledge about $T_c = \frac{\lambda_c\Lambda}{2\pi}$ with $\lambda_c = 13.87$ is used, later.)

BPS equation for ϕ

In addition to E-L equation ϕ satisfies **first-order**, BPS equation:

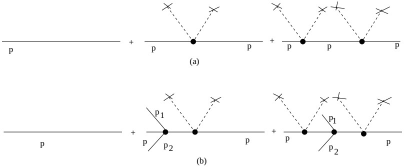
$$\partial_\tau \phi = \pm 2i \, \Lambda^3 \, t_3 \, \phi^{-1} = \pm i \, V^{1/2}(\phi) \, .$$

Because ϕ satisfies both, second-order E-L and first-order BPS equation, usual shift ambiguity in ground-state energy density, as allowed by E-L equation, **absent** *in SU(2) Yang-Mills thermodynamics.*

Effective action for deconfining phase

Coupling the coarse-grained $k = 0$ sector to ϕ, following constraints:

- perturbative renormalizability
  ['t Hooft, Veltman, Lee, and Zinn-Justin 1971-1973]
  $\Rightarrow$ form invariance of action for effective $k = 0$ gauge field a_μ from integrating fundamental $k = 0$ fields only, no higher dim. ops. constr. from a_μ only
- no energy-momentum transfer to $\phi \Rightarrow$ absence of higher dim. ops. involving a_μ **and** ϕ
- gauge invariance $\Rightarrow \partial_\mu \phi \to D_\mu \phi \equiv \partial_\mu \phi - ie[a_\mu, \phi]$ (e **effective** coupling); no momentum transfer to ϕ (unitary gauge $\phi = 2|\phi|\, t_3$), massive 1,2 modes propagate on-shell only



Effective action and ground-state estimate

unique effective action density:

$$\mathcal{L}_{\text{eff}}[a_\mu] = \text{tr}\left(\frac{1}{2}\,G_{\mu\nu}G_{\mu\nu} + (D_\mu\phi)^2 + \frac{\Lambda^6}{\phi^2}\right)\,,$$

$$\text{where } G_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - ie[a_\mu, a_\nu] \equiv G^a_{\mu\nu}\,t_a$$

ground-state estimate:

- E-L EOM from $\mathcal{L}_{\text{eff}}[a_\mu]$

$$D_\mu G_{\mu\nu} = ie[\phi, D_\nu\phi]\,.$$

- solved by zero-curvature (pure-gauge) config. a^{gs}_μ:

$$a^{\text{gs}}_\mu \;=\; \mp\delta_{\mu 4}\frac{2\pi}{e\beta}\,t_3 \quad (D_\nu\phi \equiv G_{\mu\nu} \equiv 0) \quad \Rightarrow$$

$$\rho^{\text{gs}} \;=\; -P^{\text{gs}} = 4\pi\Lambda^3\,T\,.$$

Unresolvable interaction between $k=0$ to $|k|=1$ sector lifts ρ^{gs} from zero (BPS). EOS of a cosmological constant; pressure **negative***. (Short-lived, attracting magnetic (anti)monopoles by temporary shifts of (anti)caloron holonomies from trivial to small through absorption/emission of unresolved plane-wave fluctuations.)*

Winding to unitary gauge: $\mathbf{Z}_2$ degeneracy

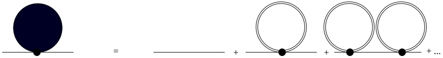
- consider gauge rotation $\tilde{\Omega}(\tau) = \Omega_{\text{gl}}\, Z(\tau)\, \Omega(\tau)$ where $\Omega(\tau) \equiv \exp[\pm 2\pi i \frac{\tau}{\beta} t_3]$, $Z(\tau) = \left(2\Theta(\tau - \frac{\beta}{2}) - 1\right) \mathbf{1}_2$, and $\Omega_{\text{gl}} = \exp[i \frac{\pi}{2} t_2]$
- $\tilde{\Omega}(\tau)$ transforms a_μ^{gs} to $a_\mu^{\text{gs}} \equiv 0$ and ϕ to $\phi = 2t^3|\phi|$
- $\tilde{\Omega}(\tau)$ is **admissible** because respects periodicity of δa_μ:

