Model of QCD Renormalization applied to (in the process) pp scattering

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GAUGE INVARIANCE BY GAUGE INDEPENDENCE

L_{QCD} → Functional derivatives on Generative Functional, Z_{QCD}, to pull down quarks and gluons →

$$Z_{QCD}[j, \bar{\eta}, \eta] \\ = N \int d[\chi] e^{\frac{i}{4}\int \chi^2} e^{D_A^{(0)}} \cdot e^{\frac{i}{2}\int \chi \cdot F + \frac{i}{2}\int A\cdot(-\partial^2)\cdot A} e^{i\int \bar{\eta}\cdot G_c[A]\cdot\eta + L[A]}|_{A=\int D_c^{(0)}\cdot j}$$

$$2n - point\ functions$$

$$= N \int d[\chi] e^{\frac{i}{4}\int \chi^2} e^{D_A^{(0)}} e^{\frac{i}{2}\int \chi\cdot F + \frac{i}{2}\int A\cdot(D_c^{(0)})^{-1}\cdot A} G_c(1|gA) G_c(2|gA) e^{L[A]}|_{A=0}$$

$$e^{D_a} F_1[A] = exp[\frac{i}{2}\int \bar{Q}\cdot D_c^{(0)}\cdot(1 - \bar{K}\cdot D_c^{(0)})^{-1}\cdot\bar{Q} - \frac{1}{2} Tr\ ln(1 - D_c\cdot\tilde{K})] \\ \cdot\ exp[\frac{1}{2}\int A\cdot\bar{K}\cdot(1 - D_c^{(0)}\cdot\bar{K})^{-1}\cdot A + i\int \bar{Q}\cdot(1 - \bar{K}\cdot D_c^{(0)})^{-1}\cdot A]$$

$$D_c^{(0)}\cdot(1 - \bar{K}\cdot D_c^{(0)})^{-1}$$

$$= D_c^{(0)}\cdot[1 - (\hat{K} + (D_c^{(0)})^{-1})\cdot D_c^{(0)}]^{-1}$$

$$= -(\tilde{K}_{\mu\nu}^{ab} + g f^{abc}\chi_{\mu\nu}^{c})^{-1} = -\hat{\mathbf{K}}^{-1}$$

$$e^{D_A} F_1[A] F_2[A] = exp[-\frac{i}{2}\int \bar{Q}\cdot\hat{\mathbf{K}}^{-1}\cdot\bar{Q} + \frac{1}{2}Tr\ ln\hat{\mathbf{K}} + \frac{1}{2}Tr\ ln(-D_c^{(0)})]$$
$$\cdot\ exp[\frac{i}{2}\int \frac{\delta}{\delta A'}\cdot\mathbf{D_c^{(0)}}\cdot\frac{\delta}{\delta\mathbf{A'}}]$$
$$\cdot\ exp[\frac{i}{2}\int \frac{\delta}{\delta A'}\cdot\hat{\mathbf{K}}^{-1}\cdot\frac{\delta}{\delta A'} - \int\bar{Q}\cdot\hat{\mathbf{K}}^{-1}\cdot\frac{\delta}{\delta A'}]\cdot(e^{D_A}F_2[A'])$$

$$e^{D_A} F_1[A] F_2[A] = N exp[-\frac{i}{2}\int \bar{Q}\cdot\hat{\mathbf{K}}^{-1}\cdot\bar{Q} + \frac{1}{2}Tr\ ln\ \hat{\mathbf{K}}]$$
$$\cdot\ exp[\frac{i}{2}\int \frac{\delta}{\delta A}\cdot\hat{\mathbf{K}}^{-1}\cdot\frac{\delta}{\delta A} - \int\bar{Q}\cdot\hat{\mathbf{K}}^{-1}\cdot\frac{\delta}{\delta A}]\cdot exp(L[A])$$

No gauge dependence!

(H.M.Fried, Modern Physics Letters A, 2013)

Used Halpern + Fradkin to make Generating Functional, GF

Halpern →

$$e^{-\frac{i}{4}\int \mathbf{F^2}} = N'\int d[\chi]\ e^{\frac{i}{4}\int\ (\chi^a_{\mu\nu})^2 + \frac{i}{2}\int\chi^{\mu\nu}_a\mathbf{F^a_{\mu\nu}}}$$

(Halpern, Phys. Rev. D **16**, *1977)*

Fradkin → Rewrite $G_c[A]$ and $L[A]$ into Gaussians

(Fradkin Nucl.Phys.76, 588. 1966)

No worse than Gaussian!

