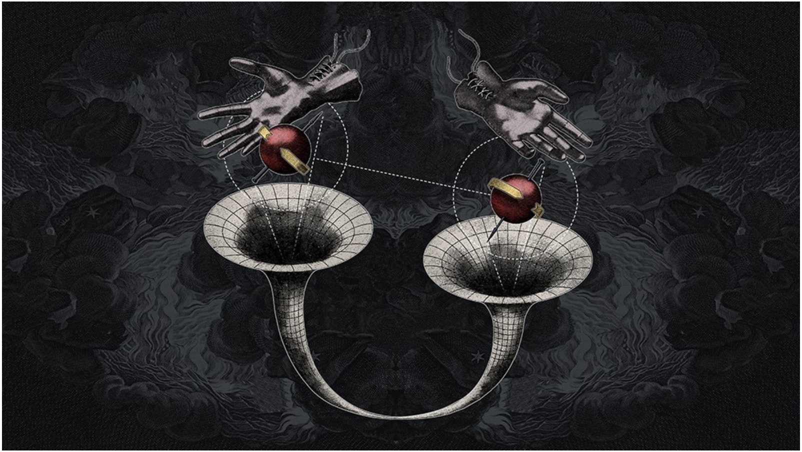
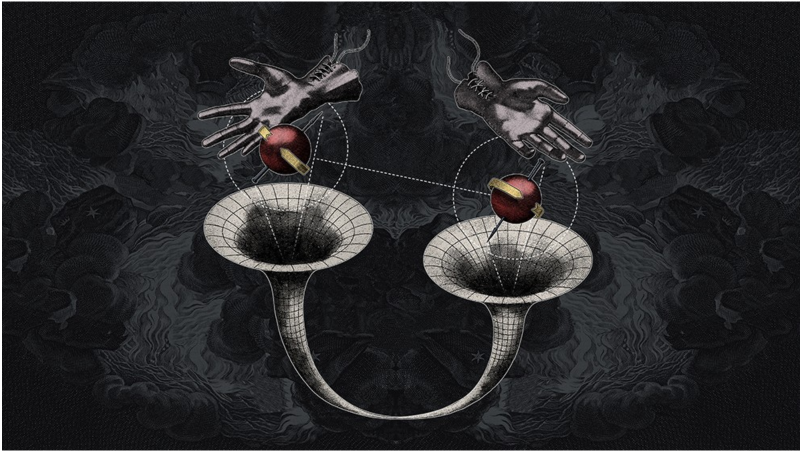


Entangled Excitations in 2d CFT

Álvaro Véliz-Osorio (Mandelstam Institute)

Kruger National Park, September 8, 2015



Entangled Excitations in 2d CFT

Álvaro Véliz-Osorio (Mandelstam Institute)

Based on: 1507.00582 with P. Caputa

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

NYT headline, May 4, 1935

What is entanglement?

" The best possible knowledge of a whole does not necessarily include the best possible knowledge of its parts."

Erwin Schrödinger

Density matrix $\rho \longrightarrow$ Von Neumann-Shannon entropy

$$S(\rho) = -\mathrm{Tr}\,(\rho \log \rho)$$

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$$S(\rho) = -\mathrm{Tr}\,(\rho \log\, \rho) = 0 \quad \text{Pure state}$$

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Quantifying entanglement

- Separate Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
- Reduced density matrix $\rho_A = \mathrm{Tr}_{\mathcal{H}_B}\rho$
- Entanglement entropy $S_A = -\mathrm{Tr}\,(\rho_A \log\, \rho_A)$

If $S_A \neq 0$ then subsystems A and B are entangled

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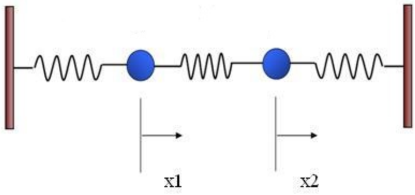
A state might appear mixed if we cannot access the full system

A simple example, coupled harmonic oscillators



Hamiltonian

$$H = \frac{1}{2}\left[p_1^2 + p_2^2 + \kappa_0\left(x_1^2 + x_2^2\right) + \kappa_1\left(x_1 - x_2\right)^2\right]$$

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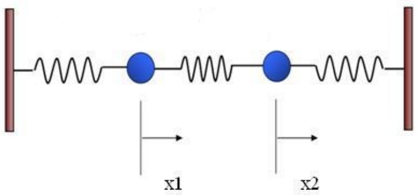

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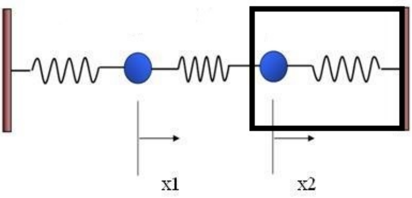
Ground state wave function

$$\psi_0\left(x_1, x_2\right) = \left(\frac{\omega_+\omega_-}{\pi^2}\right)^{1/4} \exp\left[-\frac{\left(\omega_+ x_+^2 + \omega_- x_-^2\right)}{2}\right]$$

$$x_\pm = \frac{(x_1 \pm x_2)}{2} \qquad \omega_+ = \sqrt{\kappa_0} \qquad \omega_- = \sqrt{\kappa_0 + 2\kappa_1}$$