$$
\begin{aligned}
a_\mu &\to \tilde{\Omega}(a_\mu^{\text{gs}} + \delta a_\mu)\tilde{\Omega}^\dagger + \frac{i}{e}\tilde{\Omega}\partial_\mu\tilde{\Omega}^\dagger \\
&= \Omega_{\text{gl}} \left(\Omega(a_\mu^{\text{gs}} + \delta a_\mu)\Omega^\dagger + \frac{i}{e}\left(\Omega\partial_\mu\Omega^\dagger + Z\partial_\mu Z\right)\right)\Omega_{\text{gl}}^\dagger \\
&= \Omega_{\text{gl}} \left(\Omega\delta a_\mu\Omega^\dagger + \frac{2i}{e}\delta(\tau - \frac{\beta}{2})Z\right)\Omega_{\text{gl}}^\dagger = \Omega_{\text{gl}}\Omega\, \delta a_\mu\, (\Omega_{\text{gl}}\Omega)^\dagger\,.
\end{aligned}
$$

- $\tilde{\Omega}(\tau)$ transforms Polyakov loop from $-\mathbf{1}_2$ to $\mathbf{1}_2 \Rightarrow$ ground-state estimate is (electric) $\mathbf{Z}_2$ degenerate $\Rightarrow$ **deconfining phase**

Mass spectrum; outlook resummed radiative corrections

- computation in physical and completely fixed **unitary, Coulomb gauge** ($\phi = 2t^3|\phi|$, $\partial_i a_i^3 = 0$)
- mass spectrum: $m^2 \equiv m_1^2 = m_2^2 = 4e^2 \frac{\Lambda^3}{2\pi T}$, $m_3 = 0$.
- resummation of **polarization tensor of massless mode** as



$\Rightarrow$ small linear-in-T correction to tree-level ground-state estimate [Falquez, RH, Baumbach 2010]

$$\text{tree-level:} \qquad \frac{\rho^{\text{gs}}}{T^4} = 3117.09\,\lambda^{-3}\,,$$

$$\text{one-loop resummed:} \qquad \frac{\Delta\rho^{\text{gs}}}{T^4} = 3.95\,\lambda^{-3}\,.$$

- large hierarchy between loop orders (conjecture about **termination at finite irreducible order** [RH 2006]), one-loop plus two-loop correction **practically exact**

T dependence of e: selfconsistent thermal quasiparticles

P and ρ at one loop:

$$P(\lambda) = -\Lambda^4 \left\{ \frac{2\lambda^4}{(2\pi)^6} \left[2\bar{P}(0) + 6\bar{P}(2a) \right] + 2\lambda \right\} ,$$

$$\rho(\lambda) = \Lambda^4 \left\{ \frac{2\lambda^4}{(2\pi)^6} \left[2\bar{\rho}(0) + 6\bar{\rho}(2a) \right] + 2\lambda \right\} ,$$

where

$$\bar{P}(y) \equiv \int_0^\infty dx\, x^2 \log\left[1 - \exp(-\sqrt{x^2 + y^2}) \right] ,$$

$$\bar{\rho}(y) \equiv \int_0^\infty dx\, x^2 \frac{\sqrt{x^2 + y^2}}{\exp(\sqrt{x^2 + y^2}) - 1} ,$$

and $a \equiv \frac{m}{2T} = 2\pi e \lambda^{-3/2}$. For later use introduce function $D(2a)$ as

$$\partial_{y^2} \bar{P}\Big|_{y=2a} = -\frac{1}{4\pi^2} \int_0^\infty dx\, \frac{x^2}{\sqrt{x^2 + (2a)^2}} \frac{1}{e^{\sqrt{x^2+(2a)^2}} - 1} \equiv -\frac{1}{4\pi^2} D(2a) .$$

