Torsional Newton-Cartan geometry in Field Theory, Gravity and Holography

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based on work with:

Jelle Hartong and Elias Kiritsis:

1409.1519 (PLB), 1409.1522 (PRD), 1502.00228 (JHEP) & to appear

Jelle Hartong 1504.0746 (JHEP)

and

Morten Holm Christensen, Jelle Hartong, Blaise Rollier

1311.4794 (PRD) & 1311.6471 (JHEP)

Outline

- Why Newton-Cartan (NC) ? (-> non-relativistic space-time)
  - holography,
  - field theory
  - gravity
- What is NC (& its torsionful generalization TNC) geometry ?
  - NC from gauging the Bargmann algebra
- How do non-relativistic field theories couple to NC ?
- What theory of gravity does one get when making TNC dynamical ?
  - connection to Horava-Lifshitz gravity
- Outlook

Motivation (Holography)

AdS/CFT has been very successful tool in studying strongly coupled (conformal) relativistic systems

-> holography beyond original AdS-setup ?

- How general is holographic paradigm ?
  (nature of quantum gravity, black hole physics, cosmology)

- Examples of potentially holographic descriptions based on non-AdS space-times:
  Lifshitz, Schrödinger, warped AdS3 (Kerr/CFT), flat space-time.

- simplest example appears to be Lifshitz spacetimes

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{1}{r^2}\left(dr^2 + d\vec{x}^2\right)$$

characterized by anisotropic (non-relativistic) scaling between time and space

$$t \to \lambda^z t\,, \qquad \vec{x} \to \lambda \vec{x}$$

[Kachru,Liu,Mulligan]

* introduced originally to study strongly coupled systems with critical exponent z

Motivation (Holography) cont'd

- for standard AdS setup: boundary geometry is Riemannian just like the bulk geometry

- not generic: in beyond-AdS holography bdry. geometry typically non-Riemannian

Christensen,Hartong,Rollier,NO (1311)
Hartong,Kiritsis,NO (1409)

-> need new approach: prime (simplest) example to gain traction = Lifshitz
(lessons can subsequently be applied to other cases)

Result: torsional Newton-Cartan geometry is the boundary geometry found in large class of examples in EPD model

-> Lifshitz holography dual to field theories on TNC space-time

by making the resulting non-Riemannian geometry dynamical one gains access to other bulk theories of gravity (than those based on Riemannian gravity)
- apply holography (e.g. HL gravity)
- interesting in their own right

Different Holographic setups

bulk: Riemannian gravity (GR)

non-Riemmanian gravity (e.g. HL gravity)

AdS

non-AdS

non-AdS

boundary: CFT

NR-FT with scaling

NR-FT with scaling

coupling to:

Riemannian geometry

non-Riemannian

Motivation (Field Theory)

- in relativistic FT: very useful to couple to background (Riemannian) geometry
  -> compute EM tensors, study anomalies, Ward identities, etc.

- background field methods for systems with non-relativistic (NR) symmetries require NC geometry (with torsion)

  -> there is full space-time diffeomorphism invariance when coupling to the right background fields

- Recent examples

  * Son's approach to the effective field theory for the FQHE
  [Son, 2013], [Geracie, Son, Wu, Wu, 2014]

  * non-relativistic (NR) hydrodynamics [Jensen,2014]

Motivation (Gravity)

- interesting to make NC geometry dynamical
  -> "new" theories of gravity

Hartong,NO (1504)

will see: $\boxed{\text{dynamical Newton-Cartan (NC) = Horava-Lifshitz (HL) gravity}}$

⇨ natural geometric framework with full diffeomorphism invariance
& possibly non-trivial consequences for HL gravity

such theories of gravity interesting as
- other bulk theories of gravity in holographic setups
- effective theories (cosmology)

Newton-Cartan makes Galilean local

- NC geometry originally introduced by Cartan to geometrize Newtonian gravity

both Einstein's and Newton's theories of gravity admit geometrical formulations which are diffeomorphism invariant

- NC originally formulated in "metric" formulation
more recently: vielbein formulation (shows underlying sym. principle better)
Andringa,Bergshoeff,Panda,de Roo

