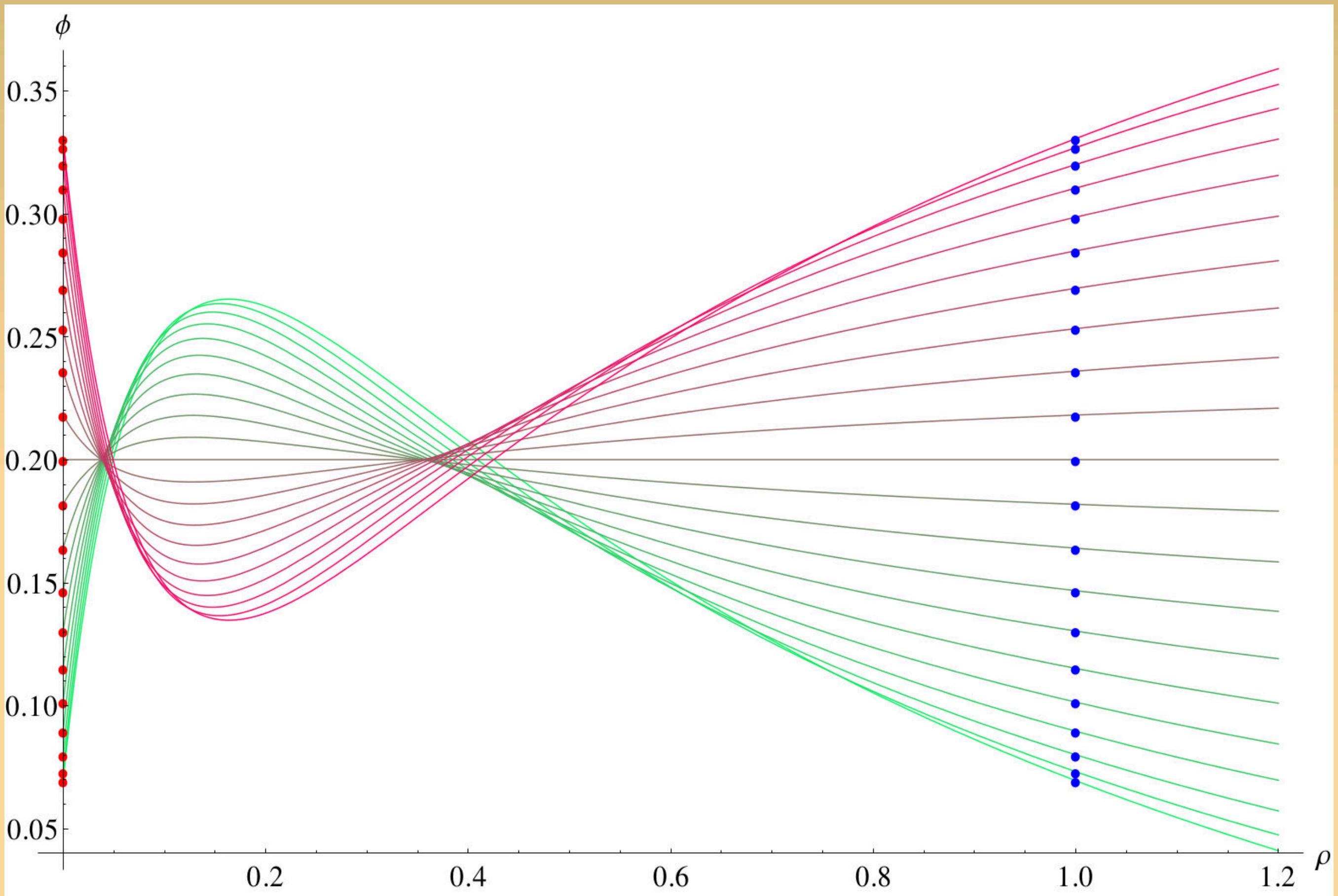
Hot Attractors



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Thanks to Vishnu Jejjala for many slides

Collaborators

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Arxiv:1410.3478 & work in progress

Outline

- Black hole thermodynamics
- The attractor mechanism (T=0)
- Hot attractors (T≠0)
- CFT take on bulk gravity results
- Summary & Prospects