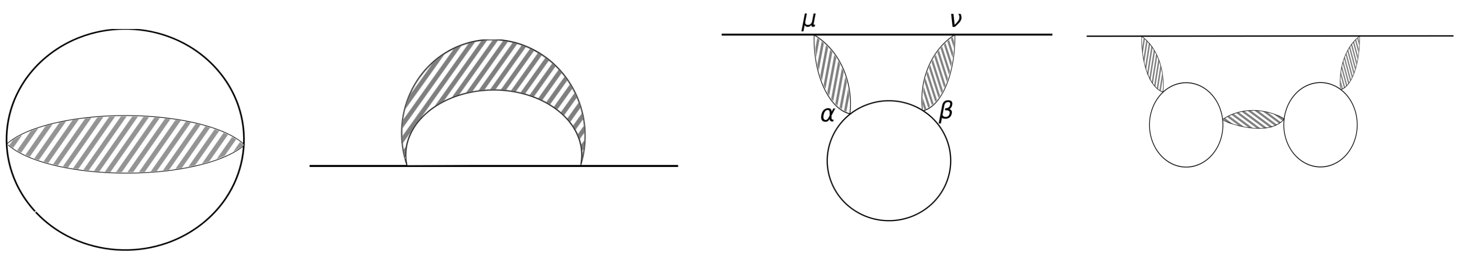
Show Gluon Bundle → Sum Gluon Exchanges Locally → **Effective Locality**

$$< x|GB|y >= \frac{1}{g} \cdot (f \cdot \chi)^{-1} \cdot \delta^4(x-y)$$

→ Halpern integral becomes ordinary integration over small δ^4 volume.

(Fried, Gabellini, Grandou, Sheu, Eur. Phys. Journal C **65** *(2010))*

Self Energy Graphs



A loop is log divergent, giving ℓ.

Each end of the Gluon Bundle connecting to Quark line gives a factor of δ

So here we have $\delta^2 \ell$ = finite? Yes! Effective Locality → can choose any δ.

Self Energies $= 0$

Use Eikonal for now

Two-body Eikonal Scattering Amplitude

$$T(s,t) = \frac{is}{2m^2}\int d^2b\; e^{iq\cdot b}[1 - e^{i\mathbf{X(s,b)}}]$$

(s and t - standard **Mandelstam** variables, $s = -(p_1+p_2)^2, t = -(p_1 - p_1')^2 = \mathbf{q^2}$,

Eikonal function $\mathbf{X(s,b)}$)

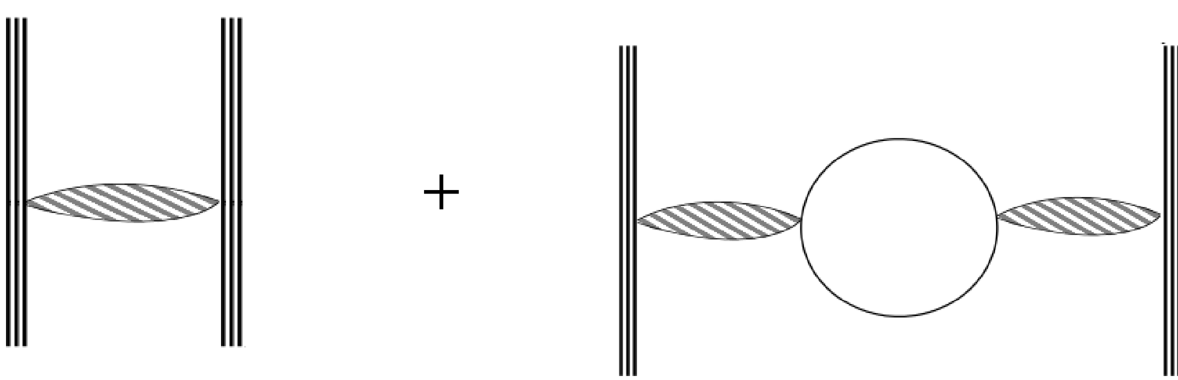
(Fried, Basics of Functional Methods and Eikonal Models, Editions Frontieres, 1990)