Riemannian geometry: tangent space is Poincare invariant

Newton-Cartan geometry: tangent space is Bargmann (central ext. Gal.) invariant

- gives geometrical framework and extension to include torsion
i.e. as geometry to which non-relativistic field theories can couple
(boundary geometry in holographic setup is non-dynamical)

* will next consider dynamical (torsional) Newton-Cartan

Riemannian geometry from gauging Poincare

Poincare = Lorentz + translations (space & time)

$$[M_{ab}\,,P_c] = \eta_{ac}P_b - \eta_{bc}P_a\,,$$
$$[M_{ab}\,,M_{cd}] = \eta_{ac}M_{bd} - \eta_{ad}M_{bc} - \eta_{bc}M_{ad} + \eta_{bd}M_{ac}\,.$$

[conformal = Poincare + dilatation + special conformal]

gauge Poincare: $$\mathcal{A}_\mu = P_a e^a_\mu + \frac{1}{2}M_{ab}{\omega_\mu}^{ab}$$

vielbein spin connection

- adjoint action $\delta\mathcal{A}_\mu = \partial_\mu\Lambda + [\mathcal{A}_\mu\,,\Lambda]$ $\Lambda = P_a\zeta^a + \frac{1}{2}M_{ab}\sigma^{ab}$

- field strength-
$$\begin{aligned}\mathcal{F}_{\mu\nu} &= \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu + [\mathcal{A}_\mu\,,\mathcal{A}_\nu]\\ &= P_a{R_{\mu\nu}}^a(P) + \frac{1}{2}M_{ab}{R_{\mu\nu}}^{ab}(M)\end{aligned}$$

Riemannian geometry from gauging Poincare (cont'd)

- find set of trafos that replace local translations by diffeomorphisms

$$\bar{\delta}\mathcal{A}_\mu = \delta\mathcal{A}_\mu - \xi^\nu \mathcal{F}_{\mu\nu} = \mathcal{L}_\xi \mathcal{A}_\mu + \partial_\mu \Sigma + [\mathcal{A}_\mu \, , \Sigma]$$

with $\Lambda = \xi^\mu \mathcal{A}_\mu + \Sigma$ $\qquad$ $\Sigma = \frac{1}{2} M_{ab} \lambda^{ab}$ $\qquad$ Lorentz

-> vielbein and spin connection transform accordingly

Lorentz invariant: $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$

covariant derivative defined via vielbein postulates -> $\nabla_\rho g_{\mu\nu} = 0$

$R_{\mu\nu}{}^a(P) \sim \Gamma^\rho_{[\mu\nu]}$ $\qquad$ encodes the torsion

$R_{\mu\nu}{}^{ab}(M)$ $\qquad$ Riemann curvature two-form

- setting torsion = zero gives Riemannian geometry with Levi-Civita connection (else Riemann-Cartan geometry)

> - GR is a diff invariant theory whose tangent space invariance group is the Poincaré group
>
> * Einstein equivalence principle -> local Lorentz invariance

Relevant non-relativistic algebras

(Galilean algebra is c to infinity limit of Poincare)

$$\underbrace{\overbrace{H, P_a, J_{ab}, G_a}^{\text{Galilean}} \quad N}_{\text{Bargmann}}$$

$$[H, G_a] = P_a \qquad [P_a, G_b] = 0$$

$$[P_a, G_b] = N\delta_{ab}$$

$$\underbrace{\overbrace{H, P_a, J_{ab}, D,}^{\text{Lifshitz}} \quad G_a, N, K(z=2)}_{\text{Schrödinger}}$$

$$[D, H] = zH \qquad [D, P_a] = P_a$$

$$[D, N] = (2-z)N$$

Schrödinger = Bargmann + dilatations (+ special conformal for z=2)

Gauging the Bargmann algebra

$$\overbrace{H, P_a, J_{ab}, G_a}^{\text{Galilean}} \quad N$$

$$\underbrace{H, P_a, J_{ab}, G_a \quad N}_{\text{Bargmann}}$$

(Galilean algebra is c to infinity limit of Poincare)