A simple example, coupled harmonic oscillators



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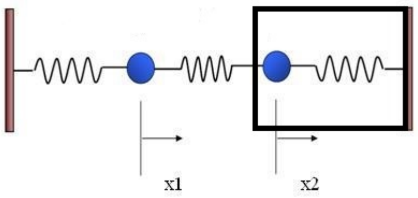
Reduced density matrix

$$\rho_2(x_2, \tilde{x}_2) = \int_{-\infty}^{\infty} \psi_0(x_1, x_2)\,\psi_0^*(x_1, \tilde{x}_2)$$

A simple example, coupled harmonic oscillators



Hamiltonian

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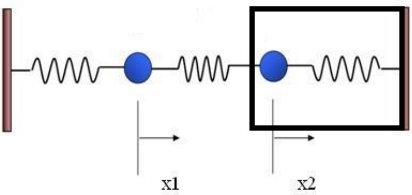
Reduced density matrix

$$\boxed{\rho_2(x_2, x_2') = \sqrt{\frac{\gamma - \beta}{\pi}}\exp\left[-\frac{\gamma(x_2^2 + \tilde{x}_2^2)}{2} + \beta x_2 \tilde{x}_2\right]}$$

$$\beta = \frac{(\omega_+ - \omega_-)^2}{4\left(\omega_+ + \omega_-\right)} \qquad \gamma - \beta = \frac{2\omega_+\omega_-}{\omega_+ + \omega_-}$$

A simple example, coupled harmonic oscillators



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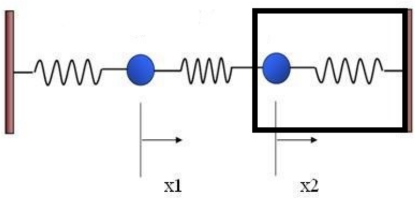
Entanglement entropy

$$\boxed{S_2 = -\log\left(1-\xi\right) - \frac{\xi}{1-\xi}\log(\xi)}$$

$$\alpha = \sqrt{\omega_+\omega_-} \qquad \xi = \frac{\beta}{\gamma+\alpha}$$

A simple example, coupled harmonic oscillators



Hamiltonian

$$H = \frac{1}{2}\left[p_1^2 + p_2^2 + \kappa_0\left(x_1^2 + x_2^2\right) + \kappa_1\left(x_1 - x_2\right)^2\right]$$

Entanglement entropy equals thermal entropy for a single oscillator

$$S_2 = S_{th}(T) = \log Z(T) + \frac{\langle E \rangle}{T}$$

$$T = T(\kappa_0, \kappa_1) = -\frac{\alpha}{\log \xi}$$

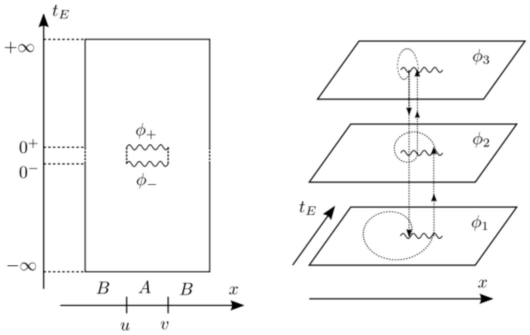
Srednicki '93

Entanglement entropy for CFT_2

Rényi entropy and entaglement entropy

$$S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr} \rho_A^n \qquad S_A = -\lim_{n \to 1} \frac{\partial}{\partial n} S_A^{(n)}$$

Replica trick



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Vacuum EE for a single interval A with length L

$$S_A^{(n)} = \frac{c}{6}\left(1+\frac{1}{n}\right)\log\left(\frac{L}{a}\right) + \dots \qquad S_A = \frac{c}{3}\log\left(\frac{L}{a}\right) + \dots$$

Holzhey, Larsen, Wilczek '94; Calabrese, Cardy '04

At finite temperature

$$S_A(\beta) = \frac{c}{3}\log\left(\frac{\beta}{\pi a}\sinh\left(\frac{\pi L}{\beta}\right)\right) + \dots$$

Calabrese, Cardy '04

Locally excited states

Physical setup:

- Start with the ground state of a CFT_2.
- At $t = 0$ insert an operator $\mathcal{O}(x, t)$ at $x = -l$.
- Compute $\Delta S_A^{(n)}(t)$ wrt to the vacuum.