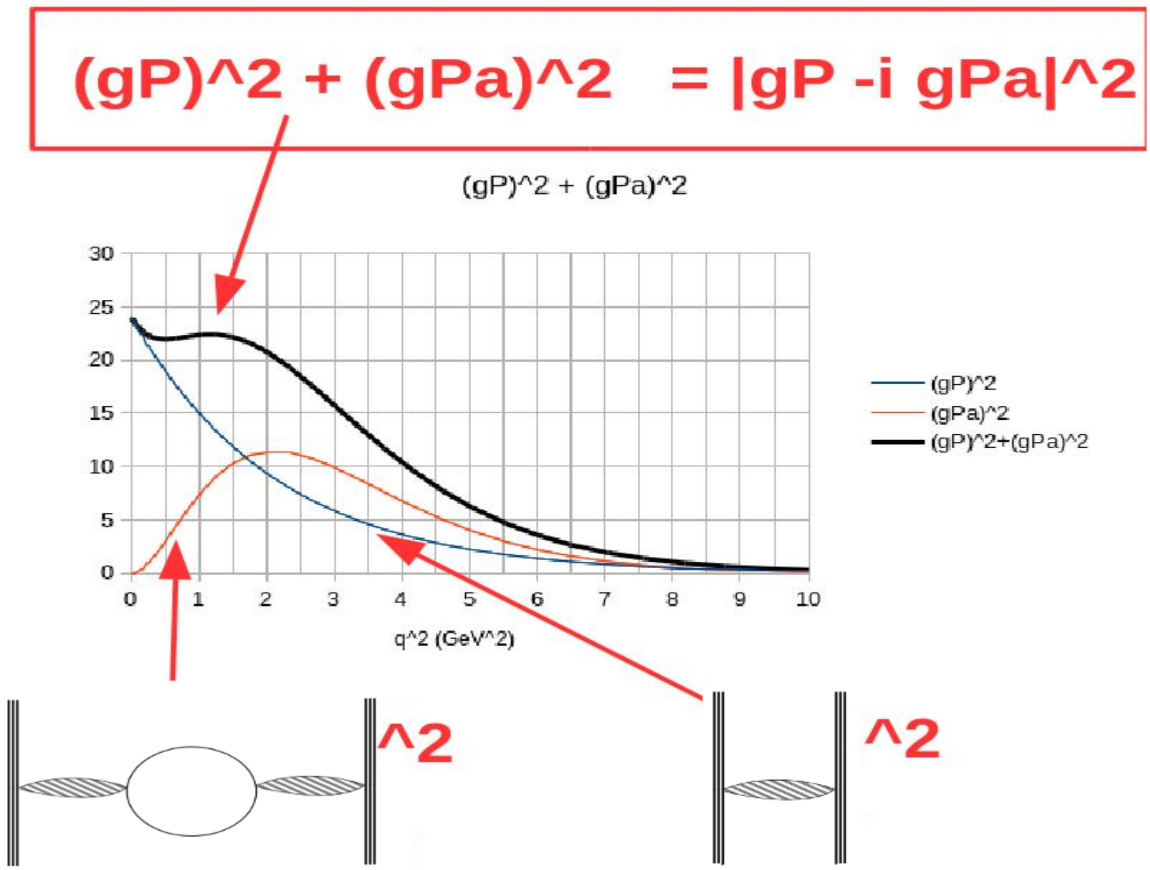
$$e^{iX(s,b)} = N' \int_0^\infty dR\; R^3 e^{i\frac{R^2}{4} + i\frac{<g><\varphi(b)>}{R}}$$

$R^2 = \sum_a (\chi^a)^2$ approximation, treating χ^a as vector in color space. But can be done in a more precise fashion...

$\rightarrow \frac{d\sigma}{dt} \approx$ constant $|g\tilde{\varphi}|^2$ + constant $|g \cdot a \cdot \tilde{\varphi}|^2$

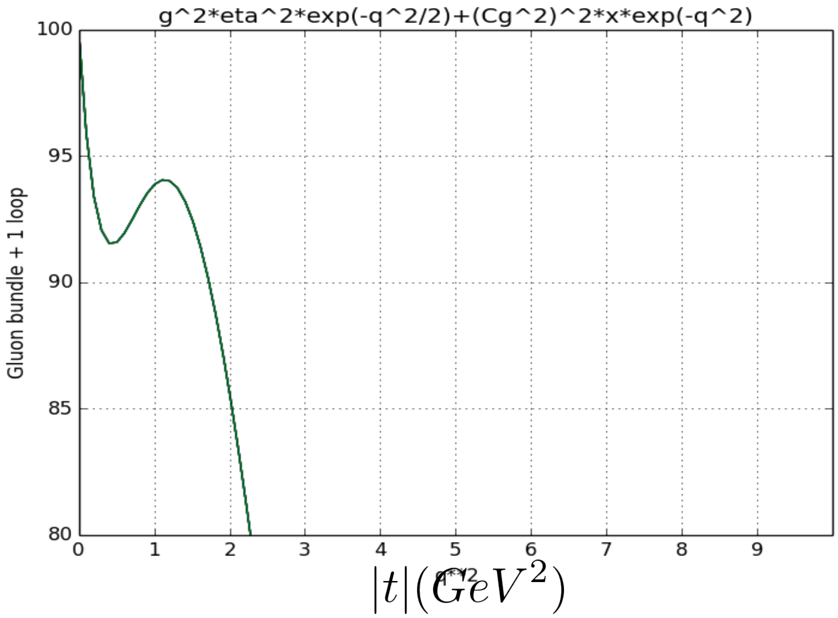
where $\tilde{\varphi} = constant \cdot e^{-(\frac{q}{2\mu})^{2-\eta}},\; a = \lambda\kappa g\vec{q}^2\tilde{\varphi},$

