Higher Loop Non-planar Anomalous Dimensions

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Planar Integrability

Motivated by AdS/CFT there has been tremendous progress in exploring the planar limit of $\mathcal{N} = 4$ super Yang-Mills theory.

Much of this progress is thanks to integrability; integrability in the planar limit follows from considering the computation of anomalous dimensions and noting

1. One can focus on single trace operators;
2. There is a bijection between states of a spin chain and single trace operators;
3. The dilatation operator is the Hamiltonian of an integrable spin chain.

Question: Is integrability present in other large N (but not planar) limits of the theory? These limits are obtained by considering operators with a dimension that grows parametrically with N.

Some points to consider

Single-traces mix with multi-traces so the spin chain language is lost; is there a new description?

Not all operators are independent.

Non-planar diagrams must be summed.

Operators are in irreps of $PSU(2,2|4)$. Large multiplicities of irreps from combinatorics of building gauge invariant operators from ϕ_i, fermions, field strengths, derivatives.

Key idea: Organizing multiple representations, summing Feynman diagrams and constructing independent operators is achieved using symmetries. (Symmetric group of $n!$ permutations of n objects; Schur-Weyl Duality between symmetric and unitary groups; $PSU(2,2|4)$.)

New description is entirely in terms of permutations.

Not all operators are independent

For $N = 2$ and a single matrix

$$\mathrm{Tr}(Z^3) = -\frac{1}{2}\left[\mathrm{Tr}(Z)^3 - 3\mathrm{Tr}(Z)\mathrm{Tr}(Z^2)\right]$$

There are also relations for operators built using many matrices - these are more complicated.

Some points to consider

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New description is entirely in terms of permutations.

Language for Multitraces: permutations

Consider a collection of n distinct $N \times N$ matrices, $(M_1)^i_j, \cdots (M_n)^i_j$. Any multitrace structure

$$\mathrm{Tr}(M_1 M_k M_l)\mathrm{Tr}(M_2)\mathrm{Tr}(M_3 M_m M_n M_p)\cdots$$

can be written using the permutation $\sigma = (1, k, l)(2)(3, m, n, p)\cdots$ as

$$(M_1)^{i_1}_{i_{\sigma(1)}}(M_2)^{i_2}_{i_{\sigma(2)}}\cdots(M_n)^{i_n}_{i_{\sigma(n)}} \equiv \mathrm{Tr}\left(\sigma M_1 \otimes M_2 \otimes \cdots M_n\right)$$

For every permutation there is a unique multitrace structure.

The language of permutations provides a convenient description which treats all mulitrace trace structures on equal footing.

Language for Multitraces: conjugacy classes

$\frac{1}{2}$-BPS sector: Gauge-invariant BPS operators are traces and products of traces built using a single matrix.

$n = 1 :\ \mathrm{Tr}(Z)$
$n = 2 :\ \mathrm{Tr}(Z^2);\ (\mathrm{Tr}Z)^2$
$n = 3 :\ \mathrm{Tr}(Z^3);\ \mathrm{Tr}(Z^2)\mathrm{Tr}(Z);\ (\mathrm{Tr}Z)^3$

In general (bose statistics)

$$\mathrm{Tr}(\sigma Z^{\otimes n}) = Z^{i_1}_{i_{\sigma(1)}} Z^{i_2}_{i_{\sigma(2)}} \cdots Z^{i_n}_{i_{\sigma(n)}}$$

$$= \mathrm{Tr}(\sigma\gamma^{-1} Z^{\otimes n}\gamma) = \mathrm{Tr}(\gamma\sigma\gamma^{-1} Z^{\otimes n})$$

Any multitrace operator composed from k fields corresponds to a $\sigma \in S_k$. Permutations in the same conjugacy class determine the same operator.

