HIGHER SPIN DUALITY:at FINITE TEMPERATURE

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Overview

- I. Construction of HS Theory from CFT
  - In the bulk with emergent AdS dimension:off
  - Off Shell, Full Nonlinearity 1/N
  - World ‘sheet’ picture: Dipole
- II. At Finite Temperature
  - Thermofield Double
  - Rindler (HS) Gravity
  - Black Hole

Higher Spin Duality

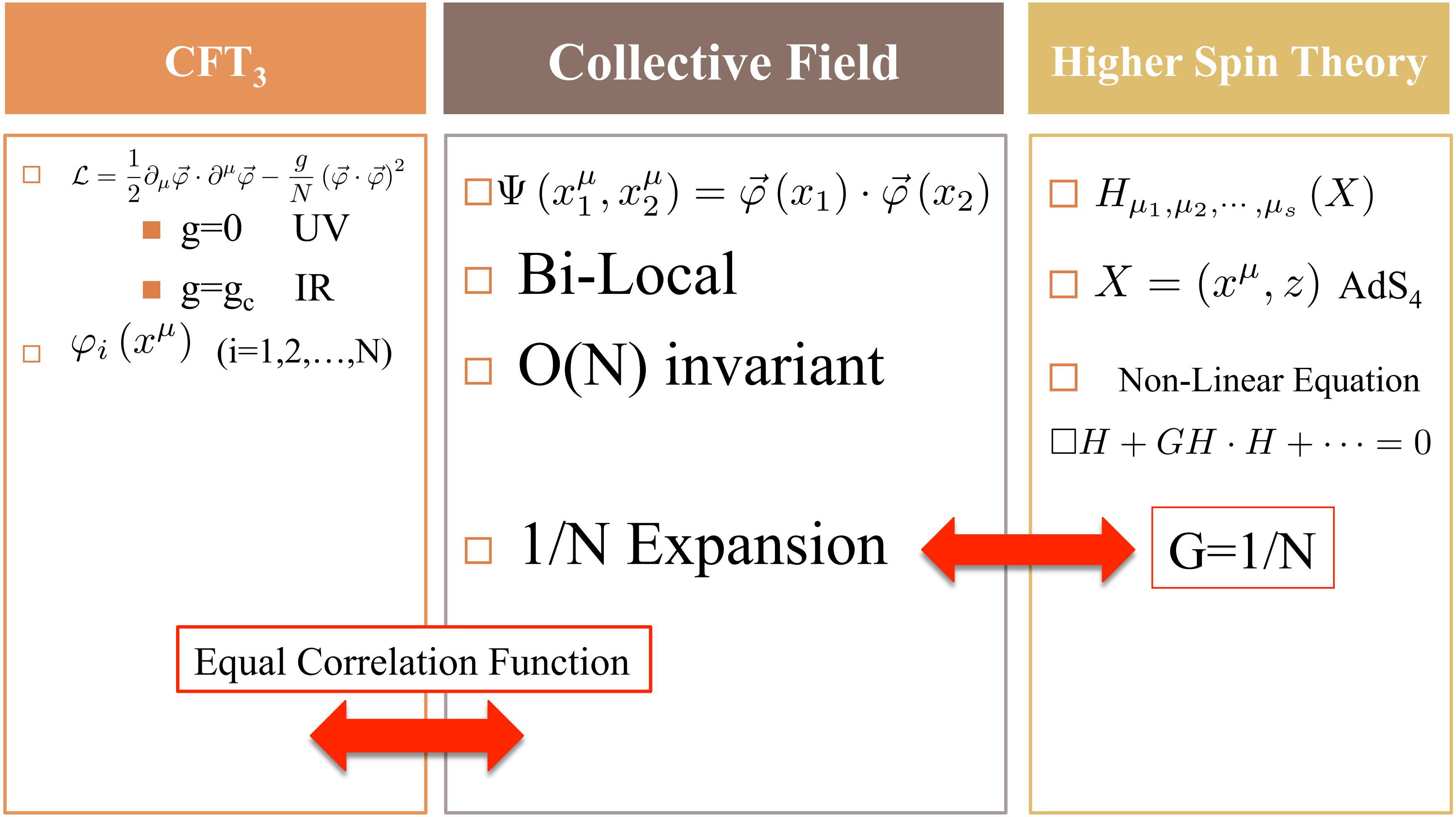
CFT3 : Vector O(N) , U(N)

- O(N), U(N) symmetry
- Boson : φ_i (i=1,…,N) [I.R.Klebanov & A.M. Polyakov,]
- Fermion : ψ_i(i=1,…,N) [E. Sezgin & P. Sundell,]
- Correlators [S. Giombi & X. Yin, 2009]

Higher Spin Theory

- s=0,1,2,3,…
- Fronsdal,Fradkin, Vasiliev (1980~1996)
- Not a String Theory
- Maybe a Sub-sector
- Consistent

Higher Spin Duality via Collective Fields



CFT$_3$

☐ $\mathcal{L} = \frac{1}{2}\partial_\mu\vec{\varphi}\cdot\partial^\mu\vec{\varphi} - \frac{g}{N}\left(\vec{\varphi}\cdot\vec{\varphi}\right)^2$

- ■ g=0 UV
- ■ g=g$_c$ IR

☐ $\varphi_i\left(x^\mu\right)$ (i=1,2,…,N)

Collective Field

☐ $\Psi\left(x_1^\mu, x_2^\mu\right) = \vec{\varphi}\left(x_1\right)\cdot\vec{\varphi}\left(x_2\right)$