$$[H, G_a] = P_a \qquad [P_a, G_b] = 0$$

$$\Downarrow$$

$$[P_a, G_b] = N\delta_{ab}$$

gauge Bargmann and impose curvature constraints

$$R_{\mu\nu}(H) = R_{\mu\nu}{}^a(P) = R_{\mu\nu}(N) = 0.$$

$$\boxed{\text{independent fields:} \quad \tau_\mu,\ e_\mu^a,\ m_\mu}$$

$$h_{\mu\nu} = e_\mu^a e_\nu^b \delta_{ab}$$

= gauge fields of Hamiltonian, spatial translations and central charge

From Bargmann to NC

Andringa,Bergshoeff,Panda,de Roo

Newtonian gravity is a diff invariant theory whose tangent space is Bargmann (make Bargmann local)

symmetry	generators	gauge field	parameters	curvatures
time translations	H	τ_μ	$\zeta(x^\nu)$	$R_{\mu\nu}(H)$
space translations	P_a	$e_\mu{}^a$	$\zeta^a(x^\nu)$	$R_{\mu\nu}{}^a(P)$
boosts	G_a	$\omega_\mu{}^a$	$\lambda^a(x^\nu)$	$R_{\mu\nu}{}^a(G)$
spatial rotations	J_{ab}	$\omega_\mu{}^{ab}$	$\lambda^{ab}(x^\nu)$	$R_{\mu\nu}{}^{ab}(J)$
central charge transf.	N	m_μ	$\sigma(x^\nu)$	$R_{\mu\nu}(N)$

curvature constraints $\quad R_{\mu\nu}(H) = R_{\mu\nu}{}^a(P) = R_{\mu\nu}(N) = 0.$

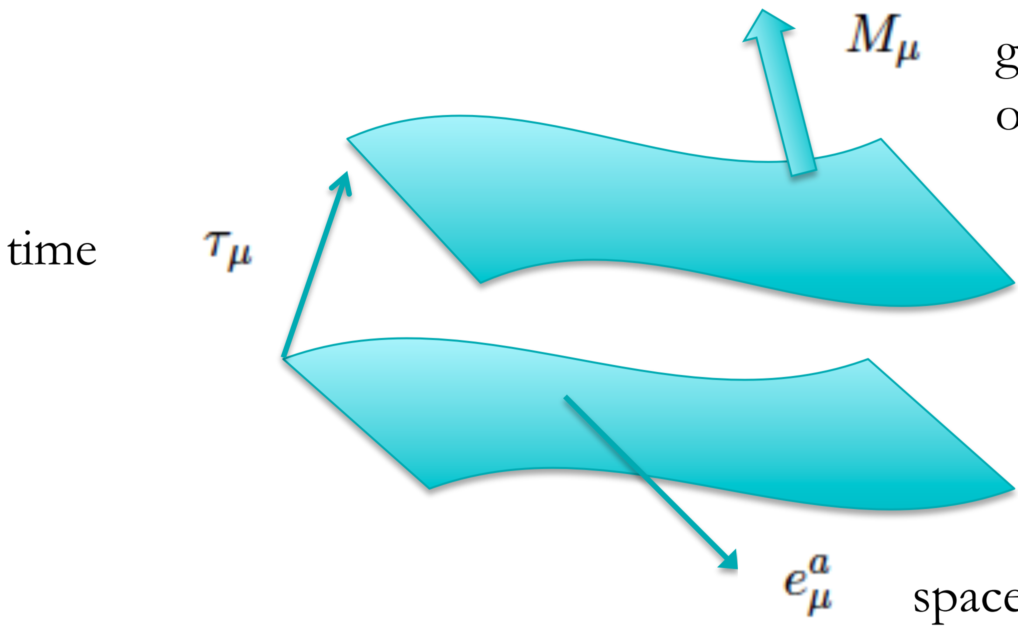
leaves as independent fields: $\quad \tau_\mu,\ e^a_\mu,\ m_\mu$

$$h_{\mu\nu} = e^a_\mu e^b_\nu \delta_{ab}$$

transforming as

$$\begin{aligned} \delta\tau_\mu &= \mathcal{L}_\xi \tau_\mu \\ \delta e^a_\mu &= \mathcal{L}_\xi e^a_\mu + \lambda^a \tau_\mu + \lambda^a{}_b e^b_\mu \\ \delta m_\mu &= \mathcal{L}_\xi m_\mu + \partial_\mu \sigma + \lambda_a e^a_\mu \end{aligned}$$