Legendre transformation and evolution equation

- for $m(T)$ to respect Legendre trafo (fundamental partition function) between P and $\rho \Leftrightarrow \partial_m P = 0$
- $\Rightarrow$ first-order **evolution equation**

$$\partial_a \lambda = -\frac{24\lambda^4 a}{(2\pi)^6} \frac{D(2a)}{1 + \frac{24\lambda^3 a^2}{(2\pi)^6} D(2a)} .$$

or

$$1 = -\frac{24\lambda^3}{(2\pi)^6} \left(\lambda \frac{da}{d\lambda} + a \right) a \, D(2a) .$$

- $\Rightarrow a(\lambda) \propto \lambda^{-\frac{3}{2}}$ for $\lambda \to \infty$
  $\Rightarrow$ for $\lambda \gg 1$ a must fall below unity
- **fixed points of evolution equation**:

  repulsive at $a = 0$ ($\lambda \to \infty$)

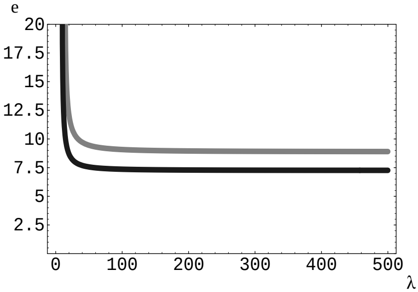
  attractive at $a = \infty$ ($\lambda = \lambda_c$)

Solution to evolution equation

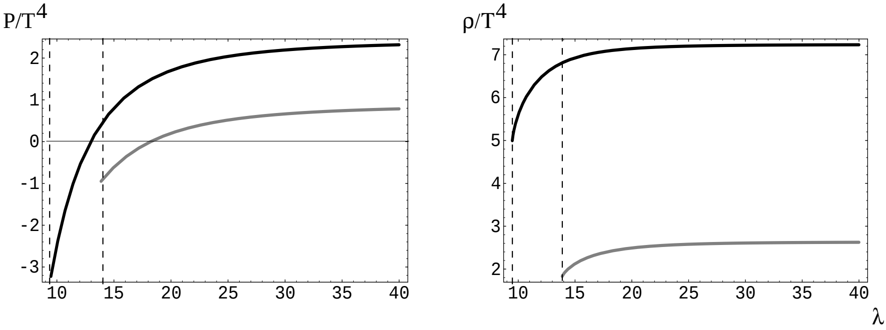
- $a \ll 1$ [Dolan, Jackiw 1974] $\Rightarrow 1 = -\frac{\lambda^3}{(2\pi)^4}\left(\lambda\frac{da}{d\lambda} + a\right)a$; solution ($a(\lambda_i) = a_i \ll 1$):

$$a(\lambda) = 4\sqrt{2}\pi^2\lambda^{-3/2}\left(1 - \frac{\lambda}{\lambda_i}\left[1 - \frac{a_i^2\lambda_i^3}{32\pi^4}\right]\right)^{1/2}\,.$$

  $\Rightarrow$ attractor $a(\lambda) = 4\sqrt{2}\pi^2\lambda^{-3/2}$ as long as $a \ll 1$
  $\Rightarrow e = \sqrt{8}\pi$ as long as $a \ll 1$ ($S = \frac{8\pi^2}{e^2} = 1 \Rightarrow$ interpretation of $\hbar$ in terms of caloron winding number, later)
- full solution for $e(\lambda) \Rightarrow \lambda_c = 13.87$:



T dependence of P and ρ



- notice **negativity** of P shortly above λ_c
- relative correction to one-loop quasiparticle P and ρ by radiative effects: $< 1\%$