Fourier Transform: Duality between irreps and conjugacy classes

Characters of symmetric group are a complete set of functions on the conjugacy classes. Comparing

$$\sum_{\sigma\in S_n}\chi_R(\sigma)\chi_S(\sigma)\propto\delta_{RS}\qquad\sum_R\chi_R(\sigma)\chi_R(\psi)\propto\delta([\sigma][\psi])$$

$$\int dx e^{ikx}e^{-ik'x}\propto\delta(k-k')\qquad\int dk e^{ikx}e^{-ikx'}\propto\delta(x-x')$$

motivates

$$\chi_R(Z)=\sum_{\sigma\in S_n}\frac{1}{n!}\chi_R(\sigma)\mathrm{Tr}(\sigma Z^{\otimes n})$$

$$\mathrm{Tr}(\sigma Z^{\otimes n})=\sum_R\chi_R(\sigma)\chi_R(Z)$$

Schur Polynomials

$$\chi_R(Z) \propto (P_R)^{i_1\, i_2 \cdots\, i_n}_{j_1\, j_2 \cdots\, j_n} Z^{j_1}_{i_1} Z^{j_2}_{i_2} \cdots Z^{j_n}_{i_n}$$

$$\chi_R(Z) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_R(\sigma) \mathrm{Tr}(\sigma Z^{\otimes n})$$

R specifies an irrep of S_n. $\chi_R(\sigma)$ is the character of σ in irrep R.

$$\chi_{\ydiagram{3}} = \frac{1}{6}\left[\mathrm{Tr}(Z)^3 + 3\mathrm{Tr}(Z)\mathrm{Tr}(Z^2) + 2\mathrm{Tr}(Z^3)\right]$$

$$\chi_{\ydiagram{2,1}} = \frac{1}{6}\left[2\mathrm{Tr}(Z)^3 - 2\mathrm{Tr}(Z^3)\right]$$

$$\chi_{\ydiagram{1,1,1}} = \frac{1}{6}\left[\mathrm{Tr}(Z)^3 - 3\mathrm{Tr}(Z)\mathrm{Tr}(Z^2) + 2\mathrm{Tr}(Z^3)\right]$$

The relation between traces at $N = 2$ reads $\chi_{\ydiagram{1,1,1}} = 0$.

Wick Contraction as a Permutation

From the basic Wick contraction

$$\langle Z^i{}_j (Z^\dagger)^k{}_l \rangle = \delta^i_l \delta^k_j$$

we find (A and B are arbitrary coefficients)

$$\langle\, A^{i_1\, i_2 \cdots\, i_n}_{j_1\, j_2 \cdots\, j_n} Z^{j_1}_{i_1} Z^{j_2}_{i_2} \cdots Z^{j_n}_{i_n} \;\; B^{k_1\, k_2 \cdots\, k_n}_{l_1\, l_2 \cdots\, l_n} (Z^\dagger)^{l_1}_{k_1} (Z^\dagger)^{l_2}_{k_2} \cdots (Z^\dagger)^{l_n}_{k_n} \,\rangle$$

$$= \sum_{\sigma \in S_n} \mathrm{Tr}(A \sigma B \sigma^{-1})$$

A nice choice for A and B follow from noting that projection operators obey

$$\big[P_A, \sigma\big] = 0 \qquad P_A P_B = \delta_{AB} P_A$$

Thus, choosing A and B to be projectors

$$\sum_{\sigma \in S_n} \mathrm{Tr}(P_A \sigma P_B \sigma^{-1}) = n! \delta_{AB} \mathrm{Tr}(P_A) = n! \delta_{AB} d_A$$

Schur Polynomials

Number of Schur polynomials agrees with finite N counting. Finite N cut off is implemented by saying R has no more than N rows.

$$\langle \chi_R(Z)\chi_S(Z)^\dagger \rangle = f_R \delta_{RS}$$

$$\mathrm{Tr}(\sigma Z^{\otimes n}) = \sum_R \chi_R(\sigma)\chi_R(Z)$$

(Corley, Jevicki, Ramgoolam, hep-th/0111222)

Summary

1. Each permutation gives a gauge invariant operator.
2. Bose symmetry implies permutations in same conjugacy class give same gauge invariant operator.
3. Mathematical structure: conjugacy classes $\leftrightarrow$ irreps of S_n. Fourier transform to find irreps of S_n $\leftrightarrow$ gauge invariant operators.
4. Use Wick contraction $\leftrightarrow$ permutation to get exact two point function.

Output: A new basis that diagonalizes the free field two point function.

Could consider fermions and then operators built with many matrices, as well as move beyond tree level.