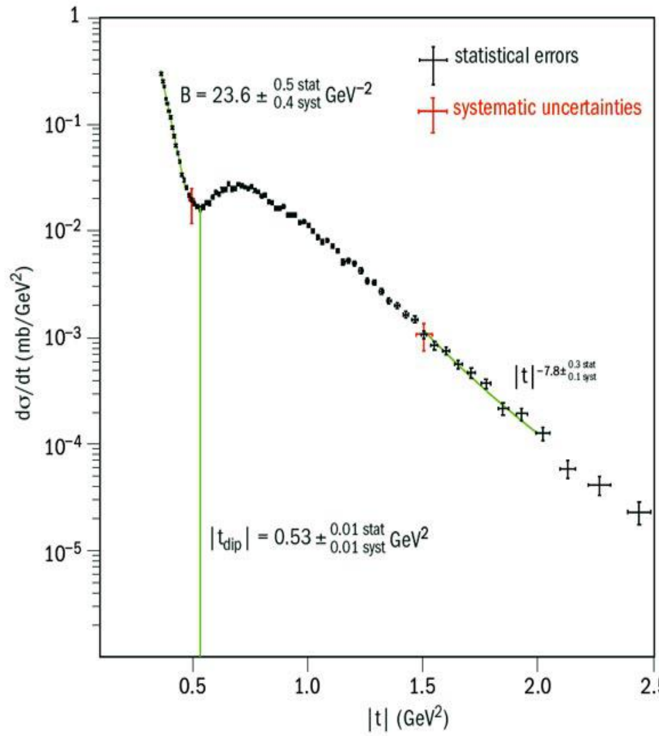




mu = 1GeV/c^2, zeta = 0.1
g=10,
lambda =0.18,
K=1,

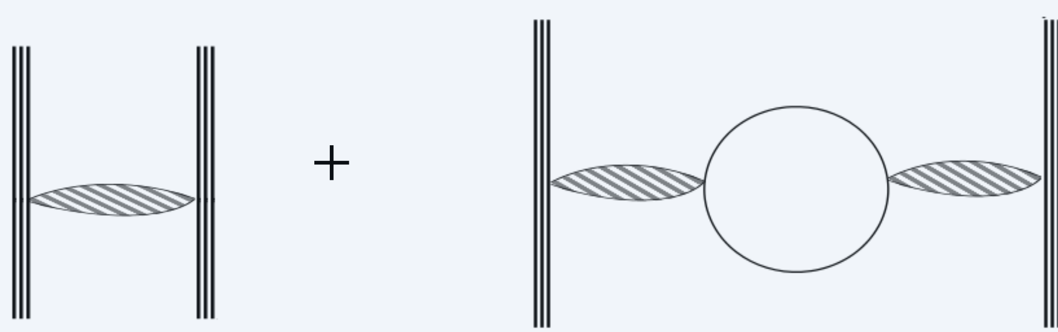
(Fried, Gabellini, Grandou, Sheu, pht. arxiv 1412.2072)





(TOTEM, EPL 101 (2013) 21002.
doi: 10.1209/0295-5075/101/21002)

Therefore Diffraction dip is from



GB falls off inverse pion mass. Loops give stretch → nucleon-nucleon binding enabled by loops. Gives you extra change of sign in nucleon potential comes from the loop! Not yukawa!

To do

1) More loops. Still, only chain graphs are non-zero.

The $I(q^2)$ looks like this :

2) Odd number of GB's to loops = 0

3) Four GB's on a loop. In QED, it is finite.

4) R^2 approximation. Solved without this approximation?

5) As a function of energy. The dip moves.