Counting powers of $\hbar$

- re-instating $\hbar$ but keeping $c = k_B = 1$
  $\Rightarrow$ (dimensionless) exponential (fluctuating fields only) in effective partition function

$$-\frac{\int_0^\beta d\tau d^3x\, \mathcal{L}'_{\text{eff}}[a_\mu]}{\hbar},$$

  is re-cast as

$$-\int_0^\beta d\tau d^3x \,\text{tr}\left(\frac{1}{2}(\partial_\mu \tilde{a}_\nu - \partial_\nu \tilde{a}_\mu - ie\sqrt{\hbar}[\tilde{a}_\mu, \tilde{a}_\nu])^2 - e^2\hbar[\tilde{a}_\mu, \tilde{\phi}]^2\right),$$

  $\tilde{a}_\mu \equiv a_\mu/\sqrt{\hbar}$, $\tilde{\phi} \equiv \phi/\sqrt{\hbar}$ assumed **not to depend** on $\hbar$ (see for example [Brodsky and Hoyer 2011; Iliopoulos, Itzykson, and Martin 1975, Holstein and Donoghue 2004])
- This re-formulation of (effective) action implies that loop expansion is expansion in ascending powers of $\hbar$.
- $[\tilde{a}_\mu]$ is length^{-1} $\Rightarrow$ $[e] = [1/\sqrt{\hbar}]$

Action of just-not-resolved (anti)caloron

- Thus $e = \frac{\sqrt{8}\pi}{\sqrt{\hbar}}$ **almost everywhere**.
- Since only (anti)calorons of $\rho \sim |\phi|^{-1}$ contribute to ϕ in effective theory $\Rightarrow$ **effective coupling** e admissible in calculation of **fundamental** (anti)caloron action:

$$S_{C/A} = \frac{8\pi^2}{e^2} = \hbar \quad \text{(almost everywhere)}\,.$$

Implications: Planck's quantum=caloron action

1) universality, constancy (quantization) of $\hbar$: no dependence on YM scale Λ, associated with *one unit of topological charge*

2) pointlike vertices between effective plane waves induced by just-not-resolved, *Euclidean* nonpropagating field configuration

$\Rightarrow$ irreconcilability of Euclidean and Minkowskian signatures as source of indeterminism in scattering event

3) because effective vertices are dominated by (anti)calorons with $\rho \sim |\phi|^{-1}$

$\Rightarrow$ **no interaction between (fundamental) plane waves** if potential momentum transfer $\gg |\phi|$

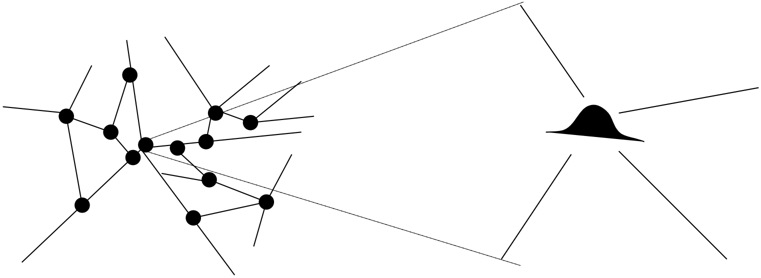
$\Rightarrow$ **absence** of plane-wave offshellness $\gg |\phi|$

$\Rightarrow$ adds **justification to renormalization programme of PT**

hypothetically resolving an effective vertex:

resolution fixed (here by T through $\phi(T)$):

$\Rightarrow$ **no plane-wave interactions beyond that resolution (UV finiteness)**

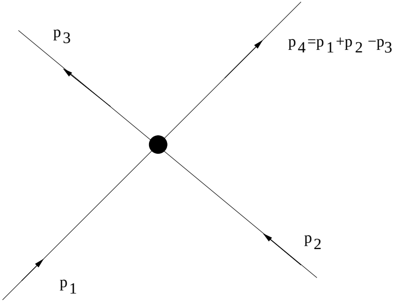


(a) (b)

radiative corrections in eff. th. *caloron mediation of vertex*

(zero-mode induced fermionic vertex on (anti)instanton: ['t Hooft 1976])