Single Adjoint Fermionic Matrix

The Grassman nature of ψ implies that the trace of an even number of fields vanishes

$$\mathrm{Tr}(\psi^4) = \psi^i_j \psi^j_k \psi^k_l \psi^l_i = -\psi^j_k \psi^i_j \psi^k_l \psi^l_i = \psi^j_k \psi^k_l \psi^i_j \psi^l_i$$

$$= -\psi^j_k \psi^k_l \psi^l_i \psi^i_j = -\mathrm{Tr}(\psi^4)$$

In general

$$\mathrm{Tr}(\sigma \gamma \psi^{\otimes n} \gamma^{-1}) = \mathrm{sgn}(\gamma) \mathrm{Tr}(\sigma \psi^{\otimes n})$$

Previously we used characters which reflected bose symmetry as $\chi_R(\gamma^{-1}\sigma\gamma) = \chi_R(\sigma)$. We now need functions that obey

$$\chi^F_R(\gamma^{-1}\sigma\gamma) = \mathrm{sgn}(\gamma)\chi^F_R(\sigma)$$

Single Adjoint Fermionic Matrix

The character is a trace. To construct the functions

$$\chi^F_R(\gamma^{-1}\sigma\gamma) = \mathrm{sgn}(\gamma)\chi^F_R(\sigma)$$

modify the trace

$$\chi^F_R(\alpha) = \sum_{m,m'} S^{[1^n]\,R\,R}{}_{m'\,m}\Gamma^R_{m\,m'}(\alpha)$$

Now take a Fourier transform

$$\chi^F_R(\psi) = \sum_{\alpha\in S_n} \chi^F_R(\alpha)\mathrm{Tr}_{V^{\otimes n}}(\alpha\psi^{\otimes n})$$

Wick contractions are now signed permutations

$$\langle(\psi^{\otimes n})^I_J(\psi^{\dagger\otimes n})^K_L\rangle = \sum_{\sigma\in S_n} \mathrm{sgn}(\sigma)\sigma^I_L(\sigma^{-1})^K_J$$

Fermionic Schur Polynomials

Number of fermionic Schur polynomials agrees with finite N counting. Finite N cut off is implemented by saying R has no more than N rows. Only self conjugate irreps R participate.

$$\langle \chi^F_R(\psi)\chi^F_S(\psi)^\dagger \rangle = f_R \delta_{RS}$$

$$\mathrm{Tr}(\sigma \psi^{\otimes n}) = \sum_R \chi^F_R(\sigma)\chi^F_R(\psi)$$

(dMK, Diaz, Nokwara, arXiv:1212.5935)

Including a second matrix

$$\mathrm{Tr}(\sigma Z^{\otimes n} \otimes Y^{\otimes m}) = Z^{i_1}_{i_{\sigma(1)}} Z^{i_2}_{i_{\sigma(2)}} \cdots Z^{i_n}_{i_{\sigma(n)}} Y^{i_{n+1}}_{i_{\sigma(n+1)}} Y^{i_{n+2}}_{i_{\sigma(n+2)}} \cdots Y^{i_{m+n}}_{i_{\sigma(m+n)}}$$

Any multitrace operator built using n Zs and m Ys corresponds to a $\sigma \in S_{n+m}$. Permutations related by $\gamma\sigma_1\gamma^{-1} = \sigma_2$ with $\sigma_1, \sigma_2 \in S_{n+m}$ and $\gamma \in S_n \times S_m$ determine the same operator.

Bose symmetry implies that we now need a set of functions that obeys

$$f(\gamma\sigma\gamma^{-1}) = f(\sigma) \quad \sigma \in S_{n+m} \quad \gamma \in S_n \times S_m$$

Restricted Characters

$$f(\gamma\sigma\gamma^{-1}) = f(\sigma) \quad \sigma \in S_{n+m} \quad \gamma \in S_n \times S_m$$

Restricting S_{n+m} irrep R to $S_n \times S_m$, we find irreps (r, s) with multiplicity indexed by α. Modify the trace in the character by restricting row index to the α copy of (r, s) and column index to the β copy of (r, s) to get the restricted character $\chi_{R,(r,s)\alpha\beta}(\sigma)$:

$$\sum_{\sigma \in S_{n+m}} \chi_{R,(r,s)\alpha\beta}(\sigma)\chi_{S,(t,u)\gamma\delta}(\sigma) \propto \delta_{RS}\delta_{rt}\delta_{su}\delta_{\alpha\gamma}\delta_{\beta\delta}$$

$$\sum_{R,(r,s)\alpha\beta} \chi_{R,(r,s)\alpha\beta}(\sigma)\chi_{R,(r,s)\alpha\beta}(\psi) \propto \delta([\sigma]_r[\psi]_r)$$