Conclusion

$$L_{QCD} \rightarrow$$
$$Gauge\ Independence \rightarrow$$
$$Halpern + Fradkin \rightarrow Gaussian \rightarrow$$
$$Eikonal \rightarrow Nuclear\ scattering$$

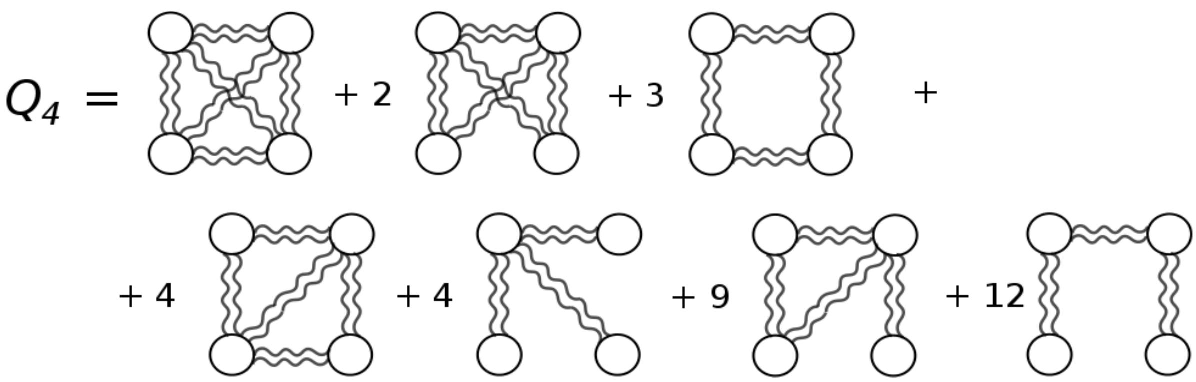
First example of nucleon binding directly from QCD.
(we are only looking at qualitative features. 1 Flavor of quarks, no E&M no spin. Can be added in by hand later)

First application of Non-perturbative + Gauge Invariant + Finite + Exact QCD to nucleon scattering.

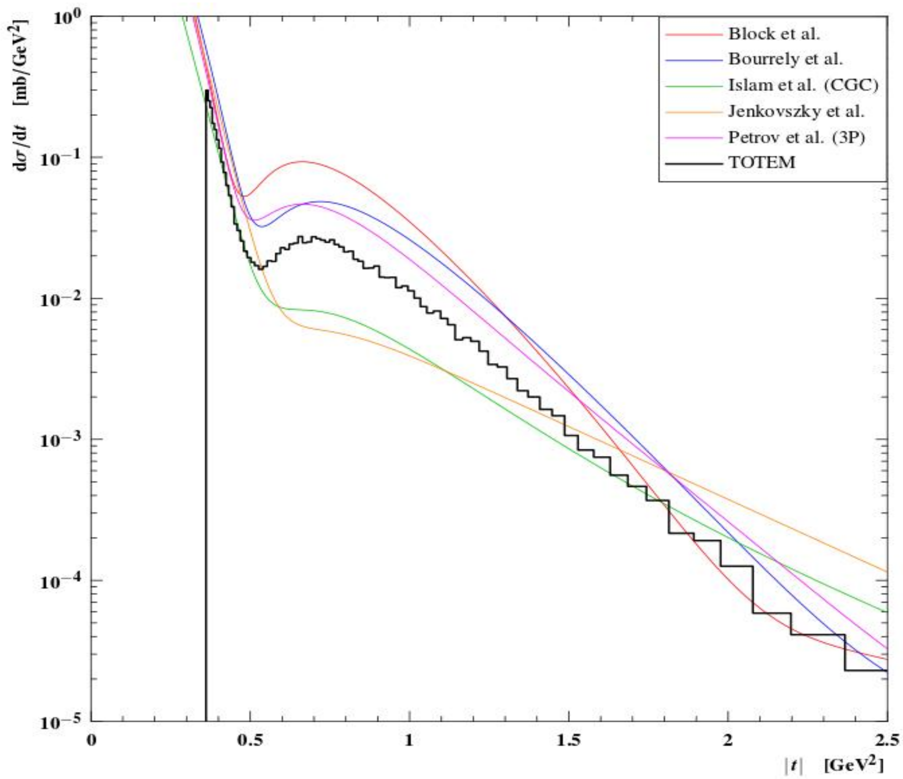
Thank you !

Additional materials

Example cluster expansion for Q_4 term:



Others comparison to pp-scattering:



APPENDIX

1 Fradkin's Representations for $\mathbf{G}_{\rm c}[A]$ and $\mathbf{L}[A]$

The exact functional representations of these two functionals of $A(x)$ are perhaps the most useful tools in all of QFT, for they allow that A-dependence of these functionals to be extracted from inside ordered exponentials; and because they, themselves, are Gaussian in their dependence upon $A(x)$, they permit the functional operations of the Schwinger/Symanzik generating functional (Gaussian functional integration, or functional linkage operation) to be performed exactly. This corresponds to an explicit sum over all Feynman graphs relevant to the process under consideration, with the results expressed in terms of functional integrals over the Fradkin variables; and in the present QCD case, because of EL, those non-perturbative results can be extracted and related to physical measurements.