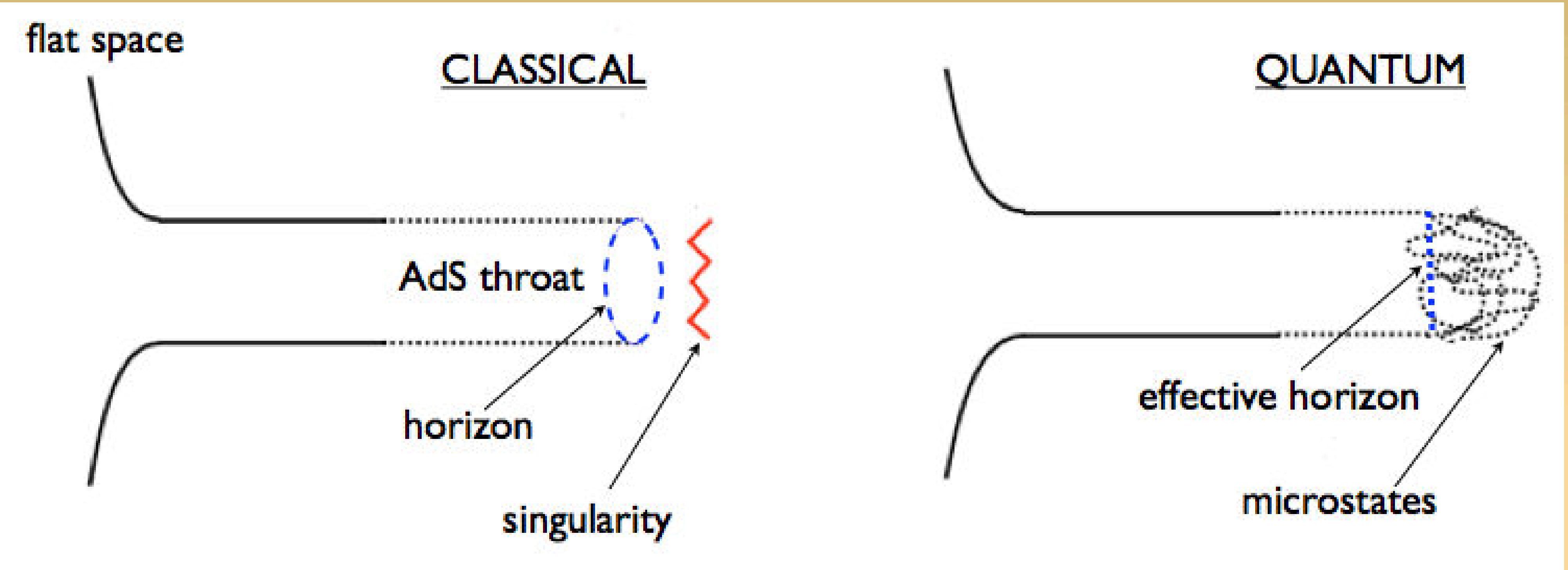
Blackhole thermodynamics

- 0^{th} Law
  - T is constant throughout body in thermal equilibrium
  - Event horizon surface gravity constant
- 1^{st} Law
  - $dE = TdS - pdV + \mu dN$
  - $dM = T_H dS_{BH} + \Omega dJ + \Phi dQ$
- 2^{nd} Law
  - $\Delta S \geq 0$
  - $\Delta S_{BH} + \Delta S_{\mathrm{Universe}} \geq 0$

$$T_H = \frac{\kappa}{2\pi}$$

$$S_{BH} = \frac{A}{4G_N}$$

Gravitational Thermodynamics



- Horizons and singularities are *effective* notions in gravity that arise only as a consequence of a thermodynamic averaging over microstates, or *coarse-graining*
- Origin of entropy lies in the inability of a semi-classical observer to distinguish different quantum microstates

Lunin, Mathur

Inner horizon “thermodynamics”

- BHs can have two horizons (at $r_\pm$ say)
  - r_- : inner (Cauchy)
  - r_+ : outer (Event)
- eg. Reissner-Nördstrom, Kerr
- S_{BH} associated with outer horizon

Inner horizon "thermodynamics"

- 1st Law

$$dM = T_+ dS + \Omega_+ dJ + \Phi_+ dQ \quad \text{(Outer)}$$

$$-dM = T_- dS - \Omega_- dJ - \Phi_- dQ \quad \text{(Inner)}$$

Curir, 1979

Inner horizon “thermodynamics”

- 1st Law

$$dM = T_+ dS + \Omega_+ dJ + \Phi_+ dQ \quad \text{(Outer)}$$

$$dM = T_- dS + \Omega_- dJ + \Phi_- dQ \quad \text{(Inner)}$$

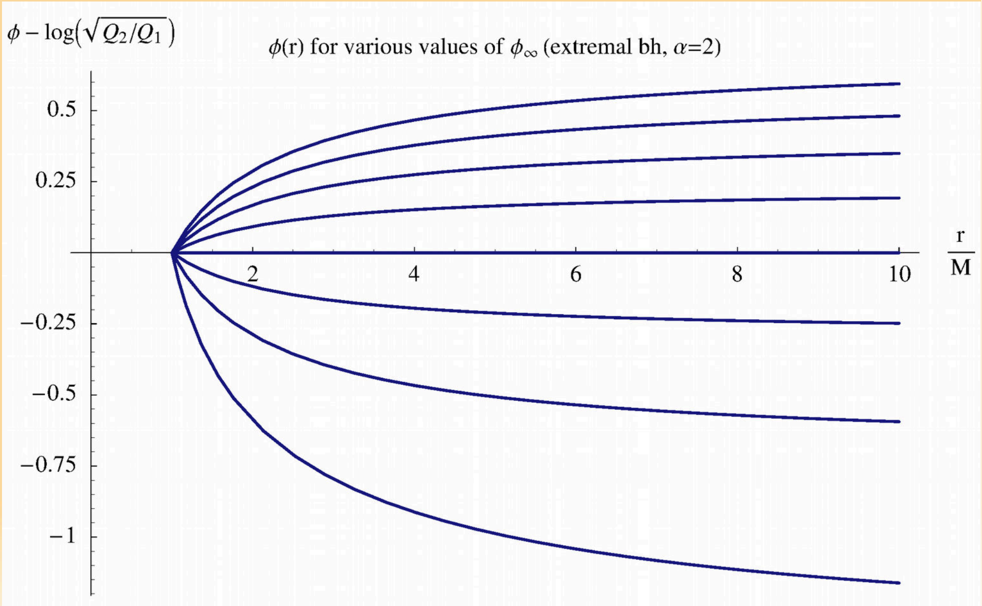
Curir, 1979

  - to preserve form we take T_- negative
- 2nd Law
  - ?? what can entropy of inner horizon mean ??
    - recent work by Martinec for μ-states
    - GJN – to appear