Newton-Cartan geometry



gauge field of central extension of Galilean algebra (Bargmann)

spatial metric:

$$h_{\mu\nu} = e^a_\mu e^b_\nu \delta_{ab}$$

NC geometry = no torsion $\longrightarrow$ $\tau_\mu = \partial_\mu t$ notion of absolute time

TTNC geometry = twistless torsion $\longrightarrow$ $\tau_\mu = \mathrm{HSO}$ preferred foliation in equal time slices

TNC geometry no condition on τ_μ

- in TTNC: torsion measured by $a_\mu = \mathcal{L}_{\hat{v}} \tau_\mu$
  geometry on spatial slices is Riemannian

Adding torsion to NC

Christensen,Hartong,Rollier,NO
Hartong,Kiritsis,NO/Hartong,NO
Bergshoeff,Hartong,Rosseel

- inverse vielbeins (v^μ, e^μ_a)

$$v^\mu \tau_\mu = -1\,, \qquad v^\mu e^a_\mu = 0\,, \qquad e^\mu_a \tau_\mu = 0\,, \qquad e^\mu_a e^b_\mu = \delta^b_a$$

can build Galilean boost-invariants

$$\begin{aligned}\hat{v}^\mu &= v^\mu - h^{\mu\nu} M_\nu\,,\\ \bar{h}_{\mu\nu} &= h_{\mu\nu} - \tau_\mu M_\nu - \tau_\nu M_\mu\,,\\ \tilde{\Phi} &= -v^\mu M_\mu + \tfrac{1}{2} h^{\mu\nu} M_\mu M_\nu\,,\end{aligned}$$

-introduce Stueckelberg scalar chi
(to ensure N-invariance):

$$M_\mu = m_\mu - \partial_\mu \chi.$$

affine connection of TNC (inert under G,J,N)

$$\Gamma^\rho_{\mu\nu} = -\hat{v}^\rho \partial_\mu \tau_\nu + \frac{1}{2} h^{\rho\sigma}\left(\partial_\mu \bar{h}_{\nu\sigma} + \partial_\nu \bar{h}_{\mu\sigma} - \partial_\sigma \bar{h}_{\mu\nu}\right)$$

with torsion $\Gamma^\rho_{[\mu\nu]} = -\frac{1}{2}\hat{v}^\rho(\partial_\mu \tau_\nu - \partial_\nu \tau_\mu)$

$$\nabla_\mu \tau_\nu = 0\,, \qquad \nabla_\mu h^{\nu\rho} = 0\,,$$

analogue of metric compatibility

torsion in NC (recent activity)

- NC introduced in problem of FQH — Son (1306)

- TNC first observed as bdry geometry in z=2 Lifshitz holography & generalized to large class with general z — Christensen,Hartong,Rollier,NO (1311); Hartong,Kiritsis,NO (1409)

- TTNC introduced in FQH — Geracie,Son,Wu,Wu(1407))

- TNC from gauging Schrödinger algebra — Bergshoeff,Hartong,Rosseel (1409)
- TNC from gauging Bargmann (with torsion) — Hartong,NO (1504)

- coupling of non-relativistic field theories to TNC (independent of holography) — Jensen (1408); Hartong,Kiritsis,NO (1409)

- TNC related to warped geometry that couples to 2D WCFT — Hofmann,Rollier (1411)

- other approaches — Banerjee,Mitra,Mukherjee (1407), Brauner,Endlich,Monin,Penco(1407); Bekaert,Morand (1412)

- recent activity using NC/TNC in CM (strongly-correlated electron system, FQH) — Gromov,Abanov][Moroz,Hoyos][Geracie,Son] [Wu,Wu],[Geracie,Golkar,Roberts] ,….

- (T)NC from non-rel limits — Jensen,Karch (1412) , Bergshoeff,Rosseel,Zojer (1505)

Coupling FTs to TNC

[Hartong,Kiritsis,NO]

- action functional $S = S[\hat{v}^\mu, h^{\mu\nu}, \tilde{\Phi}]\,.$

EM tensor: $T^\mu{}_\nu$

mass current T^μ

energy current (density +flux)

momentum current

$$\delta S \sim \int d^{d+1}x e[\mathcal{E}^\mu \,\delta\tau_\mu + \mathcal{P}_\mu \,h^\mu{}_\nu \delta v^\nu + \mathcal{T}_{\mu\nu}\, h^\mu{}_\rho h^\nu{}_\sigma \delta h^{\rho\sigma} + T^\mu \,\delta m_\mu]$$

spatial stress

mass density

* important to have torsion in order to describe the most general energy current !