The causal quark Green's function (which is essentially the most customary Feynman one) can be written as~\cite{8,9}

$$\mathbf{G}_c[A] = [m + i\gamma \cdot \Pi][m + (\gamma \cdot \Pi)^2]^{-1}$$
$$= [m + i\gamma \cdot \Pi] \cdot i \int_0^\infty ds\, e^{-ism^2}\, e^{is(\gamma\cdot\Pi)^2},$$

where $\Pi = i[\partial_\mu - igA^a_\mu \tau^a]$ and $(\gamma \cdot \Pi)^2 = \Pi^2 + ig\sigma_{\mu\nu}\,\mathbf{F}^a_{\mu\nu}\tau^a$ with $\sigma_{\mu\nu} = \frac{1}{4}[\gamma_\mu, \gamma_\nu]$. Following Fradkin's method \cite{8,9} and replacing Π_μ with $i\dfrac{\delta}{\delta v_\mu}$, one obtains

$$\mathbf{G}_{\rm c}(x,y|A) = i\int_0^\infty ds\ e^{-ism^2} \cdot e^{i\int_0^s ds'}$$
$$\times \frac{\delta^2}{\delta v^2_\mu(s')} \cdot \left[m - \gamma_\mu \frac{\delta}{\delta v_\mu(s)}\right] \delta(x - y + \int_0^s ds'\ v(s'))$$
$$\times \left(\exp\left\{-ig\int_0^s ds' \left[v_\mu(s')\, A^a_\mu(y - \int_0^{s'} v)\tau^a + i\sigma_{\mu\nu}\,\mathbf{F}^a_{\mu\nu}(y - \int_0^{s'} v)\tau^a\right]\right\}\right)$$

Then, one can insert a functional `resolution of unity' of form

$$1 = \int \mathrm{d}[u]\, \delta[u(s') - \int_0^{s'} ds''\ v(s'')],$$

and replace the delta-functional $\delta[u(s') - \int_0^{s'} ds''\ v(s'')]$ with a functional integral over Ω, and then the Green's function becomes~\cite{YMS2008}

$$\mathbf{G}_c(x, y|A) = i \int_0^{\infty} ds\ e^{-ism^2}\ e^{-\frac{1}{2}\ln(2h)} \int d[u]\, e^{\frac{i}{4}\int_0^s ds'\ [u'(s')]^2}\, \delta^{(4)}(x - y + u$$
$$\times \left[m + ig\gamma_\mu A^a_\mu(y - u(s))\tau^a\right] \left(e^{-ig\int_0^s ds'\ u'_\mu(s')\ A^a_\mu(y-u(s'))\ \tau^a + g\int_0^s ds' \sigma_{\mu\nu}\ \mathbf{F}^a_{\mu\nu}(y-\right.$$

where $\mathrm{h}(\mathrm{s}_1, s_2) = \int_0^s ds'\, \Theta(s_1 - s')\Theta(s_2 - s')$. To remove the \$A\$-dependence out of the linear (mass) term, one can replace $\mathrm{i\ g\ A}^a_\mu(y - u(s))\tau^a$ with $-\dfrac{\delta}{\delta u'_\mu(s)}$ operating on the ordered exponential so that

$$\begin{aligned} &\mathbf{G}_c(x, y|A) \\ &= i \int_0^{\infty} ds\ e^{-ism^2}\ e^{-\frac{1}{2}\ln(2h)} \int d[u]\, e^{\frac{i}{4}\int_0^s ds'\ [u'(s')]^2}\, \delta^{(4)}(x - y + u(s)) \\ &\times \left[m - \gamma_\mu \frac{\delta}{\delta u'_\mu(s)}\right] \\ &\times \left(e^{-ig\int_0^s ds'\ u'_\mu(s')\ A^a_\mu(y-u(s'))\ \tau^a + g\int_0^s ds' \sigma_{\mu\nu}\cdot \mathbf{F}^a_{\mu\nu}(y-u(s'))\ \tau^a}\right)_+. \end{aligned}$$