Attractor mechanism

Ferrara, Kallosh, Strominger, 1995

- Feature of $T_H=0$ black holes
- Scalars (moduli) drawn to fixed values at horizon
  - φ_H depend on charges carried by BH
  - independent of starting point (φ_∞)



Attractor mechanism

- Feature of $T_H=0$ black holes

Ferrara, Kallosh, Strominger, 1995

- Scalars (moduli) drawn to fixed values at horizon
  - φ_H depend on charges carried by BH
  - independent of starting point (φ_∞) ie. Vevs @ ∞
- Entropy independent of φ_∞
- Not require SUSY

Ferrara, Gibbons, Kallosh 1997 (implicit)
Sen 2005
Goldstein, Iizuka, Jena, Trivedi, 2005

Hand waving explanation

- Number of microstates is determined by quantized charges
- Entropy, which counts number of microstates, cannot vary continuously
- Moduli vary continuously

→ entropy (horizon area) must be independent of background moduli

→ Moduli assume fixed values at horizon determined by charges

T_H=0 details

$$S = \frac{1}{\kappa^2}\int d^4x\,\sqrt{-g}\,\left(R - 2(\nabla\phi_i)^2 - f_{ab}(\phi_i)F^a_{\mu\nu}F^{b,\mu\nu}\right)$$

Action

$$R_{\mu\nu} - 2\partial_\mu\phi_i\partial_\nu\phi_i = 2f_{ab}(\phi_i)\left(2F^a_{\mu\lambda}F^{b,\lambda}_{\nu} - \frac{1}{2}g_{\mu\nu}F^a_{\lambda\sigma}F^{b,\lambda\sigma}\right)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,\partial^\mu\phi_i) = \frac{1}{4}(\partial_i f_{ab})F^a_{\mu\nu}F^{b,\mu\nu}$$

$$\partial_\mu(\sqrt{-g}\,f_{ab}(\phi_i)F^{b,\mu\nu}) = 0$$

EOM

$$\mathrm{d}s^2 = -a(r)^2dt^2 + a(r)^{-2}dr^2 + b(r)^2d\Omega_2^2$$

$$F^a = Q^a_m\sin\theta\ d\theta\wedge d\phi$$

$$\phi = \phi(r)$$

Ansatz

$$V_{\rm eff} = f_{ab}(\phi_i)Q^a_mQ^b_m$$

Effective potential

Goldstein, Iizuka, Jena, Trivedi, 2005

EOM for $ds^2 = -a(r)^2 dt^2 + a(r)^{-2} dr^2 + b(r)^2 d\Omega^2$

- One equation is easy to integrate

$$(a^2 b^2)'' = 2 \quad \Longrightarrow \quad a^2 b^2 = (r - r_+)(r - r_-)$$

  - $r_+ \longrightarrow r_-$ gives T_H=0.

- Others not so easy

$$-\frac{b''}{b} = (\phi')^2 \qquad \left(a^2 b^2 \phi_i'\right)' = \frac{\partial_i V_{\text{eff}}(\phi)}{2b^2} \qquad \partial_i = \frac{\partial}{\partial \phi_i}$$

- 1st order Hamiltonian constraint

$$-1 + a^2 b'^2 + \frac{a^{2\prime} b^{2\prime}}{2} = -\frac{V_{\text{eff}}(\phi_i)}{b^2} + a^2 b^2 (\phi')^2$$

EOM @ Horizon, *T*=0

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$$(a^2 b^2)'' = 2 \quad \Longrightarrow \quad a^2 b^2 = (r - r_+)(r - r_-)$$

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double horizon

Attractor Equations

Scalar EOM on horzon:

$$\partial_i V_{eff}(\phi_0) = 0, \quad \partial_i = \frac{\partial}{\partial \phi_i}$$

if $M_{ij} = \partial_i \partial_j V_{\rm eff}(\phi_0)$ has non-negative eigenvalues

$\phi_0 = \{\phi_{i0}\}$ are attractor values

then from Hamiltonian constraint:

$$b_h^2 = V_{\rm eff}(\phi_0)$$

We can now read off

$$S_{\rm BH} = \frac{1}{4} A = \pi b_h^2 = \pi V_{\rm eff}(\phi_0)$$

Attractor Equations – for later

$$\frac{\partial_i V_{eff}(\phi_0)}{b_h^2} = 0$$

$$\frac{V_{eff}(\phi_0)}{b_h^2} - 1 = 0$$

Near horizon extremal geometry:

- Generically get $AdS_2 \times S^2$

Area Law

- $A_\pm$:= area of outer/inner horizon
- $A_+ A_-$ Larsen, 1997
  - function only of quantized charges
  - independent of mass
- taking extremal limit → geometric mean law:

$$A_+ A_- = A_{\mathrm{ext}}^2$$

- Observed d=4 Kerr & Kerr-Newmann, d=5 Myers-Perry etc.