(dMK, Smolic, Smolic hep-th/0701066)

Restricted Schur Polynomials

$$\chi_{R,(r,s)\alpha\beta}(Z,Y) = \frac{1}{n!m!} \sum_{\sigma \in S_{n+m}} \mathrm{Tr}_{(r,s)\alpha\beta}(\Gamma^R(\sigma))\mathrm{Tr}(\sigma Z^{\otimes n} \otimes Y^{\otimes m})$$

R is an irrep of S_{n+m}. We can subduce the $S_n \times S_m$ irrep (r,s) from R. α, β keep track of which copy we subduce.

$$\chi_{\square\square,(\square,\square)} = \mathrm{Tr}(Z)\mathrm{Tr}(Y) + \mathrm{Tr}(ZY)$$

$$\chi_{\substack{\square\\\square},(\square,\square)} = \mathrm{Tr}(Z)\mathrm{Tr}(Y) - \mathrm{Tr}(ZY)$$

(Bhattacharyya, Collins, dMK, arXiv:0801.2061)

Wick Contractions are permutations

Wick contractions are again a sum over permutations

$$\langle Z^i{}_j (Z^\dagger)^k{}_l \rangle = \delta^i_l \delta^k_j = \langle Y^i{}_j (Y^\dagger)^k{}_l \rangle$$

$$\langle A^{i_1 \cdots i_{n+m}}_{j_1 \cdots j_{n+m}} Z^{j_1}_{i_1} \cdots Z^{j_n}_{i_n} Y^{j_{n+1}}_{i_{n+1}} \cdots Y^{j_{n+m}}_{i_{n+m}} \cdots$$

$$\cdots B^{k_1 \cdots k_{n+m}}_{l_1 \cdots l_{n+m}} (Z^\dagger)^{l_1}_{k_1} \cdots (Z^\dagger)^{l_n}_{k_n} (Y^\dagger)^{l_{n+1}}_{k_{n+1}} \cdots (Y^\dagger)^{l_{n+m}}_{k_{n+m}} \rangle$$

$$= \sum_{\sigma \in S_n \times S_m} \mathrm{Tr}(A \sigma B \sigma^{-1})$$

Restricted Schur Polynomials

Number of restricted Schur polynomials agrees with finite N counting. (Finite N cut off is implemented by saying R, r, s each have no more than N rows.)

$$\langle \chi_{R,(r,s)\mu\nu}(Z,Y)\chi_{S,(t,u)\alpha\beta}(Z,Y)^\dagger\rangle = N(R,r,s)\delta_{RS}\delta_{rt}\delta_{su}\delta_{\mu\alpha}\delta_{\nu\beta}$$

$$\mathrm{Tr}(\sigma Z^{\otimes n}Y^{\otimes m}) = \sum_{R,(r,s)\alpha\beta} \mathrm{Tr}_{(r,s)\beta\alpha}(\Gamma^R(\sigma))\chi_{R,(r,s)\beta\alpha}(Z,Y)$$

(Bhattacharyya, Collins, dMK, arXiv:0801.2061; Bhattacharyya, dMK, Stephanou, arXiv:0805.3025)

Symmetries have provided a good way to reorganize degrees of freedom of the matrix model so that we (i) have a complete basis and (ii) diagonalize the two point function.

Can the same ideas be used at loop level?

Dilatation Operator

Since we have two matrices, action of one loop dilatation operator is non-trivial

$$D = -g_{YM}^2 \mathrm{Tr}\left(\left[Z, Y\right]\left[\frac{d}{dZ}, \frac{d}{dY}\right]\right)$$

(Beisert, Kristjansen, Staudacher, hep-th/0303060)

$$D\chi_{R,(r,s)\alpha\beta} = \sum_{T,(t,u)kl} M_{R,(r,s)\alpha\beta;T,(t,u)\gamma\delta}\chi_{T,(t,u)\delta\gamma}$$

$$M_{R,(r,s)\alpha\beta;T,(t,u)\delta\gamma} = -g_{YM}^2 \sum_{R'} N_{R,R',r,s,T,t,u}$$

$$\times \mathrm{Tr}_{R\oplus T}\left(\left[\Gamma^R(1,m+1), P_{R,(r,s)\alpha\beta}\right] I_{R'\,T'}\left[\Gamma^T(1,m+1), P_{T,(t,u)\delta\gamma}\right] I_{T'\,R'}\right).$$

$$\mathrm{Tr}_{(r,s)\alpha\beta}(*) = \mathrm{Tr}_R(P_{R,(r,s)\alpha\beta} *)$$

(De Comarmond, dMK, Jefferies, arXiv:1012.3884)