Density matrix

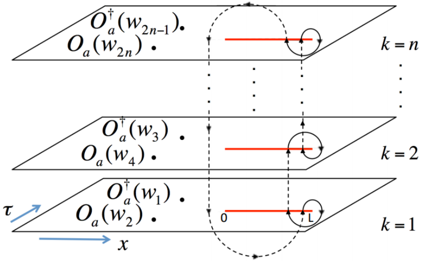
$$\begin{aligned}
\rho(t, l, L, \epsilon) &= \mathcal{N} \cdot e^{-iHt} e^{-\epsilon H} \mathcal{O}(0, -l)|0\rangle\langle 0|\mathcal{O}^\dagger(0, -l) e^{-\epsilon H} e^{iHt} \\
&\equiv \mathcal{N} \cdot \mathcal{O}(w_2, \bar{w}_2)|0\rangle\langle 0|\mathcal{O}^\dagger(w_1, \bar{w}_1)
\end{aligned}$$

Where $\epsilon \ll 1$, $\mathcal{N}$ is such that $\mathrm{Tr}\,\rho = 1$, and

$$\begin{aligned}
w_1 &= i(\epsilon - it) - l, \quad w_2 = -i(\epsilon + it) - l \\
\bar{w}_1 &= -i(\epsilon - it) - l, \quad \bar{w}_2 = i(\epsilon + it) - l
\end{aligned}$$

Locally excited states

Modified replica trick



Jump of Rényi's

$$\Delta S_A^{(n)} \equiv \frac{1}{1-n} \log \left[\frac{\langle \mathcal{O}(w_1,\bar{w}_1)\mathcal{O}^{\dagger}(w_2,\bar{w}_2)\cdots\mathcal{O}(w_{2n-1},\bar{w}_{2n-1})\mathcal{O}^{\dagger}(w_{2n},\bar{w}_{2n})\rangle_{\Sigma_n}}{\left(\langle \mathcal{O}^{\dagger}(w_1,\bar{w}_1)\mathcal{O}(w_2,\bar{w}_2)\rangle_{\Sigma_1}\right)^n} \right]$$

Nozaki, Numasawa, Takayanagi '14

Change in *purity* $\Delta S_A^{(2)}$

Uniformization

$$z_i^n = \frac{w_i}{w_i - L} \qquad\qquad \Sigma_n \to \Sigma_1$$

Four-point function $\longrightarrow$ cross ratio $z = \frac{z_{12}z_{34}}{z_{13}z_{24}}$

- Case 1: $t \in [l, L+l]$

$$z \simeq 1 - \frac{L^2\epsilon^2}{4(l-t)^2(L+l-t)^2}, \qquad \bar{z} \simeq \frac{L^2\epsilon^2}{4(l+t)^2(L+l+t)^2}$$

- Case 2: $t \notin [l, L+l]$

$$z \simeq \frac{L^2\epsilon^2}{4(l-t)^2(L+l-t)^2}, \qquad \bar{z} \simeq \frac{L^2\epsilon^2}{4(l+t)^2(L+l+t)^2}$$

Change in *purity* $\Delta S_A^{(2)}$ (primary operators)

Let $\mathcal{O}(z,\bar{z})$ be a primary operator

$$\Delta S_A^{(2)} \simeq \begin{cases} -\log \mathcal{F}_{\mathcal{O}}(0,0) & t \notin [l, L+l] \\ -\log \mathcal{F}_{\mathcal{O}}(1,0) & t \in [l, L+l] \end{cases}$$

Where

$$\langle \mathcal{O}_\alpha | \mathcal{O}_\alpha(z,\bar{z}) \mathcal{O}_\alpha(1,1) | \mathcal{O}_\alpha \rangle \equiv |z(1-z)|^{-4h} \mathcal{F}_\alpha(z,\bar{z}) \,.$$

As $z \to 0$ the identity dominates so $\mathcal{F}_{\mathcal{O}}(0,0) = 1$

He, Numasawa, Takayanagi, Watanabe '14

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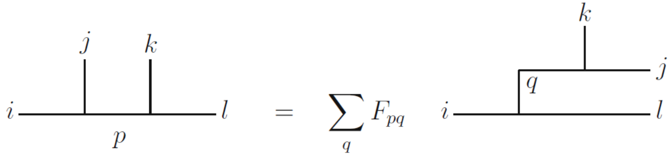
For $z \to 1$ take $z \to 1-z$, which corresponds to



He, Numasawa, Takayanagi, Watanabe '14

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$F_{00}[\mathcal{O}]$ is the inverse of the quantum dimension $d_{\mathcal{O}}$

Moore, Seiberg '88

$$\Delta S_A^{(2)} \simeq \begin{cases} 0 & t \notin [l, L+l] \\ \log d_{\mathcal{O}} & t \in [l, L+l] \end{cases}$$

He, Numasawa, Takayanagi, Watanabe '14

Quantum dimension RCFT

Given an excitation with charge a

$$d_a = \lim_{\tau \to 0} \frac{\chi_a(\tau)}{\chi_0(\tau)} \qquad \chi_a(\tau) = \mathrm{Tr}_{\mathcal{H}_a}\left(e^{i\pi\tau L_0}\right)$$

Relationship with the fusion transformation

$$\chi_a(-1/\tau) = \sum_b S_a^b \chi_b(\tau) \quad \Longrightarrow \quad d_a = \frac{S_0^a}{S_0^0}$$

Relevant for the study of (2+1) d TQFT

- Asymptotic dimension (anyons)

$$\dim \mathcal{H}_a(N) \sim d_a^N$$

- Topological entanglement entropy (Kitaev, Preskill '05)

$$S_A = \alpha L - \frac{1}{2} \log\left(\sum_a d_a\right) + \dots$$

Change in *purity* $\Delta S_A^{(2)}$ (primary operators)

Can we find a physical picture or this result?