Cvetic, Gibbons, Pope, 2010

Hot attractors

$$A_+ A_- = A_{\text{ext}}^2 \Rightarrow S_+ S_- = S_{\text{ext}}^2$$

- S_{ext} is function of conserved charges (J,Q)

  $\rightarrow$ independent of mass, asymptotic moduli
- Generalises to Wald entropy
  - **Caveat**: not work when Smarr relation violated

Castro, Dehmami, Giribet, Kastor, 2013

- $S_\pm$ depends on mass & moduli

  $\rightarrow$ There is some hot attractor mechanism/conspiracy involving both horizons so that S_+S_- indep of mass, asymptotic moduli.

BTZ: Area law = level matching

- Black hole in AdS_3 – 2 Virasoros
  - use holographic dictionary to interpret A_+A_-
- 1^{st} Law $dM = T_\pm dS + \Omega_\pm dJ$
- Gravity

$$S_\pm = \frac{A_\pm}{4G_3} = \frac{\pi r_\pm}{2G_3} \qquad T_\pm = \pm\frac{r_+^2 - r_-^2}{2\pi r_\pm L^2}$$

$$J = \frac{r_+ r_-}{4G_3 L} \qquad \Omega_\pm = \frac{r_\mp}{r_\pm L}$$

- CFT

$$T_{R,L} = \frac{r_+ \pm r_-}{2\pi L^2} \implies \frac{1}{T_\pm} = \frac{1}{2}\left(\frac{1}{T_R} \pm \frac{1}{T_L}\right)$$

$$S_\pm = \frac{\pi^2 L}{3}(c_R T_R \pm c_L T_L) \qquad c_L = c_R = \frac{3L}{2G_3}$$

$$\implies \frac{A_+ A_-}{(8\pi G_3)^2} = n_R - n_L$$

(level matching)

$T_H \neq 0$ details

- Same EOM

$$S = \frac{1}{\kappa^2}\int d^4x\,\sqrt{-g}\,\left(R - 2(\nabla\phi_i)^2 - f_{ab}(\phi_i)F^a_{\mu\nu}F^{b,\mu\nu}\right)$$

Action

$$R_{\mu\nu} - 2\partial_\mu\phi_i\partial_\nu\phi_i = 2f_{ab}(\phi_i)\left(2F^a_{\mu\lambda}F^{b,\lambda}_\nu - \frac{1}{2}g_{\mu\nu}F^a_{\lambda\sigma}F^{b,\lambda\sigma}\right)$$

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$$F^a = Q^a_m\sin\theta\;d\theta\wedge d\phi$$

$$\phi = \phi(r)$$

Ansatz

$$V_{\text{eff}} = f_{ab}(\phi_i)Q^a_mQ^b_m$$

Effective potential

$T_H \neq 0$ details

- Same EOM
- Scalars take on values at horizon that depend on values at infinitiy (not attractive)
- There are robust invariant quantities that we derive

$T_H \neq 0$ details

- As before: $a^2\, b^2 = (r - r_+)(r - r_-)$
- Horizons : a=0
- Periodicity in Euclidean time:

$$T_\pm = \frac{(a^2)'_\pm}{2\pi} = \pm \frac{r_+ - r}{4\pi b_\pm^2}$$

$\longrightarrow$

$$b_+^2 T_+ = -b_-^2 T_-$$

- Notation: non-extremality parameter

$$\Delta = \frac{r_+ - r_-}{2}$$

Deriving Geomtric Mean Law

- From $b_+^2 T_+ = -b_-^2 T_-$ and $b^2 \sim S$:

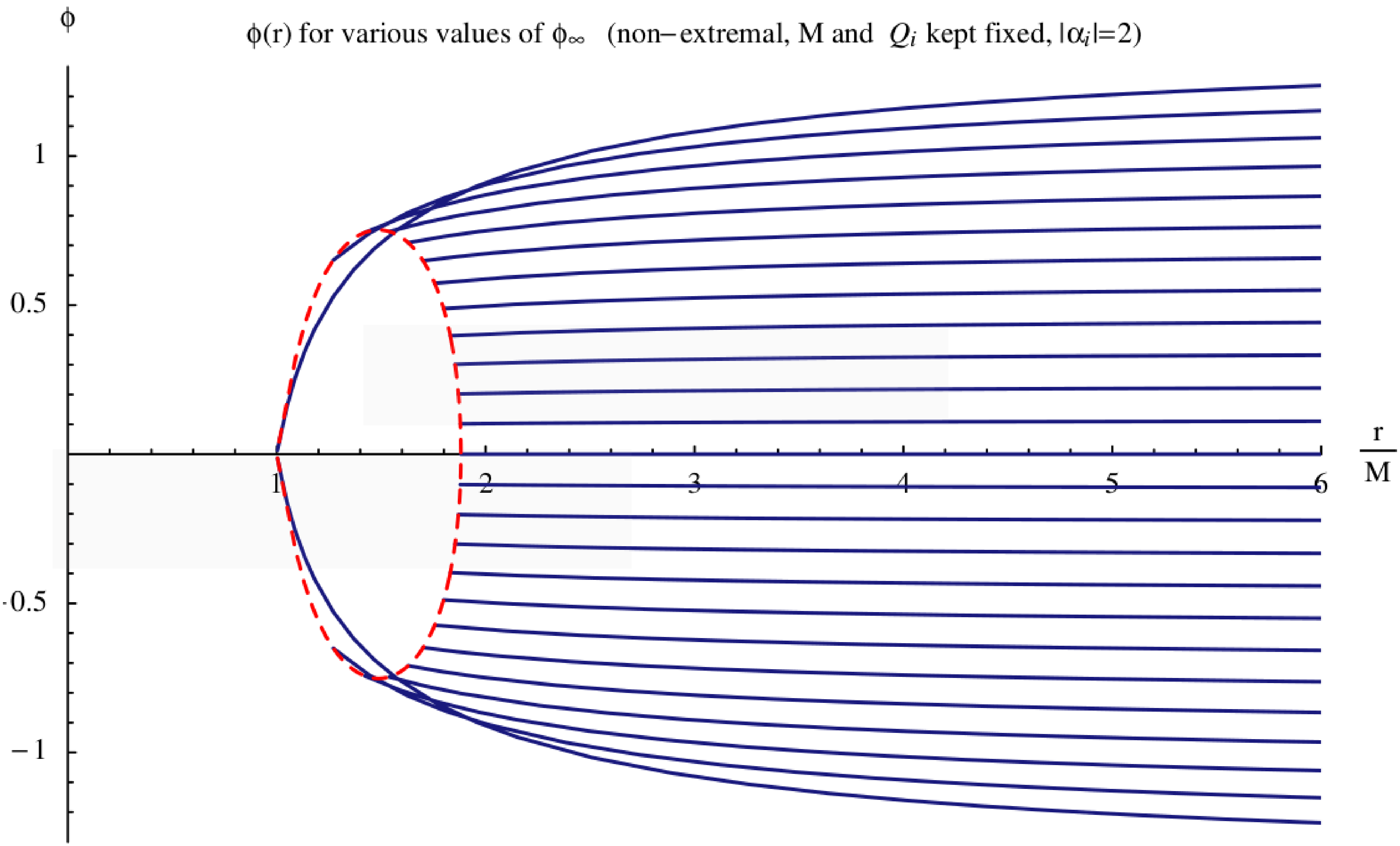
  $\rightarrow$ $S_+ T_+ + S_- T_- = 0$

- 1^{st} Law: $dM = T_\pm dS_\pm \Rightarrow T_\pm^{-1} = \left.\frac{\partial S_\pm}{\partial M}\right|_{J,Q}$

- So: $\left.\frac{\partial}{\partial M}(S_+ S_-)\right|_{J,Q} = \frac{S_+ T_+ + S_- T_-}{T_+ T_-} = 0$

- Taking $\Delta \rightarrow 0$ limit: $S_+ S_- = S_{\rm ext}^2$
- (have more general proof)

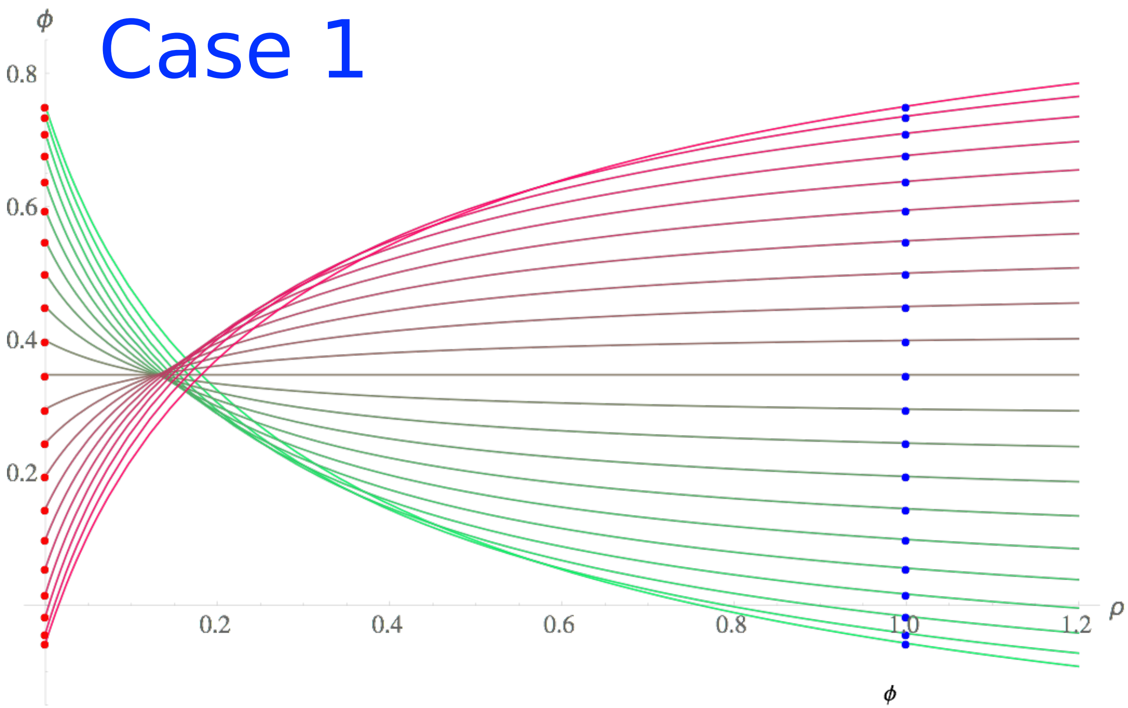
Hot black holes are unattractive



Goldstein, Iizuka, Jena, Trivedi, 2005

The Fields (particular V_{eff})