☐ Bi-Local

☐ O(N) invariant

☐ 1/N Expansion

Higher Spin Theory

☐ $H_{\mu_1,\mu_2,\cdots,\mu_s}\left(X\right)$

☐ $X = \left(x^\mu, z\right)$ AdS$_4$

☐ Non-Linear Equation

$\Box H + GH\cdot H + \cdots = 0$

G=1/N

Equal Correlation Function

Vector Type Model:Collective Action

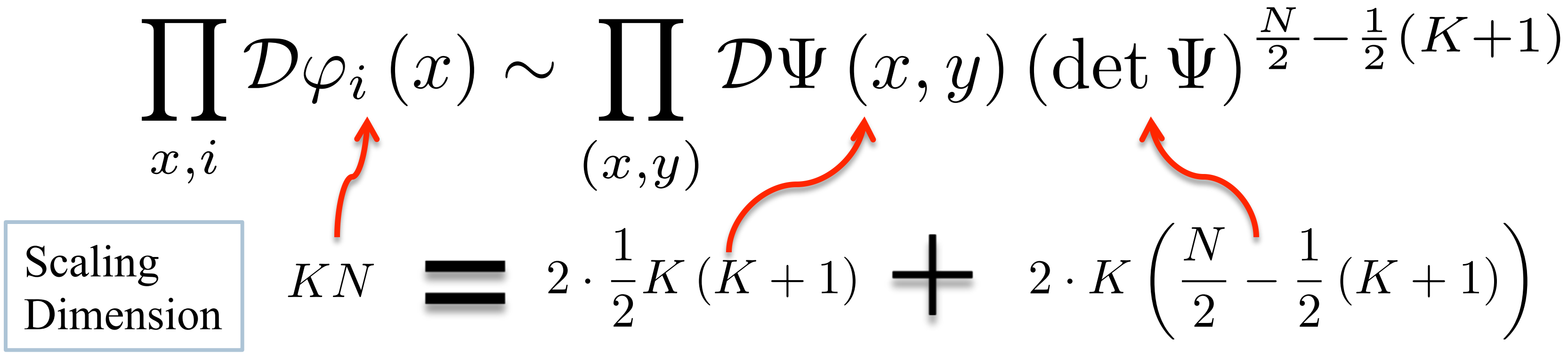
- Bi-Local field : O(N) invariant singlet

$$\sum_{i=1}^{N} \varphi_i \left(x_1^\mu\right) \varphi_i \left(x_2^\mu\right) = \Psi \left(x_1^\mu , x_2^\mu\right)$$

$$Z_{\text{col}} = \int [\prod_{(x,y)} \mathcal{D}\Psi] \underset{\substack{\uparrow \\ \text{Measure}}}{\boxed{\mu}} \exp\left(- \overbrace{\int d^3x \mathcal{L} + \underbrace{\frac{N}{2} Tr \log \Psi}_{\substack{\uparrow \\ \text{Large N Entropy}}}}^{\text{S}_{\text{col}}} \right)$$

Measure

- $\mu = (\det \Psi)^{-\kappa}$
  - $\kappa_{O(N)} = \frac{1}{2}(K+1)$: O(N) vector model
  - $\kappa_{U(N)} = K$: U(N) vector model
- Scaling Argument



$$\prod_{x,i} \mathcal{D}\varphi_i(x) \sim \prod_{(x,y)} \mathcal{D}\Psi(x,y)\,(\det\Psi)^{\frac{N}{2}-\frac{1}{2}(K+1)}$$

Scaling Dimension

$$KN = 2\cdot\frac{1}{2}K(K+1) + 2\cdot K\left(\frac{N}{2}-\frac{1}{2}(K+1)\right)$$

1/N as A Witten Expansion

- $\Psi\left(x_1, x_2\right) = \Psi_0 + \frac{1}{\sqrt{N}}\widetilde{\Psi}\left(x_1, x_2\right)$
- $S_{\text{col}} = \widetilde{\Psi}\Box_c\widetilde{\Psi} + \frac{1}{N}\left\langle V_3\right|\left|\widetilde{\Psi}\right\rangle\left|\widetilde{\Psi}\right\rangle\left|\widetilde{\Psi}\right\rangle + V_4 + \cdots$

- Recover the correlation functions

$$\left\langle\vec{\varphi}\left(x_1\right)\cdot\vec{\varphi}\left(x_1'\right)\vec{\varphi}\left(x_2\right)\cdot\vec{\varphi}\left(x_2'\right)\cdots\vec{\varphi}\left(x_n\right)\cdot\vec{\varphi}\left(x_n'\right)\right\rangle_{O(N)}$$

$$= \left\langle\widetilde{\Psi}\left(x_1, x_1'\right)\widetilde{\Psi}\left(x_2, x_2'\right)\cdots\widetilde{\Psi}\left(x_n, x_n'\right)\right\rangle_{\text{col}}$$

Strong Operator Identification (Bulk HS)

- [S. Das, AJ, 2004]

AdS4

$$\Psi\left(x_1^\mu, x_2^\mu\right) \equiv H_{\mu_1\mu_2\cdots\mu_s}\left(X\right)$$

Collective Higher Spin Theory

- Need to demonstrate

$$\left(x_1^\mu, x_2^\mu\right) \equiv X + \text{“Spin”}$$

3+3 AdS_4 Internal

☐ Issue : Gauges of HS Theory/ and Gauge Reductions

Higher Spin:Reduction to 4+2

- Fronsdal Gauge:

$$p^{\mu}k_{\mu}\left|\phi\right\rangle = 0$$: de Donder gauge $$k^{\mu}k_{\mu}\left|\phi\right\rangle = 0$$: Traceless

- To ‘Solve’ the gauge conditions: kernel

$$\left|\phi\left(x;y^0,y^1,y^2,y^3\right)\right\rangle = \mathcal{M}_1\mathcal{M}_2\mathcal{M}_3\mathcal{M}_4\left|\Phi\left(x;y^0,y^1,y^2,y^3\right)\right\rangle$$

- Constraints becomes

$$-k^0p^0\left|\Phi\left(x;y^0,y^1,y^2,y^3\right)\right\rangle = 0$$:Independent of y^0

$$\left(k^Ik_I + k^0f\left(p,k\right)\right)\left|\Phi\left(x;y^0,y^1,y^2,y^3\right)\right\rangle = 0$$: Spherical Harmonics

- Solution :

$$\left|\Phi^{sol}_{s,m}\left(x;y^1,y^2,y^3\right)\right\rangle = \Phi\left(x\right)Y_{s,m}\left(y^1,y^2,y^3\right)\left|0\right\rangle$$: AdS_4×S^2

Light Cone Map:

- Light-cone (KJin,Aj,dMello,Rodrigues, 2011 /Brodsky at al /Polchinski

$$p^+ = p_1^+ + p_2^+$$

$$p^x = p_1 + p_2$$

$$p^z = \sqrt{\frac{p_2^+}{p_1^+}}p_1 - \sqrt{\frac{p_1^+}{p_2^+}}p_2$$

$$\theta = 2\arctan\sqrt{\frac{p_2^+}{p_1^+}}$$

$$x^- = \frac{x_1^- p_1^+ + x_2^- p_2^+}{p_1^+ + p_2^+}$$

$$x = \frac{x_1 p_1^+ + x_2 p_2^+}{p_1^+ + p_2^+}$$

$$z = \frac{(x_1 - x_2)\sqrt{p_1^+ p_2^+}}{p_1^+ + p_2^+}$$

$$p^\theta = \sqrt{p_1^+ p_2^+}(x_1^- - x_2^-) + \frac{x_1 - x_2}{2}\left(\sqrt{\frac{p_2^+}{p_1^+}}p_1 + \sqrt{\frac{p_1^+}{p_2^+}}p_2\right)$$