The Displaced Corners Approximation

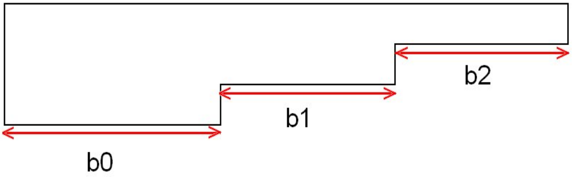


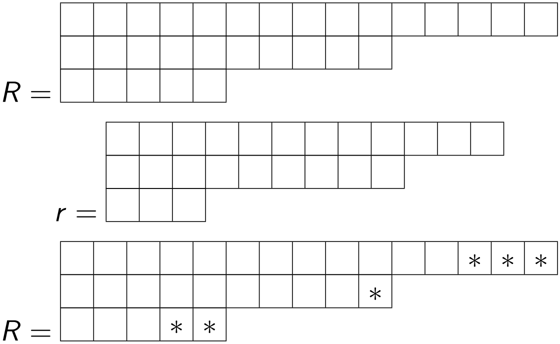
Figure: Example of a three row Young diagram.

In the displaced corners approximation we assume that b_0, b_1, b_2 are all of order N.

This limit simplifies the action of the symmetric group which is responsible for a new $U(p)$ symmetry.

A New Symmetry

$\chi_{R,(r,s)\mu\nu}$



$$\vec{m} = (3, 1, 2)$$

New symmetry leads to a further conservation law - the dilatation operator does not mix operators with different $\vec{m}$.

D in the Displaced Corners Approximation: Factorization

$$D\chi_{R,(r,s)\mu_1\mu_2} = -g_{YM}^2 \sum_{u\nu_1\nu_2} \sum_{i<j} M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2} \Delta_{ij} \chi_{R,(r,u)\nu_1\nu_2}$$

i, j run over the rows of R.

Δ_{ij} acts only on the Young diagrams R, r and $M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2}$ acts only on the labels $s\mu_1\mu_2$.

(dMK, Dessein, Giataganas, Mathwin, arXiv:1108.2761)

Action of Δ_{13}

Action of Δ_{12}

$$\Delta_{12}\chi(b_0,b_1,b_2) = (2N+2b_0+2b_1+b_2)\chi(b_0,b_1,b_2)$$
$$-\sqrt{(N+b_0+b_1+1)(N+b_0+b_1+b_2)}\chi(b_0,b_1-1,b_2+2)-$$
$$-\sqrt{(N+b_0+b_1)(N+b_0+b_1+b_2+1)}\chi(b_0,b_1+1,b_2-2))$$

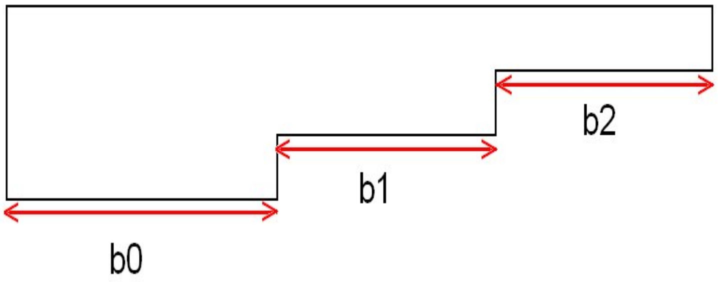


Figure: Example of labeling for a three row Young diagram.

Δ_{ij} eigenproblem

Think of $\Delta_{ij} \in u(2)$ acting on states given by restricted Schur polynomials.