To extract the A-dependence out of the ordered exponential, one may use the following identities,

$$1 = \int d[\alpha]\, \delta\left[\alpha^a(s') + g u'_\mu(s')\, A^a_\mu(y - u(s'))\right],$$
$$1 = \int d[\boldsymbol{\Xi}]\, \delta\left[\boldsymbol{\Xi}^a_{\mu\nu}(s') - g\mathbf{F}^a_{\mu\nu}(y - u(s'))\right],$$

and the ordered exponential becomes

$$\left(e^{-ig\int_0^s ds'\, u'_\mu(s')\, A^a_\mu(y-u(s'))\,\tau^a + g\int_0^s ds' \sigma_{\mu\nu}\,\mathbf{F}^a_{\mu\nu}(y-u(s'))\,\tau^a}\right)_+$$
$$= \mathcal{N}_\Omega\,\mathcal{N}_\Phi \int d[\alpha] \int d[\mathbf{\Xi}] \int d[\Omega] \int d[\mathbf{\Phi}] \left(e^{i\int_0^s ds' \left[\alpha^a(s') - i\sigma_{\mu\nu}\,\mathbf{\Xi}^a_{\mu\nu}(s')\right]\tau^a}\right)_+$$
$$\times e^{-i\int ds'\,\Omega^a(s')\,\alpha^a(s') - i\int ds'\,\mathbf{\Phi}^a_{\mu\nu}(s')\,\mathbf{\Xi}^a_{\mu\nu}(s')}$$
$$\times e^{-ig\int ds'\, u'_\mu(s')\,\Omega^a(s')\,A^a_\mu(y-u(s')) + ig\int ds'\,\mathbf{\Phi}^a_{\mu\nu}(s')\,\mathbf{F}^a_{\mu\nu}(y-u(s'))},$$

where N_Ω and N_Φ are constants that normalize the functional representations of the delta-functionals. All A-dependence is removed from the ordered exponential and the resulting form of the Green's function is exact (it entails no approximation). Alternatively, extracting the A-dependence out of the ordered exponential can also be achieved by using the functional translation operator, and one writes

$$\left(e^{+g\int_0^s ds' \left[\sigma_{\mu\nu}\,\mathbf{F}^a_{\mu\nu}(y-u(s'))\tau^a\right]}\right)_+$$
$$= e^{g\int_0^s ds'\,\mathbf{F}^a_{\mu\nu}(y-u(s'))\,\frac{\delta}{\delta\mathbf{\Xi}^a_{\mu\nu}(s')}} \cdot \left(e^{\int_0^s ds' \left[\sigma_{\mu\nu}\,\mathbf{\Xi}^a_{\mu\nu}(s')\tau^a\right]}\right)_+\Bigg|_{\mathbf{\Xi}\to 0}.$$

For the closed-fermion-loop functional $\mathrm{L}[\mathrm{A}]$, one can write~\cite{9}

$$\mathbf{L}[A] = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\, e^{-ism^2}\,\left\{\left[e^{-is(\gamma\cdot\Pi)^2}\right] - \{g=0\}\right\},$$

where the trace Tr sums over all degrees of freedom, space-time coordinates, spin and color. The Fradkin representation proceeds along the same steps as in the case of $\mathrm{G_c}[A]$, and the closed-fermion-loop functional reads

$$\mathbf{L}[A] = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\, e^{-ism^2}\, e^{-\frac{1}{2}\ln(2h)} \quad \times \int d[v]\,\delta^{(4)}(v(s))\, e^{\frac{i}{4}\int_0^s ds'\,[v'(s')]^2}$$
$$\times \int d^4x \left(e^{-ig\int_0^s ds'\, v'_\mu(s')\, A^a_\mu(x-v(s'))\,\tau^a + g\int_0^s ds' \sigma_{\mu\nu}\,\mathbf{F}^a_{\mu\nu}(x-v(s'))\,\tau^a}\right)_+$$
$$-\{g=0\}\,,$$

where the trace Tr sums over color and spinor indices. Also, Fradkin's variables have been denoted by $\mathrm{v(s')}$, instead of $\mathrm{u(s')}$, in order to distinguish them from those appearing in the Green's function $\mathrm{G_c}[A]$. One finds

$$\mathbf{L}[A] = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\, e^{-ism^2}\, e^{-\frac{1}{2}\ln{(2h)}}$$
$$\times \mathcal{N}_\Omega\, \mathcal{N}_\Phi \int d^4x \int \mathrm{d}[\alpha] \int \mathrm{d}[\Omega] \int \mathrm{d}[\mathbf{\Xi}] \int \mathrm{d}[\mathbf{\Phi}]$$
$$\times \int d[v]\, \delta^{(4)}(v(s))\, e^{\frac{i}{4}\int_0^s ds'\, [v'(s')]^2}$$