Constraints of momentum transfers in effective 4-vertex

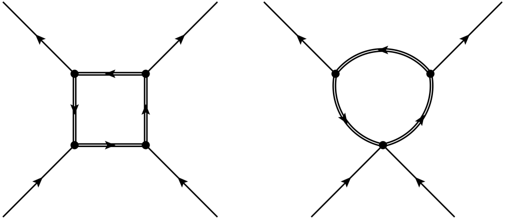


sum over nontrivial s-, t-, and u-channel contributions in physical unitary-Coulomb gauge constrained as

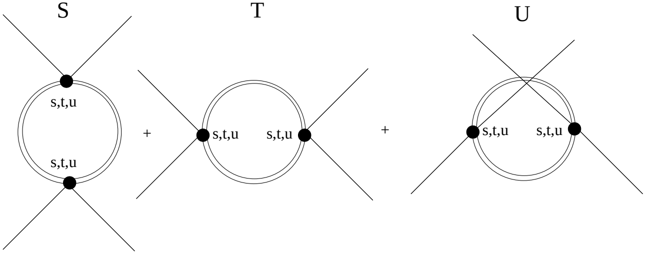
- s-channel: $|(p_1 + p_2)^2| \leq |\phi|^2$
- t-channel: $|(p_3 - p_1)^2| \leq |\phi|^2$
- u-channel: $|(p_3 - p_2)^2| \leq |\phi|^2$

Negligible photon-photon scattering

- diagrams excluded by overall on-shellness:



- coherent channel superposition in remaining diagram:



Details on photon-photon scattering

- investigate 27 combinations of s,t,u in 3 overall channels S,T,U with 4 energy-sign combinations each subject to:
  - on-shellness constraint on massive modes and
  - 4-vertex constraints
- distinguish cases for signs of loop energy $\tilde{u}_0$ and $\tilde{v}_0$:

$\tilde{u}_0 > 0;\ \tilde{v}_0 > 0$	$\tilde{u}_0 > 0;\ \tilde{v}_0 < 0$
$\tilde{u}_0 < 0;\ \tilde{v}_0 > 0$	$\tilde{u}_0 < 0;\ \tilde{v}_0 < 0$

.

- example of overall S:

Vertex 2Vertex 1	s-ch.	t-ch.	u-ch.
s- ch.	| X X |	X | X X | X	X | X X | X
t-ch.	X | X X | X	X | X | X	X | X | X
u-ch.	X | X X | X	X | X | X	X | X | X

Exclusion of Sss

- from on-shellness and momentum conservation:

$$(1 - \cos(\measuredangle \mathbf{ab})) \geq \frac{2\tilde{m}^2}{\tilde{a}_0 \tilde{b}_0} \,,$$

- from momentum transfer constraints:

$$(1 - \cos(\measuredangle \mathbf{ab})) \leq \frac{1}{2\tilde{a}_0 \tilde{b}_0} \,,$$

where $\tilde{m} \equiv \frac{m}{|\phi|} = 2e \geq 2\sqrt{8}\pi$.

$\Rightarrow$ upper bound smaller than lower bound
$\Rightarrow$ **no Sss contribution!**
$\Rightarrow$ Stt$+-$, Stu$+-$, Sut$+-$, Suu$+-$ remain.
(4 out of 36 combinations)

What about T and U?

- for both channels:

<table>
<tr><th>Vertex 2Vertex 1</th><th>s-ch.</th><th>t-ch.</th><th>u-ch.</th></tr>
<tr><td>s- ch.</td><td><table><tr><td>X</td><td>X</td></tr><tr><td>X</td><td>X</td></tr></table></td><td><table><tr><td>X</td><td>X</td></tr><tr><td>X</td><td>X</td></tr></table></td><td><table><tr><td>X</td><td>X</td></tr><tr><td>X</td><td></td></tr></table></td></tr>
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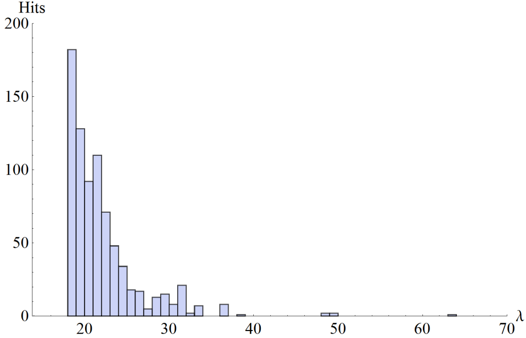
<tr><td>u-ch.</td><td><table><tr><td></td><td>X</td></tr><tr><td>X</td><td>X</td></tr></table></td><td><table><tr><td>X</td><td>X</td></tr><tr><td>X</td><td>X</td></tr></table></td><td><table><tr><td>X</td><td>X</td></tr><tr><td>X</td><td>X</td></tr></table></td></tr>
</table>