:

Covariant Map

- Bi-local Map [AJ, RdM Koch, J P Rodrigues and J Yoon, 2014]
  - From Bi-local collective field to HS field in AdS

$$\mathcal{A}_s(\vec{x},z) = \int d^2\vec{p}dp^z d\vec{p}_1 d\vec{p}_2 \; f(\vec{x},z;\vec{p},p^z;\vec{p}_1,\vec{p}_2)$$

$$\times\delta(\sqrt{2|\vec{p}_1||\vec{p}_2| - 2\vec{p}_1\cdot\vec{p}_2} - p^z)\delta^{(2)}(\vec{p}_1 + \vec{p}_2 - \vec{p})A(\vec{p}_1,\vec{p}_2)$$

$$\boxed{\begin{aligned} \vec{p} &= \vec{p}_1 + \vec{p}_2 \\ p^z &= 2\sqrt{|\vec{p}_1||\vec{p}_2|}\,\sin\left(\frac{\varphi_1 - \varphi_2}{2}\right) \\ \theta &= \arctan\left(\frac{2\vec{p}_2 \times \vec{p}_1}{(|\vec{p}_2 - |\vec{p}_1|)p^z}\right) \end{aligned}}$$

Explicit Calculations :One Loop

- Free Energy on S^3

$$F_{\rm CFT} = N\left(\frac{1}{8}\log 2 - \frac{3\zeta(3)}{16\pi^2}\right)$$

- One-loop Free Energy of HS theory in $EAdS_4$

[S. Giombi & I. Klebanov, 2013,Giombi,Klebanov &Safdi,2014]

$$Z_{\rm bulk} = e^{-\frac{1}{G}F^{(0)} - F^{(1)} - \cdots}$$

$$F^{(1)}_{\rm bulk} = \log\left[\frac{1}{\left[\det\left(-\nabla^2-2\right)\right]^{\frac{1}{2}}}\prod_{s=1}^{\infty}\frac{\left[\det^{STT}_{s-1}\left(-\nabla^2+s^2-2\right)\right]^{\frac{1}{2}}}{\left[\det^{STT}_{s}\left(-\nabla^2+s\left(s-2\right)-2\right)\right]^{\frac{1}{2}}}\right]$$

$$= \frac{1}{8}\log 2 - \frac{3\zeta(3)}{16\pi^2}$$ (Equal to $F_{\rm CFT}$ with N=1)

One Loop:

- Collective $\Box_{\mathrm{col}} = \partial_1^2 \partial_2^2$

$$\frac{1}{2} tr \log \Box_{\mathrm{col}} \sim \frac{1}{2} tr \log \partial^2 = \frac{1}{8} \log 2 - \frac{3\zeta\left(3\right)}{16\pi^2}$$

- Conclusion [A. Jevicki, K. Jin & J. Yoon, 2014]

$$Tr \log \Box_{\mathrm{col}} = Tr \log \frac{\Box_{\mathrm{scalar}+hs}}{\Box_{gh}}$$

- Sign of Stronger Operator Identities

- Bulk $\Box_{\mathrm{col}} = \Box_{\mathrm{scalar}+hs}$

WORLD Line Duality:[R. d. M. Koch, A.J.,J. P. Rodrigues & J. Yoon,2014

- ☐ We can have an understanding of HS duality in the world line (1st quantization) framework:

- ☐ HS particle : X^A, ψ^A_i with $X^A X_A = -1$
  - [E. Majorana]
  - [F.A. Berezin & M.S. Marinov,1975]
  - [P.S. Howe, S. Penati, M. Pernici & P. Townsend,1988]
  - [S.M. Kuzenko, S.L. Lyakhovich, A.Y. Segal & A.A. Sharapov,1994]
  - Sorkin &Bandos,
  - Vasiliev hep-th/0111119
  - Barnich,Grigoriev

A=-1,0,1,2,3
i=1, 2,…, N
S=N/2 : Spin

Higher Spin Particle (in AdS$_4$)

- Constraint : Gauge Symmetry

$$L = \frac{1}{2e}\left(\dot{X}^A - i\lambda_i\psi_i^A\right)\left(\dot{X}_A - i\lambda_j\psi_{jA}\right) + \frac{i}{2}\psi_i^A\left(\psi_{iA} - f_{ij}\psi_{jA}\right)$$

1st class constraints

$$X^A P_A = 0 \qquad X^A\psi_{iA} = 0 \qquad \psi_i^A\psi_{iA} = 0$$

$$P^A P_A = 0 \qquad P^A\psi_{iA} = 0$$

All Spin HS Particle

- To generate all spins (s=0,1,2,…) enlarge the internal space:

$$\psi_{iA}\ \text{(Fermionic)} \longrightarrow Y_A\ \text{(Bosonic)}$$

- SO(2,3) :

$$J_{AB} = \underbrace{X_A P_B - X_B P_A}_{\text{AdS}_4} + \underbrace{Y_A K_B - Y_B K_A}_{\text{Internal Space}}$$

$$[X, P] = 1 \qquad [Y, K] = 1$$

$$X^2 = -1$$

Massless Representation : SO(2,3)

- Casimirs :

$$\Omega_1 = E_0^2 + s^2$$

$$\Omega_2 = E_0^2 s^2$$

$D\left(E_0, s\right)$: representation

Dilatation

Spin

- Massless condition :

$$E_0 = s + 1$$

$$\widehat{L} \approx \Omega_1 - 2\sqrt{\Omega_2} \Longrightarrow \widehat{L}\left|\quad\right\rangle = 0$$