Compute the energy density

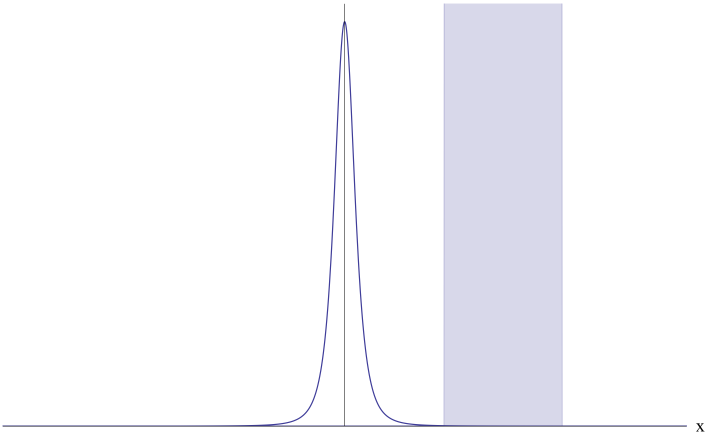
$$T_{tt}(x,\bar{x}) = -(T(x) + \bar{T}(\bar{x}))$$

In the presence of the excited state

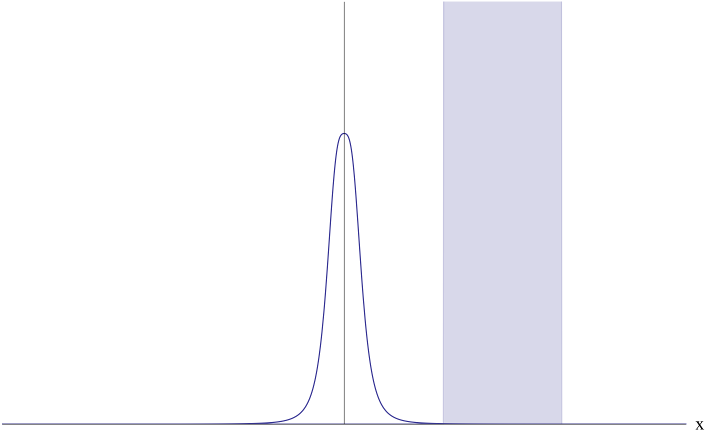
$$\begin{aligned}\langle T_{tt}(x,x)\rangle_{\mathcal{O}_\alpha} &\equiv \frac{\langle \mathcal{O}_\alpha^\dagger(w_2,\bar{w}_2)T_{tt}(x,x)\mathcal{O}_\alpha(w_1,\bar{w}_1)\rangle}{\langle \mathcal{O}_\alpha(w_1,\bar{w}_1)\mathcal{O}_\alpha^\dagger(w_2,\bar{w}_2)\rangle} \\ &= \frac{4h\,\epsilon^2}{((x+l-t)^2+\epsilon^2)^2} + \frac{4\bar{h}\,\epsilon^2}{((x+l+t)^2+\epsilon^2)^2}\end{aligned}$$

How does this quantity evolve in time?