- **again:** 4 out of 36 combinations remain in each case.

MC sampling of nonexcluded cases: Total hits

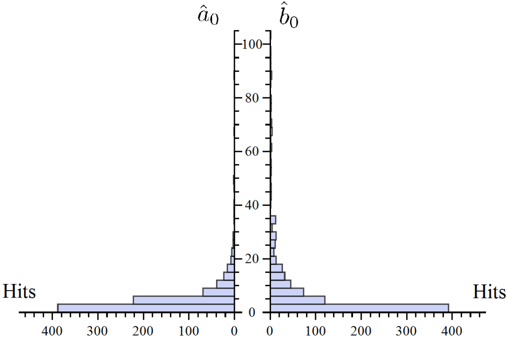
- 2×10^{11} test shots into region $\hat{a}_0 = \frac{a_0}{T}$, $\hat{b}_0 = \frac{b_0}{T} \leq 100$, $\lambda_c = 13.867 \leq \lambda \leq 100$ (noncompact arguments) and nonconstrained ang. domain.
- histogram of hits:



- in Sss analysis of Bose suppression yields factor $\leq 10^{-7}$ for $\lambda_c = 13.867 \leq \lambda \leq 30$.

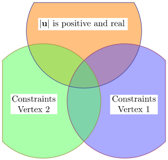
MC sampling of nonexcluded cases: Distribution of photon energies

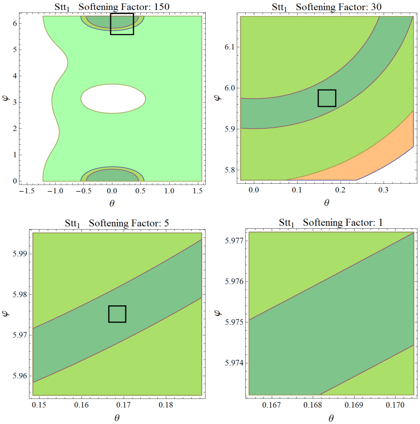
- for 2×10^{11} test shots we obtain ($\hat{a}_0 \equiv \frac{a_0}{T}$, $\hat{b}_0 \equiv \frac{b_0}{T}$)



- Hard photons do not scatter at all.
- Very feeble participation of soft photons.

Filamented algebraic varieties





Real-world implications

- **postulate that photon propagation described by SU(2) rather than U(1) gauge principles:** [RH 2005; Giacosa and RH 2005] $\Rightarrow$ blackbody anomaly (transverse polarisations), magnetic charge-density waves (longitudinal polarisations) [Schwarz, RH, and Giacosa 2006; Ludescher and RH 2008; Falquez, RH, and Baumbach 2010, 2011]
- in units $c = \epsilon_0 = \mu_0 = k_B = 1$ QED fine-structure constant α is

$$\alpha = \frac{Q^2}{4\pi\hbar}$$

$\Rightarrow$ to be **unitless**: $Q \propto 1/e$.