: on-shell

Laplacian

- Lagrangian

$$L = P^A \dot{X}_A + K^A \dot{Y}_A - \lambda\left(\tau\right)\widehat{L}$$

- Plus Constraints to have:

$$\sum_s D\left(s+1, s\right)$$

HS Equations: Fronsdal

- Tensor Fields (Symmetric, traceless) :

$$-(\Box - m^2)h_{\mu_1\cdots\mu_s} + s\nabla_{(\mu_1}\nabla^{\nu}h_{\mu_2\cdots\mu_s)\nu}$$

$$-\frac{s(s-1)}{2(d+2s-3)}g_{(\mu_1\mu_2}\nabla^{\nu_1}\nabla^{\nu_2}h_{\mu_3\cdots\mu_s)\nu_1\nu_2} = 0$$

- 5-D version (A=0,1,2,3,5)

$$X^A X_A = -1$$

$$H(X,Y) = \sum_s H_{A_1 A_2 \cdots A_s}(X)\, Y^{A_1} Y^{A_2} \ldots Y^{A_s}$$

$$\frac{\partial}{\partial X}\cdot\frac{\partial}{\partial X}H(X,Y) = 0 \qquad \frac{\partial}{\partial X}\cdot\frac{\partial}{\partial Y}H(X,Y) = 0$$

$$\frac{\partial}{\partial Y}\cdot\frac{\partial}{\partial Y}H(X,Y) = 0$$

Fronsdal Gauge

- Consider the field H(X,Y)
- $$K^2 = \frac{\partial}{\partial Y} \cdot \frac{\partial}{\partial Y}$$
- $$P \cdot K = \frac{\partial}{\partial X} \cdot \frac{\partial}{\partial Y}$$
- $$P^2 = \frac{\partial}{\partial X} \cdot \frac{\partial}{\partial X} = 0$$

$$H^{\alpha}{}_{\alpha\beta\ldots} = 0$$

Traceless Condition

$$\nabla^{\alpha} H_{\alpha\beta\ldots} = 0$$

De Donder Gauge Condition

$$\left(\Box - m^2\right) \varphi_{\mu\ldots} = 0$$

Equation of motion

Fronsdals Gauge

- First Class Constraints

$$X \cdot K = 0 \qquad (1)$$

$$P \cdot K = 0 \qquad (2) \qquad K \cdot K = 0 \quad (4)$$

$$X \cdot P + Y \cdot K = 0 \qquad (3)$$

- Gauge Conditions

$$T_{-1} = X^2 - 1 = 0$$

$$T_{-2} = X \cdot Y = 0$$

- Laplacian

$$\hat{L} \rightarrow P^2$$

$$P_X^2 \, |H\rangle = 0$$

: Fronsdal HS equation

Collective Gauge

- First Class Constraints

$$U^2 = 0 \qquad V^2 = 0$$

$$U \cdot P_U = 0 \qquad V \cdot P_V = 0$$

- Dirac Cone

$$U^2 = 0 \qquad U \cdot P_U = 0$$

Equivalences of Gauges

- I. Collective :

$$U^2 = 0 \qquad V^2 = 0$$

$$U \cdot P_U = 0 \qquad V \cdot P_V = 0$$

- II. Fronsdal :

$$X \cdot P + Y \cdot K = 0 \quad X \cdot K = 0$$

$$P \cdot K = 0 \qquad K \cdot K = 0$$

Equivalences of Gauges

- Change of coordinates:

$$U = \frac{1}{2}(X+Y) \qquad V = \frac{1}{2}(X-Y)$$

$$P_U = P + K \qquad P_V = P - K$$

- Canonical transformation

$$Y \to K \qquad K \to -Y$$

PART II:FINITE TEMPERATURE

- O(N) Vector Model : Phases
- at $T_c \sim \sqrt{N}$ [S. H. Shenker & X. Yin, 2011]
  - Lower Phase :

$$1 \ll T \ll T_c$$

$$F_{\text{low}} \simeq 4\zeta(5)T^4$$

Hamiltonian Formalism
Collective Field Theory

$$F_{\text{low}} = \sum_{\text{singlet}} \log\left(1 - e^{-\beta H}\right)$$

[AJ, K. Jin, J. Yoon, 2014]

  - Higher Phase :

$$T \gg T_c$$

$$F_{\text{high}} \simeq 4\zeta(3)NT^2$$

Action Formalism
Collective Field Theory

$$F_{\text{high}} = \frac{N}{2}\text{tr}\ \log \Box$$

[AJ, K. Jin, J. Yoon, 2014]

Bi-Locals at Single Time

Two time bi-local field : $\Psi(x_1^\mu, x_2^\mu)$

- Zero Temperature
  - can be gauge fixed to single time : $x_1^0 = x_2^0$

$$\Psi(t; \underbrace{\vec{x}_1, \vec{x}_2})$$

Bi-local in space

- Finite Temperature : ① $x_1^0 = x_2^0$ ② $x_1^0 = x_2^0 + i\beta$

$$\Psi(x_1^\mu, x_2^\mu) \begin{cases} \Psi_{11}(t; \vec{x}_1, \vec{x}_2) \\ \Psi_{12}(t; \vec{x}_1, \vec{x}_2) \\ \Psi_{22}(t; \vec{x}_1, \vec{x}_2) \end{cases}$$

- Thermo-Field Double

Thermo-field Dynamics

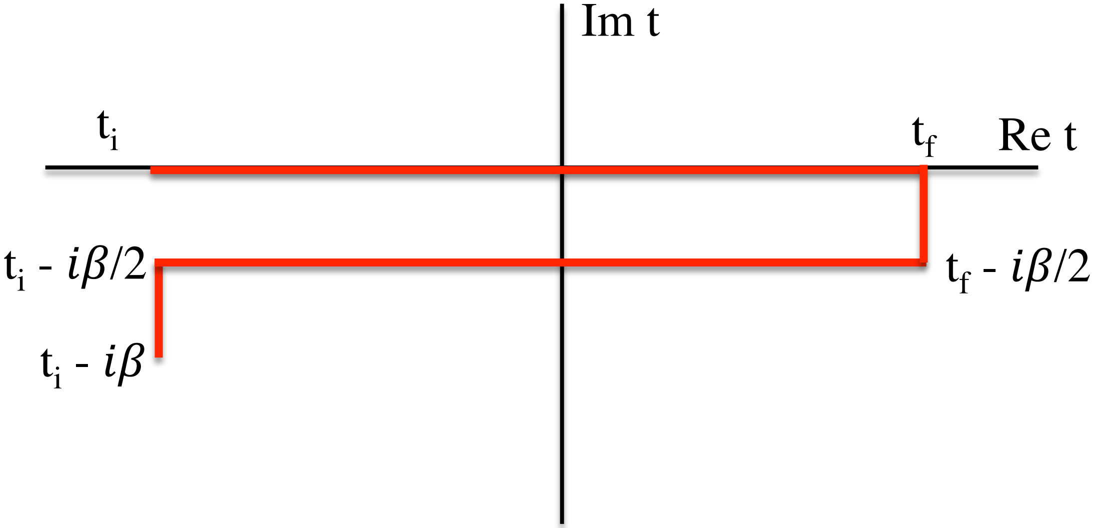
- [Y. Takahashi and H. Umezawa, 1975]
- Example : Harmonic Oscillator
- Double Hilbert space