- from the various local symmetries:

particle number conservation (if extra local U(1))

mass current= momentum current (local boosts)

symmetric spatial stress (local rotations)

Diffeomorphism and scale Ward identities

- diffeos -> on-shell WI

$$\boxed{\nabla_\nu T^\nu{}_\mu + \text{torsion terms} + \rho \nabla_\mu \tilde{\Phi} = 0}$$

* conserved currents $\partial_\nu \left(e K^\mu T^\nu{}_\mu \right) = 0\,.$

for K a TNC Killing vector:

extra force term

- if theory has scale invariance:

can use TNC analogue of dilatation connection

$$\boxed{z\mathcal{E} + \mathrm{Tr}\, T_{\text{spatial}} + 2(z-1))\rho\Phi = 0}$$

z-deformed trace WI

intermezzo: geodesics on NC space-time

- worldline action of non-rel particle of mass m on NC background

$$S = \int d\lambda L = \frac{m}{2} \int d\lambda \frac{\bar{h}_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}{\tau_\rho \dot{x}^\rho}$$

[Kuchar],
[Bergshoeff et al]

- gives the geodesic equation with NC connection $\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\rho} \frac{dx^\nu}{d\lambda} \frac{dx^\rho}{d\lambda} = 0\,,$

* reduces to Newton's law $\frac{d^2 x^i}{dt^2} + \delta^{ij} \partial_j \Phi = 0\,,$

provided we take

$$\boxed{\begin{aligned} M_t &= \partial_t M + \Phi\,, \\ M_i &= \partial_i M\,, \end{aligned}}$$

for flat NC space-time: zero Newtonian potential

$\boxed{\text{symmetries of flat NC = conformal Killing vectors (spanning Lifshitz) + extra}}$

Global symmetries for non-rel FTs on flat NC

- novel phenomenon:

notion of global symmetries depends on type of matter fields (and their couplings)

two scenarios when coupling non-rel FT to TNC background

i) theory has internal local U(1) related to particle #

ii) not

one finds for non-rel. FTs on flat NC:

-> i) mechanism that enhances Lif with:

particle # + Galilean boosts (+ special conformal)

example: Schrödinger model (+ deformations)

-> ii) no sym. enhancement (only Lif symmetry)

example: Lifshitz model

* interplay between conserved currents and space-time isometries is different compared to relativistic case: same mechanism seen in Lifshitz holography !

Dynamical Newton-Cartan geometry

so far: (T)NC geometry was non-dynamial:
- what happens when we allow it to fluctuate ?

- what is the theory of gravity that incorporates local Galilean symmetry ?

(Einstein equivalence principle, but applied to Galilean instead of Lorentz)

recently shown that: [Hartong,NO]

- dynamical NC geometry = projectable HL gravity
- dynamical TTNC geometry = non-projectable HL gravity

* Horava-Lifshitz gravity was originally introduced as non-Lorentz invariant and renormalizable UV completion of gravity
  - phenomenologically viable ?
  - interesting theoretically as alternate bulk gravity theories relevant to i) holography for strongly coupled non-relativistic systems
    ii) alternate theories in cosmology

NC/TTNC gravity

TNC geometry is a natural geometrical framework underlying HL gravity

- NC quantities combine into: $g_{\mu\nu} = -\tau_\mu \tau_\nu + \hat{h}_{\mu\nu}$
- ADM parametrization of metric used in HL gravity:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -N^2 dt^2 + \gamma_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right)$$

relation:

$$\tau_\mu \sim \text{lapse} \quad , \quad \hat{h}_{\mu\nu} \sim \text{spatial metric} \quad , \quad m_\mu \sim \text{shift} + \text{Newtonian potential}$$

some features:

- khronon field of BPS appears naturally $\tau_\mu = \psi \partial_\mu \tau$ Blas,Pujolas,Sibiryakov(2010)

  NC (no torsion): $N = N(t)$ projectable HL gravity

  TTNC: $N = N(t, x)$ non-projectable HL gravity
- U(1) extension of HMT emerges naturally as Bargmann U(1)
- new perspective (via chi field) on nature of U(1) symmetry

Horava,Melby-Thompson(2010)

Effective actions reproduce HL

- covariant building blocks:

- extrinsic curvature: $\hat{h}_{\nu\rho}\nabla_\mu \hat{v}^\rho = -K_{\mu\nu}$ spatial curvature $\check{R}_{\mu\nu\sigma}{}^\rho$.