Is realized if Q taken $\propto$ electric-magnetically dual of e:

$$Q' = \frac{4\pi}{e} \propto \sqrt{\hbar}\,, \quad Q' = NQ \ \ (\text{mixing of SU(2)'s})\,.$$

Dual interpretation of charge and flux

⇒ **magnetic monopoles** of SU(2) are **electric monopoles** in real world [RH 2005]

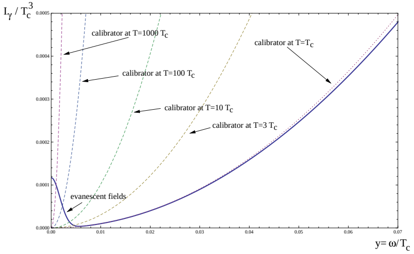
⇒ **magnetic-monopole condensate** of SU(2) is **condensate of electric monopoles** in real world (no dual Meissner effect) [Giacosa and RH 2005]

⇒ **electric charge density waves** in SU(2) are longitudinally propagating **magnetic field modes** in real world [Falquez, RH, and Baumbach 2011]

⇒ **magnetic Z_2 charge** of an SU(2) center-vortex selfintersection is **electric charge** in real world [Moosmann and RH 2008]

What is T_c?

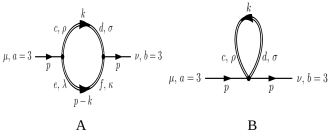
- ARCADE 2 confirms excess of line temperature for CMB frequencies $\nu \leq 3.4\,\text{GHz}$ (6σ at $\nu = 3\,\text{GHz}$) in agreement with earlier radio-frequency observations [Fixsen et al 2009]
- interpreted as onset of evanescence for $\nu \leq m_\gamma \sim 100\,\text{MHz}$ due to (partial) Meissner effect (deconfining-preconfining transition) [RH 2009]



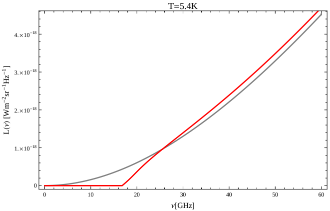
$\Rightarrow\ T_c = 2.725\,\text{K} \equiv\ T_0$, fixing $\Lambda \sim 10^{-4}\,\text{eV}$

Blackbody anomaly

- computation of (only right diagram allowed) [Schwarz,RH,Giacosa 2007; Ludescher,RH 2008; Falquez,RH,Baumbach 2010]



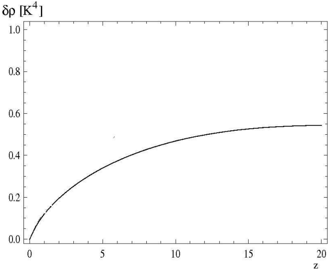
to obtain dispersion law for transverse polarisations yields



- dependence of spectral-gap frequency ν^* on T: $\frac{\nu^*}{\mathrm{GHz}} = 42.70\left(\frac{T}{K}\right)^{-0.53} + 0.21$ for $T \geq 4.3\,\mathrm{K}$

Blackbody anomaly

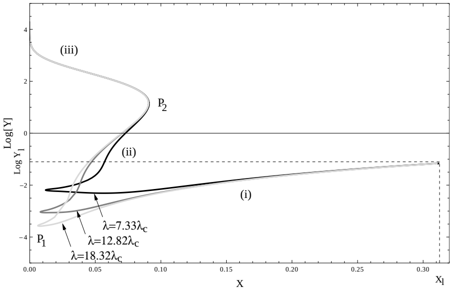
- leads to difference $\delta\rho$ between thermal energy density of SU(2) and conventional U(1) photons



- basis for approach to explanation of CMB large-angle anomalies through bias factor
  $F(\bar{T}, \delta T) = N \exp\left(-\frac{V\delta\rho}{\bar{T}}\right)$ for δT
  [RH, Nature Physics 2013]

Magnetic charge-density waves

- dispersion law for longitudinal polarisation yields

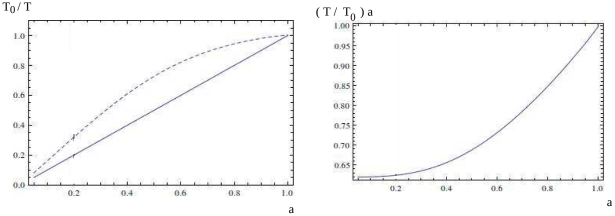


(3 branches of longitudinally propagating magnetic modes)