Case 1



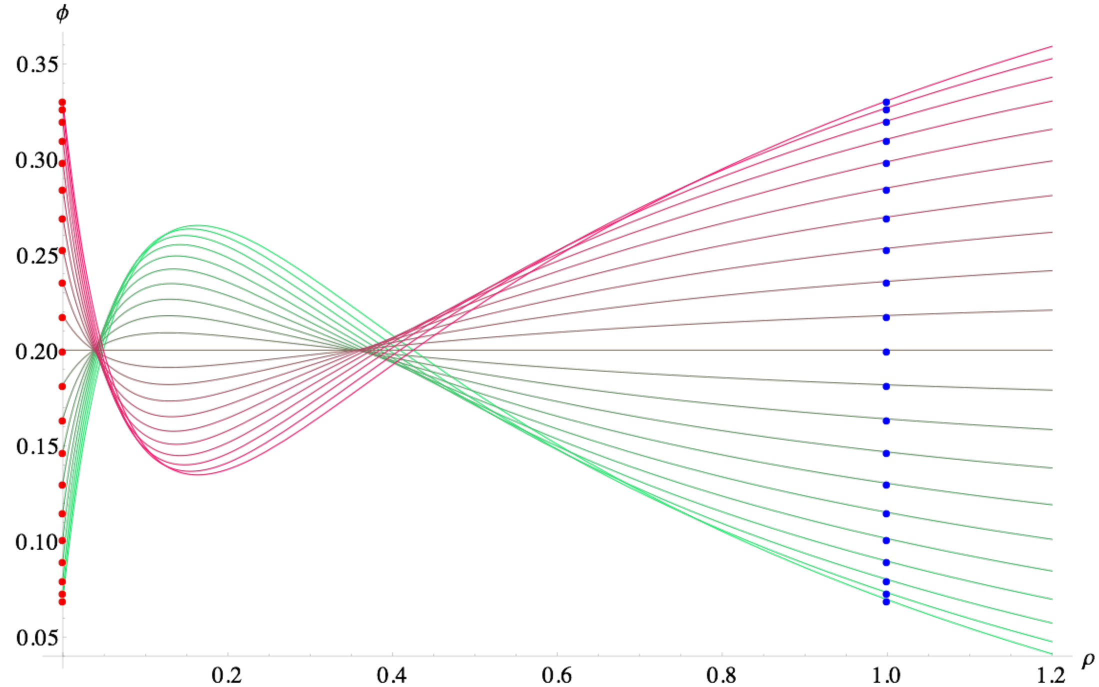
$$M \ , \ Q_i \text{ constant}$$

$$r = (r_+ - r_-)\rho + r_-$$

$$\phi_0 = \frac{1}{2}(\phi_+ + \phi_-)$$

Flows intersect attractor value once

Case 2

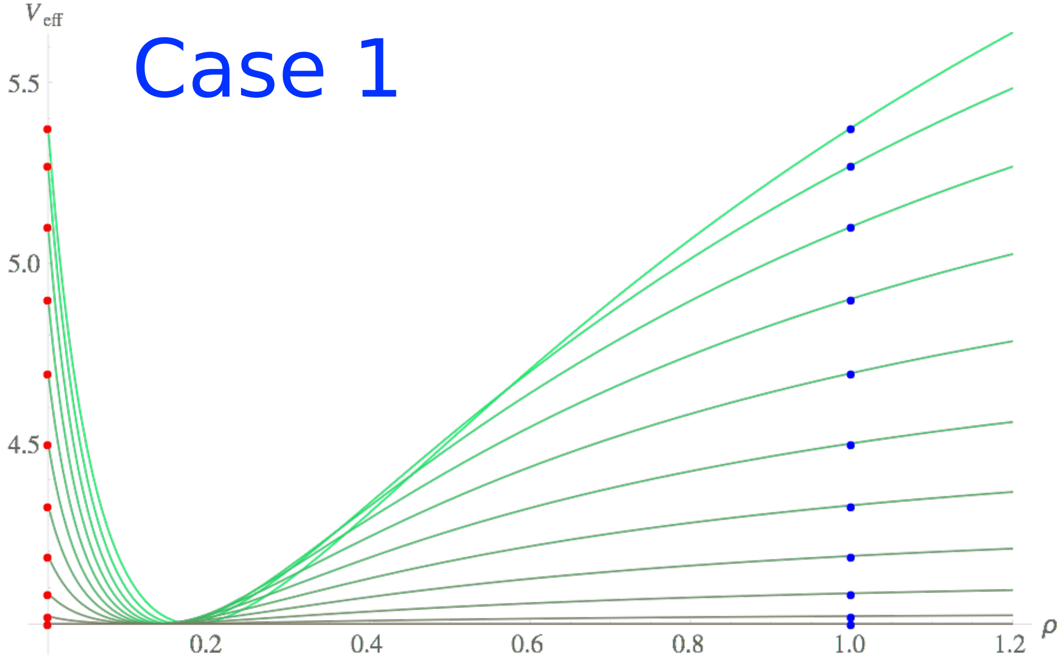


$$\phi_+ = \phi_-$$

Flows intersect attractor value twice

The Effective Potential

Case 1



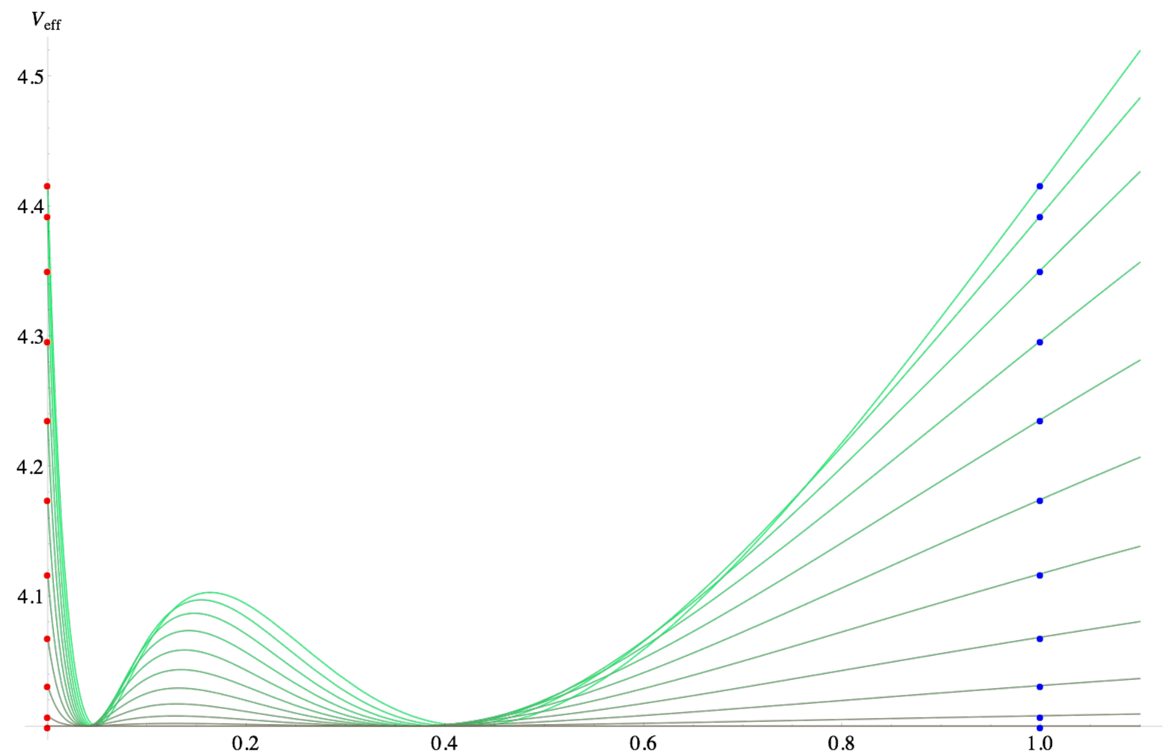
$$M\ ,\ Q_i \text{ constant}$$

$$r = (r_+ - r_-)\rho + r_-$$

Case 2

$$V_{\rm eff}(\phi_+) = V_{\rm eff}(\phi_-)$$

Minimum of potential at attractor value



Attractor Equations, T=0

$$\frac{\partial_i V_{eff}(\phi_0)}{b_h^2} = 0$$

$$\frac{V_{eff}(\phi_0)}{b_h^2} - 1 = 0$$

$T_H \neq 0$ details

- $$-1 + a^2 b'^2 + \frac{a^{2\prime} b^{2\prime}}{2} = -\frac{V_{\rm eff}(\phi_i)}{b^2} + a^2 b^2 (\phi')^2$$

$$(a^2 b^2 \phi_i')' = \frac{\partial_i V_{\rm eff}(\phi)}{2b^2}$$