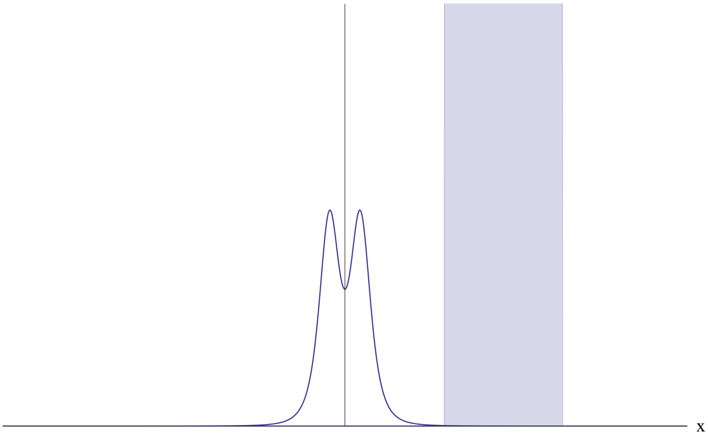
Energy time = 0



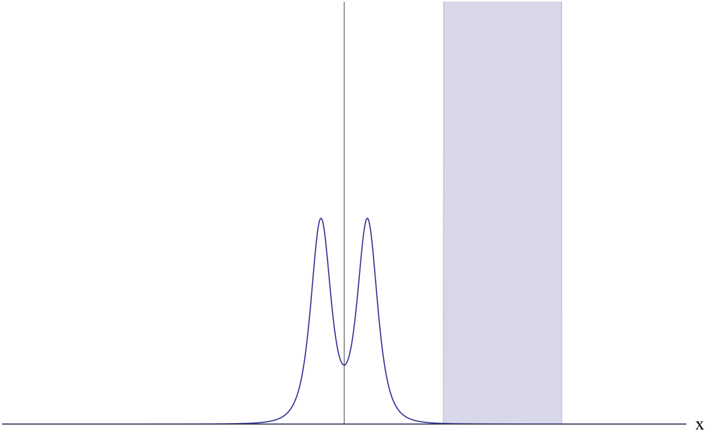
Energy time = 0.25



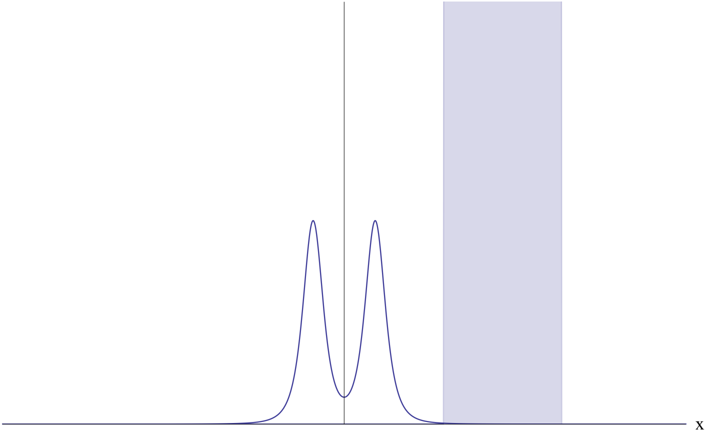
Energy time = 0.5



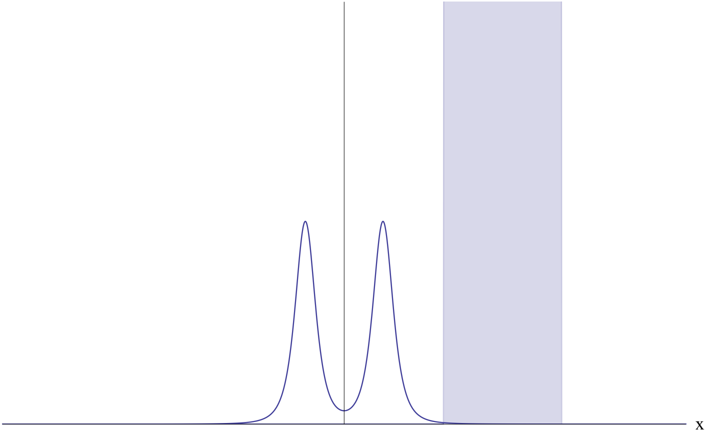
Energy time = 0.75

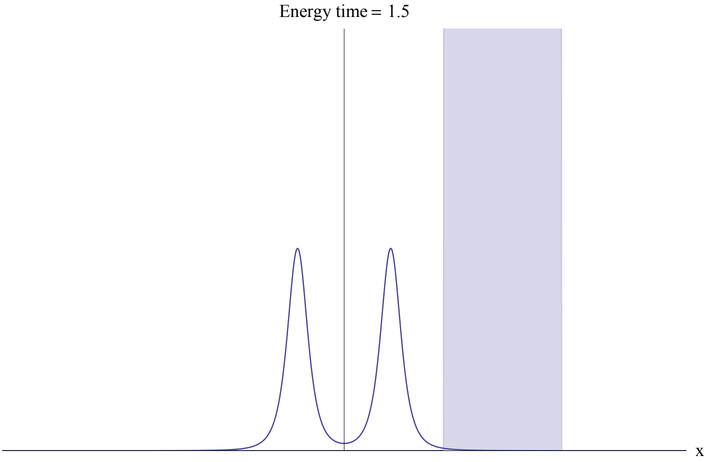