- assuming energy-free conversion of energy density of these incoherent magnetic modes into magnetic field B of cosmological coherence length:
  $B \sim 10^{-8}$ G$\sim$ upper bound from small-angle TT correlation
  [see Widrow 2002; Falquez, RH, Baumbach 2012]

CMB temperature vs. cosmological scale factor

- existence of Yang-Mills scale Λ changes $T \propto a^{-1}$ scaling of CMB temperature at late times
  [RH 2014]



- $\Rightarrow T = 0.62\, a^{-1} \times T_0\,, \quad (a \leq \frac{1}{10})$
- repeal of difference between CMB extracted redshift for instantaneous reionisation, $z_{\rm reion} = 10.8 \pm^{3.1}_{2.5}$, and that deduced from high-redshift quasar-spectra (Gunn-Peterson trough), $z_{\rm reion} \sim 6$.

Redshift of CMB decoupling (recombination)

- due to scaling violation at low z one has
  $z_{\rm dec} = \frac{1}{0.62}\,\frac{3000}{2.725} - 1 = 1775$
  in contrast to $z_{\rm dec} = 1089$ in conventional U(1) theory
- to keep successful BAO physics, matter density (DM+baryonic) needs to be the same at $z_{\rm dec} = 1775$ as conventionally was at $z_{\rm dec} = 1089$; leads to a re-scaling of conventional matter density by
  $\left(\frac{1089}{1775}\right)^3 = 0.231$
  (today's 25% matter contribution to $\rho_{\rm crit}$ conventionally is re-scaled to 5.8% !)
- But what about small-redshift DM contribution, rotation curves of galaxies?
  Possibly: slow-roll of Planck-scale axion with topologically stabilised U(1) solitons centered at spiral galaxies
  [Giacosa and RH 2005]

Cosmic neutrinos

- redshift dependence of N_{eff}
- $\frac{T_\nu}{T} = \left(\frac{16}{23}\right)^{1/3}$ instead of $\frac{T_\nu}{T} = \left(\frac{4}{11}\right)^{1/3}$
- too low value of N_{eff} today $\Rightarrow$
- way out:
  coupling of CMB and neutrino fluids inducing $T_\nu = T$ and $m_\nu = \xi T$
  (neutrino: single center-vortex loop in confining phase of an SU(2) YM theory)
  [Moosmann and RH 2008]

More material

- **Low-order radiative corrections:**
  RH 2006; Schwarz, RH, Giacosa 2007; Ludescher, RH 2008; Falquez, RH, Baumbach 2010, 2011
- **Loop expansions:**
  RH 2006
- **Stable but unresolved monopoles:**
  Keller et al. 2008
- **The two other phases:**
  RH 2005, 2007, 2011; Moosmann, RH 2008

Summary

- mini review on (thermal) Yang-Mills action
- mini review on calorons: trivial vs. nontrivial holonomy for $|k| = 1$ plus semiclassical approx.
- construction of thermal ground-state estimate: inert field ϕ; BPS and E-L; potential
- discussion of constraints on effective action: pert. renormalizability plus inertness of $\phi \Rightarrow$ unique answer
- full ground-state estimate, deconfining nature, tree-level quasiparticles
- evolution of effective coupling
- T dependence pressure and energy density
- interpretation of $\hbar$ in terms of caloron action
- photon-photon scattering

Summary, cntd.

Some physics implications:

postulate: SU(2) (10^{-4} eV) describes photon **propagation**

$\Rightarrow$ blackbody spectral anomaly at $T \sim 5 - 20\,\mathrm{K}$
$\Rightarrow$ low frequency magnetic charge-density waves
$\Rightarrow$ CMB large-angle anomalies
$\Rightarrow$ instantaneous, early reionisation
$\Rightarrow$ cosmic neutrinos
$\Rightarrow$ cold matter and Planck-scale axion

Thank you.