$$H = \omega a^\dagger a$$

$$\widetilde{H} = \omega \widetilde{a}^\dagger \widetilde{a}$$

$$H_{\text{TFD}} \equiv H - \widetilde{H}$$

- Path Integral along Contour in Complex time plane



Thermo-field Dynamics

- Thermal Vacuum $|0(\beta)\rangle$
  - : Expectation value equals Finite Temperature VEV

$$|0(\beta)\rangle \equiv e^{\theta\left(a^\dagger \widetilde{a}^\dagger - a\widetilde{a}\right)}\,|0\rangle$$

  - Inverse Temperature

$$\beta = \frac{1}{\omega}\log\coth\theta$$

$$\langle \mathcal{O}\rangle_\beta \equiv \langle 0(\beta)|\,\mathcal{O}\,|0(\beta)\rangle = \frac{1}{Z(\beta)}\mathrm{Tr}\,(e^{-\beta H}\mathcal{O})$$

Thermo-field Dynamics

- Thermal Vacuum is not annihilated by a and a†

$$a\,|0(\beta)\rangle \neq 0 \qquad \widetilde{a}\,|0(\beta)\rangle \neq 0$$

- Bogoliubov Transformation

$$\begin{aligned} a_\theta &\equiv a\cosh\theta - \widetilde{a}^\dagger\sinh\theta \\ a_\theta^\dagger &\equiv a^\dagger\cosh\theta - \widetilde{a}\sinh\theta \\ \widetilde{a}_\theta &\equiv \widetilde{a}\cosh\theta - a^\dagger\sinh\theta \\ \widetilde{a}_\theta^\dagger &\equiv \widetilde{a}^\dagger\cosh\theta - a\sinh\theta \end{aligned}$$

- Annihilate the Thermal Vacuum

$$a_\theta\,|0(\beta)\rangle = \widetilde{a}_\theta\,|0(\beta)\rangle = 0$$

TFD : O(N) Vector Model

- O(N) Vector Model $H = \sum_{i=1}^{N} \int d\vec{x} \left[\frac{1}{2}(\pi^i)^2 + \frac{1}{2}(\vec{\partial}\phi^i)^2 \right]$
  - O(N) Symmetry $\phi^a \longrightarrow U^{ab}\phi^b \qquad U \in O(N)$
- TFD of O(N) Vector Model : Doubled Vector field: $\phi^a \quad \widetilde{\phi}^a$

$$H_{\rm TFD} \equiv H - \widetilde{H}$$

- O(N) Symmetry of TFD $O(N) \times O(N)$
  - $\phi^a \quad \widetilde{\phi}^a \xrightarrow{\quad ? \quad} U^{ab}\phi^b \;\; V^{ab}\widetilde{\phi}^b \qquad U, V \in O(N)$
  - $O(N)$

    $\phi^a \quad \widetilde{\phi}^a \xrightarrow{\quad ? \quad} U^{ab}\phi^b \;\; U^{ab}\widetilde{\phi}^b \qquad U \in O(N)$

O(N)×O(N) Collective TFD

- Collective TFD with $O(N) \times O(N)$
  - Invariant variables
$$\Psi(t;\vec{x},\vec{y}) = \phi^i(t,\vec{x})\phi^i(t,\vec{y})$$
$$\widetilde{\Psi}(t;\vec{x},\vec{y}) = \widetilde{\phi}^i(t,\vec{x})\widetilde{\phi}^i(t,\vec{y})$$
  - Collective TFD Hamiltonian(Doubled)
$$H_{TFD} = H_{col} - \widetilde{H}_{col}$$
  - Classical Solution is not finite temperature two-point function.
$$\Psi_0(t;\vec{x},\vec{y}) \neq \langle \varphi^i(t,\vec{x})\varphi^i(t,\vec{y})\rangle_\beta$$
  - Only applicable in Lower Phase: Thermal AdS4 background

High Temperature Phase

- Singlet Constraint : Diagonal $O(N)$ singlet

$$\phi^j \quad \widetilde{\phi}^j \quad \xrightarrow{\qquad\qquad} \quad U^{jk}\phi^k \quad U^{jk}\widetilde{\phi}^k$$

$$J^{ij} + \widetilde{J}^{ij}\,|\Phi\rangle = 0$$

- Invariant Collective Fields

$$\phi^i(t,\vec{x})\phi^i(t,\vec{y}) \qquad \phi^i(t,\vec{x})\widetilde{\phi}^i(t,\vec{y}) \qquad \widetilde{\phi}^i(t,\vec{x})\widetilde{\phi}^i(t,\vec{y})$$

- Hamiltonian of TFD

$$H_{\mathrm{TFD}} = H_{\mathrm{vec}} - \widetilde{H}_{\mathrm{vec}}$$

O(N) Collective TFD

- Collective TFD with Diagonal O(N) Symmetry

- Invariant Variables

$$\phi^i(t,\vec{x})\phi^i(t,\vec{y}) \quad \phi^i(t,\vec{x})\widetilde{\phi}^i(t,\vec{y}) \quad \widetilde{\phi}^i(t,\vec{x})\widetilde{\phi}^i(t,\vec{y})$$

- Define Collective Field

$$\Psi((\vec{x},i),(\vec{y},j)) \equiv \begin{pmatrix} \phi^a(\vec{x})\phi^a(\vec{y}) & i\phi^a(\vec{x})\widetilde{\phi}^a(\vec{y}) \\ i\widetilde{\phi}^a(\vec{x})\phi^a(\vec{y}) & -\widetilde{\phi}^a(\vec{x})\widetilde{\phi}^a(\vec{y}) \end{pmatrix}$$