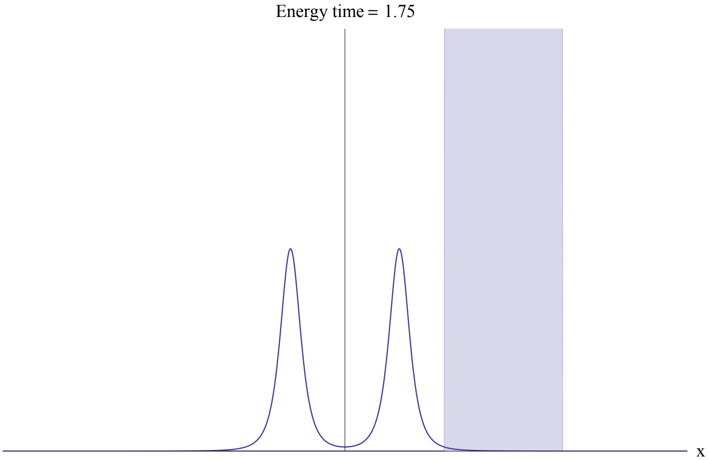
Energy time = 1



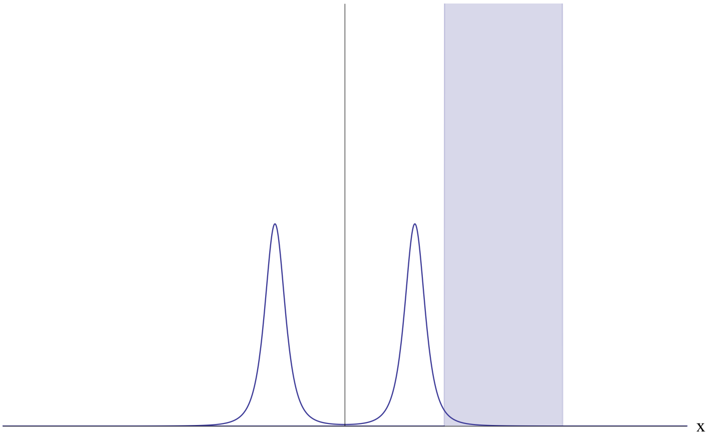
Energy time = 1.25



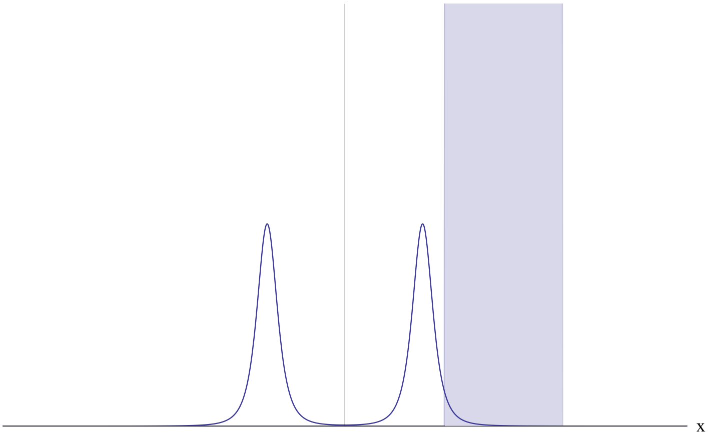

Energy time = 1.5
x


Energy time = 1.75
x

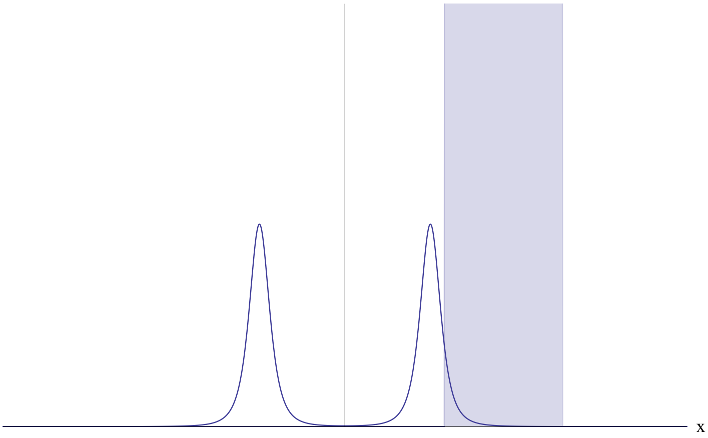
Energy time = 2.25



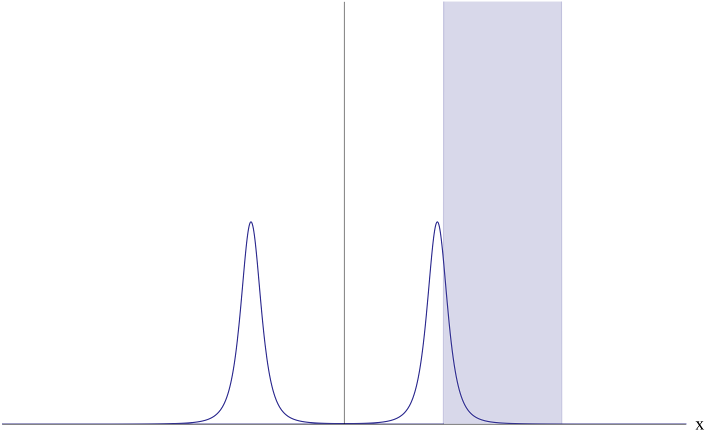