$$\times\ e^{-i\int ds'\, \Omega^a(s')\,\alpha^a(s') - i\int ds'\, \mathbf{\Phi}^a_{\mu\nu}(s')\,\mathbf{\Xi}^a_{\mu\nu}(s')}\ \cdot \left(e^{i\int_0^s ds'\,\left[\alpha^a(s') - i\sigma_{\mu\nu}\,\mathbf{\Xi}^a_{\mu\nu}(s')\right]\tau^a}\right)_+$$
$$\times e^{-ig\int_0^s ds'\, v'_\mu(s')\,\Omega^a(s')\, A^a_\mu(x - v(s')) - 2ig\int d^4z\,\left(\partial_\nu \mathbf{\Phi}^a_{\nu\mu}(z)\right) A^a_\mu(z)}$$
$$\times e^{+ig^2\int ds'\, f^{abc}\,\mathbf{\Phi}^a_{\mu\nu}(s')\, A^b_\mu(x - v(s'))\, A^c_\nu(x - v(s'))} - \{g = 0\}\,,$$

where the same properties as those of $\mathrm{G_c}[A]$ can be read off readily.

21 Vanishing 'Bundle self-energy' diagrams of Figures 1 and 2.

For simplicity and clarity, we first consider the non-spin dependence of the Fradkin representation of $\mathrm{G_c}[A]$, and then discuss the spin terms separately. Because of the Effective Locality, the four-dimensional delta-function multiplying the $(\mathrm{f}\cdot\chi)^{-1}$factor of the GB of Figure 1 is given by $\delta^{(4)}(u(s_1) - u(s_2))$. This suggests but does not necessarily require that $\mathrm{S}_1 = s_2$; but that condition is obtained by considering the time-like and longitudinal integrals separately, $\delta(u_0(s_1) - u_0(s_2))$ and $\delta(u_L(s_1) - u_L(s_2))$. Suppose now that there are a set of points S_ℓ for which the argument of the time-like $\delta_{(0)}$ vanishes, and a set of points S_m for which the argument of the longitudinal $\delta_{(L)}$-function vanishes,

$$\delta_{(0)} = \sum_\ell \frac{\delta(s_1 - s_\ell)}{|u'_0(s_\ell)|}\Bigg|_{u_0(s_\ell) = u_0(s_2)},$$

$$\delta_{(L)} = \sum_m \frac{\delta(s_s - s_m)}{|u'_L(s_m)|}\Bigg|_{u_L(s_1) = u_L(s_m)}.$$

Their product is then given by

$$\sum_{\ell,m} \frac{\delta(s_1 - s_\ell)\delta(s_1 - s_m)}{|u_0'(s_\ell)\ u_L'(s_m)|}\Bigg|_{\substack{u_0(s_\ell)=u_0(s_m)\\ u_L(s_\ell)=u_L(s_m)}}$$

and it is the subsidiary conditions which are most relevant. Since u_0 and u_L are continuous but completely independent functions, the probability of finding sets of points s_ℓ and s_m at which u_0 takes on the same value, and at which u_L simultaneously has the same value, would appear to be less than ϵ. However, there are two s-values for which this is possible, where initial conditions specify that $u_\mu(0) = 0$, and that $u_\mu(s) = -z_\mu$. Therefore, only $s_1 = s_2 = 0$, or else $s_1 = s_2 = s$. Then, for either case, $s_1 = s_2$, and the coefficients $u'_\mu(s_1)$ and $u'_\nu(s_2)$ are symmetric in μ and ν, and are multiplying $(f{\cdot}\chi)^{-1}|_{\mu\nu}$ which is antisymmetric in those indices; and the result is zero.

The spin dependence for this particular process will also vanish, but for two different reasons. Those terms coming from the linear A-dependence of the $G_c[A]$ representation will have gradient terms differentiating the y dependence of the δ-functions representing EL, but that y-dependence trivially cancels for this 'self-energy' process, and hence those terms give a zero result. The antisymmetric spin dependence coming from quadratic A-terms finds itself multiplying a different set of $u'_\mu(s_1)$ and $u'_\nu(s_2)$ coefficients; and then the analysis of the previous paragraph again rules out any non-zero contribution.

The vanishing of the Bundle Diagram of Fig. \ref{Fig:2} may be inferred from that of Fig. \ref{Fig:1}, by imagining the two ends of the quark line of Fig. 1 to be wrapped around and form a closed loop; and then, without performing the loop integrations, the result is zero. Or, one may follow the argument used in the text following eq. (\ref{Eq:6}) for chain-graph loops but applied to this single loop containing an internal GB; and again the result is zero.

Tsang, Peter H.: “Model of QCD Renormalization applied to pp scattering”

We use a new realistic, gauge invariant (via being gauge independent!), analytic formulation of QCD to look at high energy pp-scattering. The differential cross section is derived by means of exchange of Gluon Bundles between quarks with Eikonal approximation. A Gluon Bundle in our formulation is the sum of infinite number of gluons exchanged between quarks.