- Correct Hamiltonian of TFD of O(N) Vector Model

$$H_{\text{TFD}} = \frac{2}{N}\text{Tr}\left[\Pi \star \Psi \star \Pi\right] + \frac{N}{8}\text{Tr}\left[\Psi^{-1}\right] + \frac{N}{2}\text{Tr}\left[-\nabla^2 \star \Psi\right] + \Delta V$$

RINDLER-ADS / HYPERBOLIC BH

- D=4 Rindler-AdS:

$$ds^2 = \frac{1}{\rho^2}\left[-(1-\rho^2)d\tau^2 + \frac{d\rho^2}{1-\rho^2} + \frac{dx^2 + d\sigma^2}{\sigma^2}\right]$$

- Boundary Metric is given by $\rho = 0\,, \; d\rho = 0.$

$$ds^2 = -d\tau^2 + \frac{dx^2 + d\sigma^2}{\sigma^2}$$

Hyperbolic Black Holes

- D=4 Hyperbolic BH:

$$ds^2 = -f(r)d\tau^2 + \frac{dr^2}{f(r)} + r^2\left(\frac{dx^2 + d\sigma^2}{\sigma^2}\right)$$

$$f(r) = r^2 - \frac{\mu}{r} - 1$$

- BH Temperature:

$$\beta = \frac{4\pi r_+}{3r_+^2 - 1}$$

- Horizon location r_+ is determined by $f(r_+) = 0$

Boundary:FREE O(N) RINDLER VECTOR MODEL

- From D=3 Minkowski to Rindler Spacetime:

$$S = -\frac{1}{2}\int d^3x\ \partial^\mu \phi_i \partial_\mu \phi_i$$

- Transformation to right Rindler wedge:

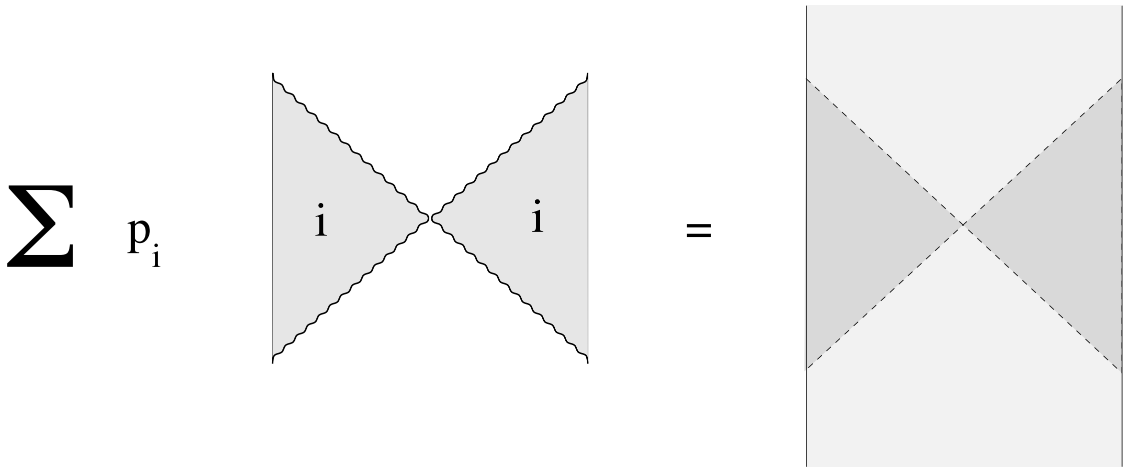
$$\begin{cases} t = \sigma \sinh \tau \,, \\ y = \sigma \cosh \tau \,. \end{cases}$$

- Rindler Metric:

$$ds^2 = -\sigma^2 d\tau^2 + d\sigma^2 + dx^2$$

DOMAIN ADS/CFT QUESTION

- Quantum Superposition of Complementary Rindler Wedges Yields Pure AdS Spacetime
  [Czech, Karczmarek, Nogueira& Raamsdonk '12]



- This Correspondence is a Special Case of the Hyperbolic BH/ Entangled Hyperbolic CFT's Duality [Emparan '99]

EVANESCENT MODES IN ADS-BH

- (d+1)-dimensional Pure AdS:

$$\frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}g^{\mu\nu}\partial_\nu\phi) - m^2\phi = 0$$

$$\mathrm{d}s^2 = \frac{-\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2 + \mathrm{d}z^2}{z^2}$$

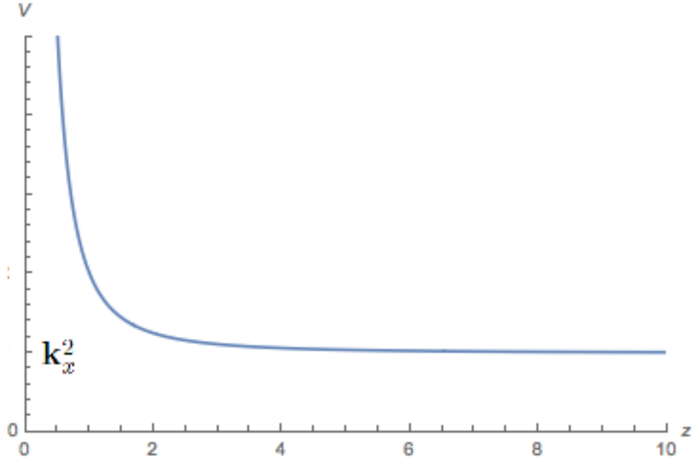
- z-Component of the Mode $\phi_{\omega\mathbf{k}_x} = u(z)z^{\frac{d-1}{2}}e^{-i\mathbf{k}_x\cdot\mathbf{x}-i\omega t}$ satisfies

$$-\frac{d^2u(z)}{dz^2} + V(z)u(z) = \omega^2 u(z)$$

$$V(z) = \mathbf{k}_x^2 + \frac{\nu^2 - 1/4}{z^2}$$

Effective Potential of Pure AdS

- The Effective Potential



- The Frequency is Bounded by $\omega^2 \geq \mathbf{k}_x^2$
- Bulk Radial Momentum $q = \sqrt{\omega^2 - \mathbf{k}_x^2}$ is real