Energy time = 2.5



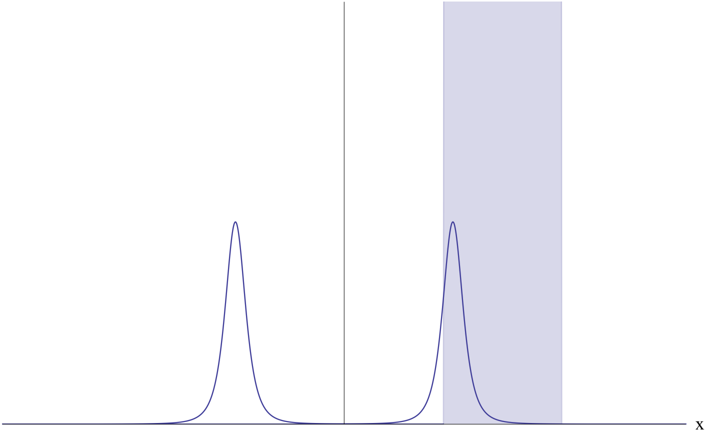
Energy time = 2.75



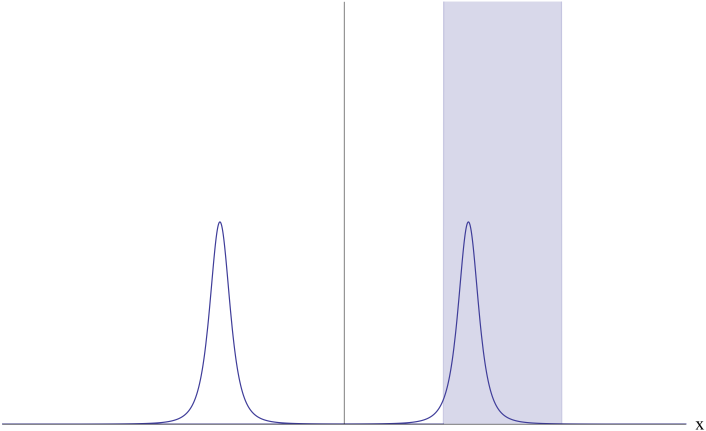
Energy time = 3



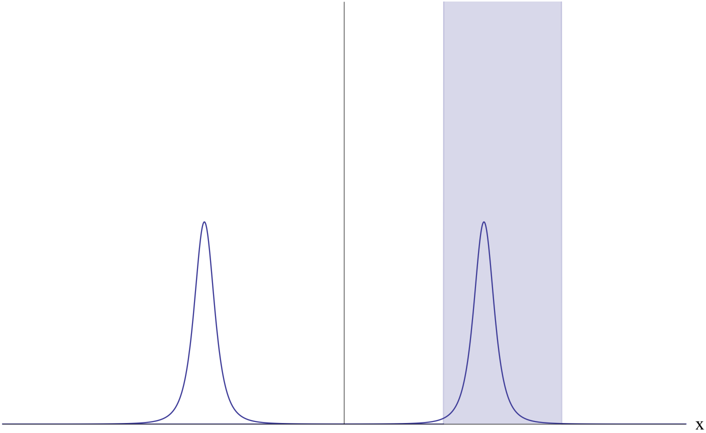
Energy time = 3.5



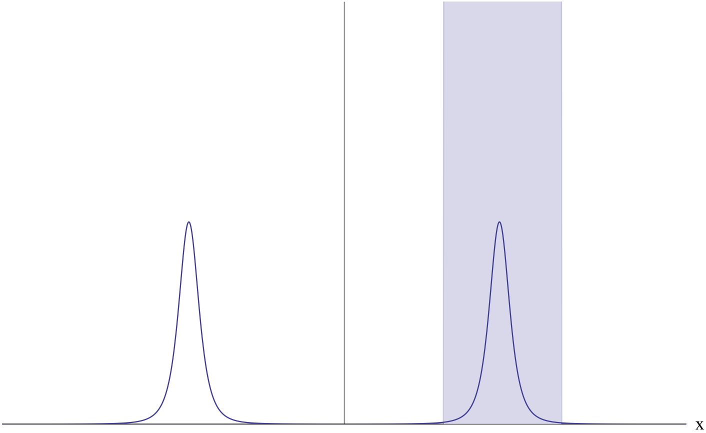
Energy time = 4

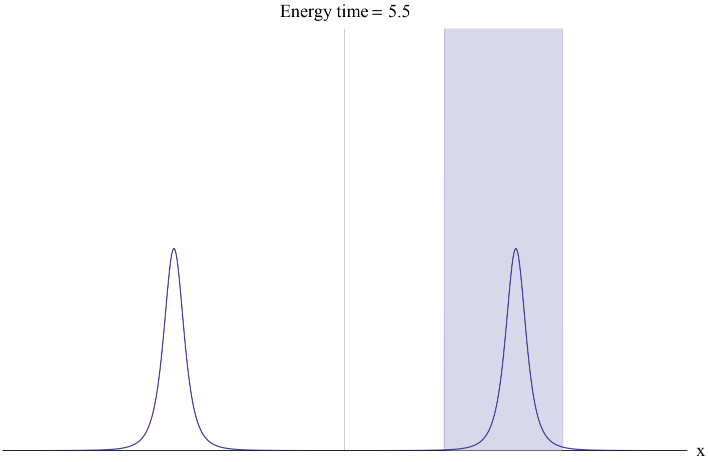