- covariant derivative, torsion vector a_μ , inverse spatial metric $h^{\mu\nu}$

- tangent space invariant integration measure $e = \det(\tau_\mu, e_\nu^a)$

-> construct all terms that are relevant or marginal (up to dilatation weight d+z)

- in 2+1 dimensions for $1 < z \leq 2$

$$\boxed{S = \int d^3x e \left[C \left(h^{\mu\rho} h^{\nu\sigma} K_{\mu\nu} K_{\rho\sigma} - \lambda \left(h^{\mu\nu} K_{\mu\nu} \right)^2 \right) - \mathcal{V} \right]}$$

kinetic terms (2nd order)

potential:

$$-\mathcal{V} = 2\Lambda + c_1 h^{\mu\nu} a_\mu a_\nu + c_2 \mathcal{R} + \delta_{z,2} \left[c_{10} \left(h^{\mu\nu} a_\mu a_\nu \right)^2 + c_{11} h^{\mu\rho} a_\mu a_\rho \nabla_\nu \left(h^{\nu\sigma} a_\sigma \right) \right.$$
$$\left. + c_{12} \nabla_\nu \left(h^{\mu\rho} a_\rho \right) \nabla_\mu \left(h^{\nu\sigma} a_\sigma \right) + c_{13} \mathcal{R}^2 + c_{14} \mathcal{R} \nabla_\mu \left(h^{\mu\nu} a_\nu \right) + c_{15} \mathcal{R} h^{\mu\nu} a_\mu a_\nu \right]$$

Perspectives for HL gravity

new perspectives on HL:
- different vacuum (flat NC space-time): reexamine issues with HL gravity
- IR effective theory for non-relativistic field theories
- insights into non-relativistic quantum gravity corner of $(\hbar, G_N, 1/c)$ cube ?

- relevance for cosmology ?

alternate theories of gravity in cosmological scenarios, effective theories for inflation

- examine TNC gravity (general torsion)
  * relation with vector khronon of [Janiszweski,Karch]

TNC in NR hydro & fluid/gravity correspondence

- TNC of growing interest in cond-mat (str-el, mes-hall) literature

developments in Lifshitz holography can drive development of
tools to study dynamics and hydrodynamics of non-rel. systems

Lifshitz hdyro: [Hoyos,Kim,Oz]
Galilean: [Jensen]

(in parallel to progress in the last many years in relativistic fluids and
superfluids inspired from the fluid/gravity correspondence in AdS)

TNC right ingredients to start constructing effective TNC theories
and their coupling to matter (e.g. QH-effect)

- organizing principle for derivative expansion of stress tensor/mass current (transport coefficients)
- consider boosted Lifshitz black branes & perturb

[Kiritsis,Matsuo[, in progress: [Hartong,NO,Sanchioni]

Outlook

- employ similar techniques to Schrödinger, warped AdS, flat space holography

  [Andrade,Keeler,Peach,Ross,].[Hofman,Rollier][Armas,Blau,Hartong(in progress)]

- adding charge (Maxwell in the bulk)

  adding other exponents (hyperscaling, matter scaling)

  [Kiritsis,Goutereaux][Gath,Hartong,Monteiro,NO]
  [Khveshchenko][Karch][Hartnoll,Karch]

- applications to non-rel. hydrodynamics:

  fluid/gravity: black branes with zero/non-zero particle no. ? Galilean perfect fluids

  Lifsthiz: [Hoyos,Kim,Oz] Galilean: [Jensen]
  in progress: [Kiritsis,Matsuo[,[Hartong,NO,Sanchioni]

- flat space holography: gauging of Caroll group and ultra-relativistic gravity

  [Hartong]

- NC supergravity, NC in string theory [Bergshoeff et al]

- revisit HL gravity using TNC language/connections with NR String Theory

- effective TNC theories and their coupling to matter (e.g. QH-effect)

  [Son] et al

The end