⇨ Propagating Modes

AdS-Schwarzschild Black Hole

- (d+1)-dimensional AdS-Schwarzschild BH:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2 \mathrm{d}\Omega_{d-1}^2$$

$$f(r) = 1 + r^2 - \left(\frac{r_0}{r}\right)^{d-2}$$

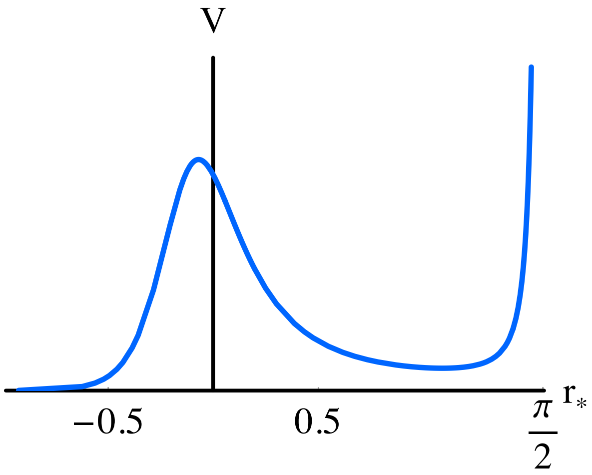
- Substituting $\phi(t,r,\Omega) = r^{-\left(\frac{d-1}{2}\right)} u(r)\, Y(\Omega)\, e^{-i\omega t}$

$$\frac{d^2u}{dr_*^2} + (\omega^2 - V(r))u = 0$$

$$V(r) = f\left[\frac{(d-1)}{2}\frac{f'}{r} + \frac{(d-1)(d-3)}{4}\frac{f}{r^2} + \frac{l(l+d-2)}{r^2} + m^2\right]$$

Effective Potential and Evanescent Mode

- The Effective Potential [Rey& Rosenhaus '14]



  - r_* is the tortoise coordinate defined by $dr_* = f^{-1} dr$
  - Near Horizon $r_* = -\infty$, $\omega \to 0$
- Bulk Radial Momentum $q = \sqrt{\omega^2 - \mathbf{k}_x^2}$ can be imaginary

⇒ Evanescent Modes

Cont.:TFD FLUCTUATIONS

- Large N, Background :

$$\frac{1}{2}\Psi_0 \star \nabla^2 \star \Psi_0 = -\frac{1}{8}\mathbb{I}$$

- Solution with one free parameter F

$$\Psi_0((\vec{x},i),(\vec{y},j)) = \int \frac{d\vec{p}}{(2\pi)^2 2|\vec{p}|} \begin{pmatrix} \cosh F(\vec{p}) e^{i\vec{p}\cdot(\vec{x}-\vec{y})} & i\sinh F(\vec{p}) e^{i\vec{p}\cdot(\vec{x}-\vec{y})} \\ i\sinh F(\vec{p}) e^{i\vec{p}\cdot(\vec{x}-\vec{y})} & -\cosh F(\vec{p}) e^{i\vec{p}\cdot(\vec{x}-\vec{y})} \end{pmatrix}$$

- Agreement with finite temperature two-point function

$$\Psi_0((\vec{x},i),(\vec{y},j)) = \langle \Psi((\vec{x},i),(\vec{y},j)) \rangle_\beta = \begin{pmatrix} \langle \phi^a(\vec{x})\phi^a(\vec{y}) \rangle_\beta & i\left\langle \phi^a(\vec{x})\widetilde{\phi}^a(\vec{y}) \right\rangle_\beta \\ i\left\langle \widetilde{\phi}^a(\vec{x})\phi^a(\vec{y}) \right\rangle_\beta & -\left\langle \widetilde{\phi}^a(\vec{x})\widetilde{\phi}^a(\vec{y}) \right\rangle_\beta \end{pmatrix}$$

$$F(\vec{p}) = 2\tanh^{-1} e^{-\beta|\vec{p}|} \quad \text{cf. In QM, } 2\theta = 2\tanh^{-1} e^{-\beta\omega}$$

Fluctuation

- Expand Collective Field around Background

$$\Psi(t;(\vec{x},i),(\vec{y},j)) = \Psi_0((\vec{x},i),(\vec{y},j)) + \frac{1}{\sqrt{N}}\eta(t;(\vec{x},i),(\vec{y},j))$$

$$\Pi(t;(\vec{x},i),(\vec{y},j)) = \sqrt{N}\pi(t;(\vec{x},i),(\vec{y},j))$$

- Quadratic Hamiltonian

$$H^{(2)} = 2\mathrm{Tr}\left[\pi\star\Psi_0\star\pi\right] + \frac{1}{8}\mathrm{Tr}\left[\Psi_0^{-1}\star\eta\star\Psi_0^{-1}\star\eta\star\Psi_0^{-1}\right]$$

O(N) Singlet Spectrum

- Diagonal Subgroup of O(N) x O(N):

- Oscillators:

$$\alpha_{RR}(\vec{p}_1,\vec{p}_2) = \sum_{i=1}^{N} b_i^R(\vec{p}_1)\, b_i^R(\vec{p}_2)$$

$$\alpha_{LL}(\vec{p}_1,\vec{p}_2) = \sum_{i=1}^{N} b_i^L(\vec{p}_1)\, b_i^L(\vec{p}_2)$$

$$\gamma_{RL}(\vec{p}_1,\vec{p}_2) = \sum_{i=1}^{N} b_i^R(\vec{p}_1)\, b_i^L(\vec{p}_2)$$

Oscillating Mode

- $\alpha^\dagger(\vec{p}_1, \vec{p}_2)$ create a mode with

$$\boxed{\begin{aligned} p^0 &= |\vec{p}_1| + |\vec{p}_2| \\ \vec{p} &= \vec{p}_1 + \vec{p}_2 \end{aligned}}$$

- $\tilde{\alpha}^\dagger(\vec{p}_1, \vec{p}_2)$ create a mode with

$$\boxed{\begin{aligned} p^0 &= -|\vec{p}_1| - |\vec{p}_2| \\ \vec{p} &= -\vec{p}_1 - \vec{p}_2 \end{aligned}}$$

$$(p^z)^2 = (p^0)^2 - \vec{p}^2$$

- On shell Condition :

$$\boxed{(p^z)^2 = (p^0)^2 - \vec{p}^2 \geqq 0}$$

for both modes

Evanescent Mode

- $\gamma^\dagger(\vec{p}_1, \vec{p}_2)$ create a mode with

$$\boxed{\begin{aligned} p^0 &= |\vec{p}_1| - |\vec{p}_2| \\ \vec{p} &= \vec{p}_1 - \vec{p}_2 \end{aligned}}$$

$$\Longrightarrow \boxed{(p^z)^2 = (p^0)^2 - \vec{p}^2 \leq 0}$$

- p^z : pure imaginary
- This mode exponentially decay along z direction
- Evanescent mode
  - [S. Leichenauer and V. Rosenhaus, 2013], [S. J. Rey and V. Rosenhaus, 2014]