- BC: $a(r_\pm) = 0$

$$(a^2)'(r_\pm) b^2(r_\pm) = \pm\Delta$$

- Find averaged attractor eqns

$$\int_{r_-}^{r_+} dr \left(\frac{\partial_i V_{eff}}{b^2}\right) = 0$$

$$\int_{r_-}^{r_+} dr \left(\frac{V_{eff}}{b^2} - 1\right) = 0$$

Marketing

- First order fluctuations of moduli between horizons satisfy

$$\partial_z(z^2-1)\partial_z\delta\phi = \lambda^2\delta\phi$$

where $$\lambda^2 = \frac{\partial_\phi^2 V}{2b_h^2}\Bigg|_{\phi=\phi_h} = \frac{m^2}{b_h^2}$$

- equations of scalar in AdS_2
  - is this coincidence or sign of something deeper ?
    - To appear

Comparing $T_H=0$ and $T_H \neq 0$

Extremal

Moduli independent area

$$\left\langle \frac{V_{\text{eff}}(\phi)}{b^2} - 1 \right\rangle_{\mathrm{AdS}_2 \times S^2} = 0$$

$$\left\langle \frac{\partial_{\phi_i} V_{\text{eff}}(\phi)}{b^2} \right\rangle_{\mathrm{AdS}_2 \times S^2} = 0$$