Standard raising/lowering operator gives

$$J^+|\lambda,\Lambda\rangle = c_+|\lambda+1,\Lambda\rangle\,, \qquad c_+ = \sqrt{(\Lambda+\lambda+1)(\Lambda-\lambda)}\,,$$
$$J^-|\lambda,\Lambda\rangle = c_-|\lambda-1,\Lambda\rangle\,, \qquad c_- = \sqrt{(\Lambda+\lambda)(\Lambda-\lambda+1)}\,.$$

$$\begin{aligned}\Delta_{12}\chi(b_0,b_1,b_2) &= (2N+2b_0+2b_1+b_2)\chi(b_0,b_1,b_2)\\ &-\sqrt{(N+b_0+b_1+1)(N+b_0+b_1+b_2)}\chi(b_0,b_1-1,b_2+2)\\ &-\sqrt{(N+b_0+b_1)(N+b_0+b_1+b_2+1)}\chi(b_0,b_1+1,b_2-2))\end{aligned}$$

$$\Delta_{ij} \to \left(\frac{\partial}{\partial x_i}-\frac{\partial}{\partial x_j}\right)^2 - \frac{(x_i-x_j)^2}{4}$$

$$x_i = \frac{r_i - r_p}{\sqrt{N+r_p}}$$

(dMK, Kemp, Smith, arXiv:1111.1058)

D in the Displaced Corners Approximation: Factorization

$$D\chi_{R,(r,s)\mu_1\mu_2} = -g_{YM}^2 \sum_{u\nu_1\nu_2} \sum_{i<j} M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2} \Delta_{ij} \chi_{R,(r,u)\nu_1\nu_2}$$

i, j run over the rows of R.

Δ_{ij} acts only on the Young diagrams R, r and $M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2}$ acts only on the labels $s\mu_1\mu_2$.

(dMK, Dessein, Giataganas, Mathwin, arXiv:1108.2761)

Y Eigenproblem: $M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2}$

Diagonalization of $M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2}$ means diagonalizing $r(r-1)/2$ matrices simultaneously. Matrix elements are given by Clebsch-Gordan coefficients of $U(r)$. Can we solve this problem by reorganizing degrees of freedom?

The operators we consider are dual to giant gravitons (one for each row of R) with strings attached (one for each Y).

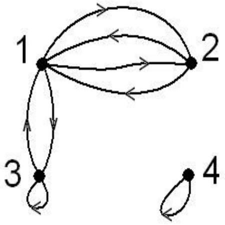
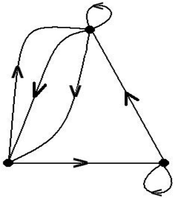


Figure: R has 4 long rows; there are 8 Y fields

Graph as a permutation



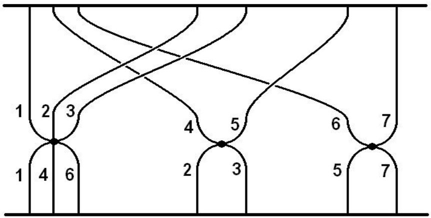


Figure: The graph determines an element of $H \setminus S_{m_1+m_2+m_3}/H$ where $H = S_{m_1} \times S_{m_2} \times S_{m_3}$. $\sigma = (1)(24)(356)(7)$

$$\frac{1}{|H|^2} \sum_{\alpha_1,\alpha_2 \in H} \sum_{\sigma \in S_n} \delta(\alpha_2 \sigma^{-1} \alpha_1 \sigma) = \sum_{s \vdash m} (M^s_{1_H})^2$$

Fourier transform applied to the double coset

Complete set of functions on the double coset (modify trace!!)

$$C^{s;\mu\nu}(\sigma) = \sum_{ij} \sqrt{d_s}\Gamma^s_{ij}(\sigma) B_{j\mu} B_{i\nu} \qquad \left[\sim e^{ikx} \right]$$

$$\frac{1}{|H|} \sum_{\gamma \in H} \Gamma^s_{ik}(\sigma) = \sum_{\mu} B_{i\mu} B_{k\mu}$$

$$O_{R,r}(\sigma) = \sum_{s,\mu,\nu} C^{s;\mu\nu}(\sigma) \chi_{R,(r,s)\mu\nu}(Z, Y)$$

$$DO_{R,r}(\sigma) = -g^2_{YM} \sum_{i<j} n_{ij}(\sigma) \Delta_{ij} O_{R,r}(\sigma)$$

(dMK, Ramgoolam, arXiv:1204.2153)

Y Eigenproblem

Example: (from Young diagrams with 4 rows and 8 labeled boxes; $\vec{m} = (3, 2, 2, 1)$)

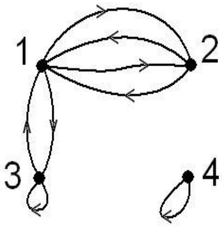


Figure: Example of a pictorial labeling.