BI-LOCAL MAP FOR RINDLER-ADS

- Bi-local Map from bi-local Rindler-CFT to Rindler-AdS:

$$\begin{aligned}
p^{\tau} &= \sigma\sqrt{1-\rho^2}\,\left(|\vec{p}_1| + |\vec{p}_2|\right), \\
p^{x} &= p_1^x + p_2^x\,, \\
p^{\sigma} &= \rho\sqrt{2|\vec{p}_1||\vec{p}_2| - 2\,\vec{p}_1\cdot\vec{p}_2} + \sqrt{1-\rho^2}\,(p_1^\sigma + p_2^\sigma)\,, \\
p^{\rho} &= \sigma\sqrt{2|\vec{p}_1||\vec{p}_2| - 2\,\vec{p}_1\cdot\vec{p}_2} - \frac{\sigma\rho}{\sqrt{1-\rho^2}}\,(p_1^\sigma + p_2^\sigma)\,, \\
\theta &= \arctan\left(\frac{2\,(p_2^x\,p_1^\sigma - p_2^\sigma\,p_1^x\,)}{(|\vec{p}_2| - |\vec{p}_1|)\,\left[\rho p^\sigma + \left(\frac{1-\rho^2}{\sigma}\right)p^\rho\right]}\right).
\end{aligned}$$

- Rindler-AdS on-shell condition:

$$0 = g^{\mu\nu}\partial_\mu\partial_\nu = -\frac{(p^\tau)^2}{\sigma^2(1-\rho^2)} + \left(\frac{1-\rho^2}{\sigma^2}\right)(p^\rho)^2 + (p^x)^2 + (p^\sigma)^2$$

Diagonal Modes and Propagating Modes

- Diagonal Oscillators:

$$\begin{aligned}
\alpha^\dagger_{RR}(\vec{p}_1,\vec{p}_2)\ :\quad p^\tau &= \sigma\sqrt{1-\rho^2}\Big(|\vec{p}_1|\,+\,|\vec{p}_2|\Big)\,,\\
p^x &= p_1^x\,+\,p_2^x\,,\\
p^\sigma &= \rho\sqrt{2\,|\vec{p}_1||\vec{p}_2|-2\,\vec{p}_1\cdot\vec{p}_2}\;+\;\sqrt{1-\rho^2}\,(p_1^\sigma+p_2^\sigma)\,,\\
p^\rho &= \sigma\sqrt{2\,|\vec{p}_1||\vec{p}_2|-2\,\vec{p}_1\cdot\vec{p}_2}\;-\;\frac{\sigma\rho}{\sqrt{1-\rho^2}}(p_1^\sigma+p_2^\sigma)\\
&= 2\sigma\sqrt{|\vec{p}_1||\vec{p}_2|}\,\sin\left(\frac{\varphi_1-\varphi_2}{2}\right)\;-\;\frac{\sigma\rho}{\sqrt{1-\rho^2}}(p_1^\sigma+p_2^\sigma)\,,
\end{aligned}$$

$$\begin{aligned}
\alpha^\dagger_{LL}(\vec{p}_1,\vec{p}_2)\ :\quad p^\tau &= -\,\sigma\sqrt{1-\rho^2}\Big(|\vec{p}_1|\,+\,|\vec{p}_2|\Big)\,,\\
p^x &= -\,p_1^x\,-\,p_2^x\,,\\
p^\sigma &= \rho\sqrt{2\,|\vec{p}_1||\vec{p}_2|-2\,\vec{p}_1\cdot\vec{p}_2}\;-\;\sqrt{1-\rho^2}\,(p_1^\sigma+p_2^\sigma)\,,\\
p^\rho &= \sigma\sqrt{2\,|\vec{p}_1||\vec{p}_2|-2\,\vec{p}_1\cdot\vec{p}_2}\;+\;\frac{\sigma\rho}{\sqrt{1-\rho^2}}(p_1^\sigma+p_2^\sigma)\\
&= 2\sigma\sqrt{|\vec{p}_1||\vec{p}_2|}\,\sin\left(\frac{\varphi_1-\varphi_2}{2}\right)\;+\;\frac{\sigma\rho}{\sqrt{1-\rho^2}}(p_1^\sigma+p_2^\sigma)\,.
\end{aligned}$$

Off-Diagonal Modes and Evanescent Modes

- **Off-Diagonal Oscillators:**

$$
\begin{aligned}
\gamma^{\dagger}_{RL}(\vec{p}_1, \vec{p}_2) \;:\quad p^{\tau} &= \sigma\sqrt{1-\rho^2}\Big(|\vec{p}_1| - |\vec{p}_2|\Big)\,, \\
p^{x} &= p_1^{x} - p_2^{x}\,, \\
p^{\sigma} &= \rho\sqrt{2\,\vec{p}_1\cdot\vec{p}_2 - 2\,|\vec{p}_1||\vec{p}_2|} + \sqrt{1-\rho^2}\,(p_1^{\sigma} - p_2^{\sigma})\,, \\
p^{\rho} &= \sigma\sqrt{2\,\vec{p}_1\cdot\vec{p}_2 - 2\,|\vec{p}_1||\vec{p}_2|} - \frac{\sigma\rho}{\sqrt{1-\rho^2}}(p_1^{\sigma} - p_2^{\sigma}) \\
&= \underline{2i\sigma\sqrt{|\vec{p}_1||\vec{p}_2|}\,\sin\left(\frac{\varphi_1-\varphi_2}{2}\right)} - \frac{\sigma\rho}{\sqrt{1-\rho^2}}(p_1^{\sigma} - p_2^{\sigma})\,.
\end{aligned}
$$

↑

Imaginary Value in Radial Direction Momentum

⇨ Evanescent Modes

CONCLUSION

- Thermal Gravitational Backgrounds :Characterized Presence of the Evanescent Modes

- Bi-local Construction of Rindler-AdS Spacetime from the Two Entangled O(N) Vector Model CFT's
- Indeed Produces the Evanescent Modes

- This Result Supports the Proposed More General Duality between AdS Black Hole / Two Entangled CFT's