Non-extremal

Moduli independent product of areas

$$\left\langle \frac{V_{\text{eff}}(\phi)}{b^2} - 1 \right\rangle_{\text{Region 2}} = 0$$

$$\left\langle \frac{\partial_{\phi_i} V_{\text{eff}}(\phi)}{b^2} \right\rangle_{\text{Region 2}} = 0$$

Harrison Transformation

- AdS_2 black hole lifts to BTZ in AdS_3
- Can interpret entropy in AdS_3 via Cardy
- Computation of entropy moduli indep.
- Harrison transformation maps asymptotically flat black holes to asymptotically AdS keeping same near horizon
- Can use here?
  - can map constant scalar solution to asymptotically $AdS_2 \times S^2$
  - given geometric mean law, sufficient to consider constant scalar case

Harrison Transformation

- Recall BTZ: T=0 $\longrightarrow$ $T_R = \frac{r_h}{\pi L^2}$ $T_L = 0$

- Take $a \to \Lambda(r)\, a \ , \quad b \to \frac{b}{\Lambda(r)} \ , \quad \Lambda(r) = \frac{r_+ r_-}{r}$
  - (symmetry of EOM)

$\longrightarrow$ BH in $AdS_2 \times S^2$ (scalars const) :

$$ds^2 = -\frac{(r-r_-)(r-r_+)}{\ell^2}dt^2 + \frac{\ell^2}{(r-r_+)(r-r_-)}dr^2 + \ell^2 d\Omega_2^2 \ , \qquad l^2 = V_{\text{eff}}(\phi_0)$$

- Lift to BH in $AdS_3 \times S^2$

$$ds^2 = -\frac{(r-r_-)(r-r_+)}{\ell^2}dt^2 + \frac{\ell^2}{(r-r_+)(r-r_-)}dr^2 + r^2(dy + \frac{r_+ r_-}{\ell r}dt)^2 + \ell^2 d\Omega_2^2 \ .$$

Brown & Henneaux

- Consider AdS_3
- With suitable B.C., Large diffeos generate Vir×Vir

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c_L}{12}(m^3 - m)\delta_{m+n,0}$$

$$[\overline{L}_m, \overline{L}_n] = (m-n)\overline{L}_{m+n} + \frac{c_R}{12}(m^3 - m)\delta_{m+n,0}$$

$$[L_m, \overline{L}_n] = 0$$

- Central charge is $c_L = c_R = \frac{3}{2G_3\sqrt{-\Lambda}} = \frac{3\ell}{2G_3}$
- Precursor to AdS_3/CFT_2

CTF & Area Law

- Use holographic dictionary (units: $8G_3 = 1$)

$$M = \frac{r_+^2 + r_-^2}{\ell^2} \quad \Longrightarrow \quad M\ell = L_0 + \overline{L}_0$$

$$J = \frac{2r_+ r_-}{\ell} \quad \Longrightarrow \quad J = L_0 - \overline{L}_0$$

- Partition function Z(τ) of CFT is same as partition function of gravity theory on (a quotient of) AdS_3 whose boundary geometry is same as that of torus
- This modular parameter is given by Euclidean BTZ geometry

$$Z(q, \overline{q}) = \mathrm{Tr}\, q^{L_0} \overline{q}^{\overline{L}_0}$$

$$\tau = y + it_E \;, \;\; q = e^{2\pi i \tau}$$

CTF & Area Law

- $L_0 + \overline{L}_0$ generates time translation
  - periodicity of t_E gives T

$$Z(q,\overline{q}) = \mathrm{Tr}\, q^{L_0} \overline{q}^{\overline{L}_0}$$

$$\tau = y + i t_E \ , \ q = e^{2\pi i \tau}$$

- $L_0 - \overline{L}_0$ generates space translation
  - use SL(2,**Z**) to shift Re τ

    → symmetry if $L_0 - \overline{L}_0$ quantized
- black holes of different mass at fixed quantum numbers have different temperatures
  - $L_0 + \overline{L}_0$ changes
  - $L_0 - \overline{L}_0$ stays same as vary mass (ortho. circle)
    - $L_0 - \overline{L}_0 \sim r_+ r_- = r^2_{ext} \sim A_{ext}$

Hot attractive branes

- Look at black brane Gauged SUGRA solutions in 5D
- Harrison Txn → Asymptotically Lifschitz spaces (not AdS_2)
  - Work in progress

Summary

- We have adapted the attractor mechanism for non-supersymmetric, non-extremal black holes to derive for a class of black holes: $A_+ A_- = A^2_{ext}$
  - $T=0 \rightarrow$ average over $AdS_2 \times S^2$
  - $T \neq 0 \rightarrow$ average over region between horizons
- CFT interpretation stems from constancy of $L_0 - \bar{L}_0$ as mass varies

Prospectus

- Study the geometry of solutions space – can we extract invariants?
- Can we describe non-extremal black holes in a first order formalism?
- How seriously should we take inner horizon thermodynamics?
  - Is there an underlying theory of statistical mechanics for inner horizon?
  - What degrees of freedom (if any) does the area of the inner horizon count?
- Harrison→ non-Extremal BH in of $AdS_2 X S^2$
  - Entanglement entropy?