$$DO(b_0, b_1, b_2, b_3) = -g_{YM}^2(4\Delta_{12} + 2\Delta_{13})O(b_0, b_1, b_2, b_3)$$

(dMK, Dessein, Giataganas, Mathwin, arXiv:1108.2761)

Open Spring Theory

$$D = -g_{YM}^2 \sum_{i<j} n_{ij}(\sigma) \left[\left(\frac{\partial}{\partial x_i} - \frac{\partial}{\partial x_j} \right)^2 - \frac{(x_i - x_j)^2}{4} \right]$$

$$\Delta = \Delta_0 + g_{YM}^2 \sum_i n_i \omega_i = \Delta_0 + \lambda \sum_i \frac{n_i}{N} \omega_i$$

Continuous spectrum at large N!
(dMK, Kemp, Smith, arXiv:1111.1058)

Higher Loops: Restrictions imposed by Symmetry

Dilatation operator commutes with R-symmetry generators.
Consider an $su(2)$ subgroup of the $su(4)$ R-symmetry.

$$\left[J_+, J_-\right] = J_3, \qquad \left[J_3, J_\pm\right] = \pm 2 J_\pm \,.$$

When acting on the restricted Schur polynomials

$$J_+ = \mathrm{Tr}\left(Y\frac{d}{dZ}\right), \;\; J_- = \mathrm{Tr}\left(Z\frac{d}{dY}\right), \;\; J_3 = \mathrm{Tr}\left(Y\frac{d}{dY}\right) - \mathrm{Tr}\left(Z\frac{d}{dZ}\right)$$

From the algebra, eigenvalues of the $su(2)$ generators are integers
$\Rightarrow$ they don't pick up λ corrections

Restrictions imposed by $SU(2)$ Symmetry Algebra

Use the explicit form for the $J_{\mp}$, J_3. Work with two rows.

Ansatz:

$$DO^{(n,m)}_{r_1,j,j^3} = \sum_{a=-p}^{p} \sum_{b=-p}^{p} \beta^{(n,m)}_{r_1,j,j^3}(a,b) O^{(n,m)}_{r_1+a,j+b,j^3}$$

Now the requirement $\left[D_p, J_{\pm}\right] = \left[D_p, J_3\right] = 0$ gives recursion relations for the $\beta^{(n,m)}_{r_1,j,j^3}(a,b)$. Also $D = D^{\dagger}$.

No unique solution:

$$D \to \kappa_1 D + 2k_0 \mathbf{1}$$

k_0 fixed by requiring lowest eigenvalue is zero. Using distant corners approximation the form of $M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2}$ is fixed to all loops. (dMK, Graham, Messamah)

Restrictions imposed by $SU(2)$ Symmetry Algebra: Result

To obtain restrictions on Δ_{ij} one needs the exact form for $J_\pm$. Can be computed for small m.

$$D\chi_{R,(r,s)\mu_1\mu_2} = -\left(\sum_{n=1} c_n\lambda^n\right)\sum_{u\nu_1\nu_2}\sum_{i<j} M^{(ij)}_{s\mu_1\mu_2;u\nu_1\nu_2}\Delta_{ij}\chi_{R,(r,u)\nu_1\nu_2}$$

(dMK, Hasina-Tahiridimbisoa)

Summary

Families of operators with definite scaling dimension labeled by permutations - giant gravitons with open strings attached.

Dilatation operator reduces to decoupled harmonic oscillators.

$$O_{\vec{n}}(\sigma) = \sum_{s,\mu,\nu} \sum_{ij} \sqrt{d_s} \sum_r \Gamma^s_{ij}(\sigma) B_{j\mu} B_{i\nu} \psi_{HO,\vec{n}}(r) \chi_{R,(r,s)\mu\nu}(Z,Y)$$

$$\chi_{R,(r,s)\alpha\beta}(Z,Y) = \frac{1}{n!m!} \sum_{\psi \in S_{n+m}} \mathrm{Tr}_{(r,s)\alpha\beta}(\Gamma^R(\psi)) \mathrm{Tr}(\psi Z^{\otimes n} \otimes Y^{\otimes m})$$

$$R \vdash m+n \quad r \vdash n \quad s \vdash m \quad \sigma \in H \setminus S_m / H$$

$$\Delta = \Delta_0 + f(\lambda) \sum_i n_i \omega_i$$

The action of the dilatation operator is tightly constrained by symmetry.

Thanks for your attention!