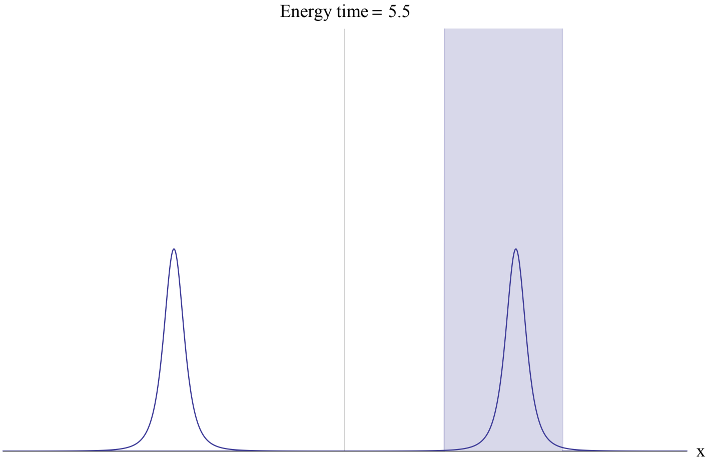
Energy time = 4.5



Energy time = 5

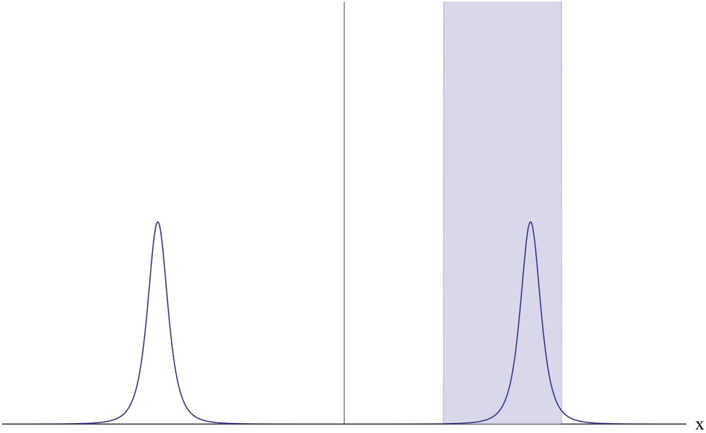


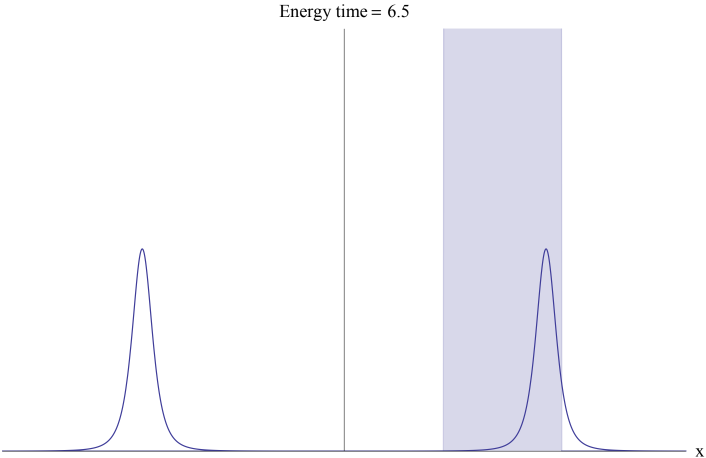

Energy time = 5.5
x



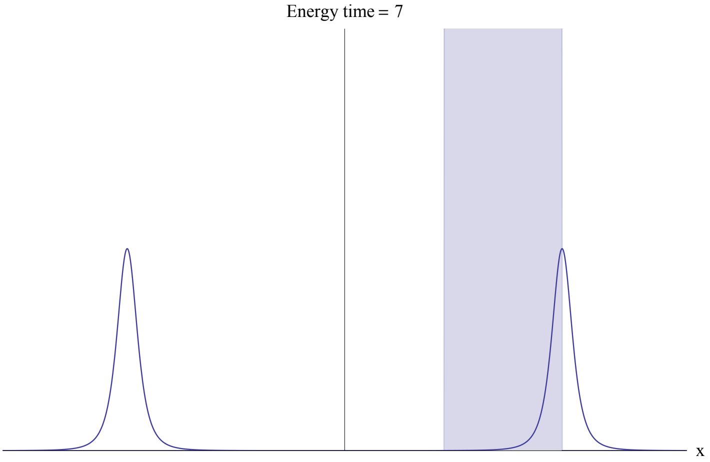
Energy time = 5.5
x

Energy time = 6

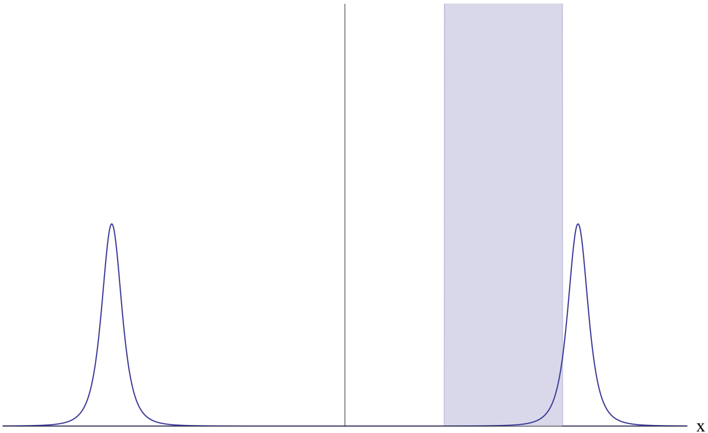




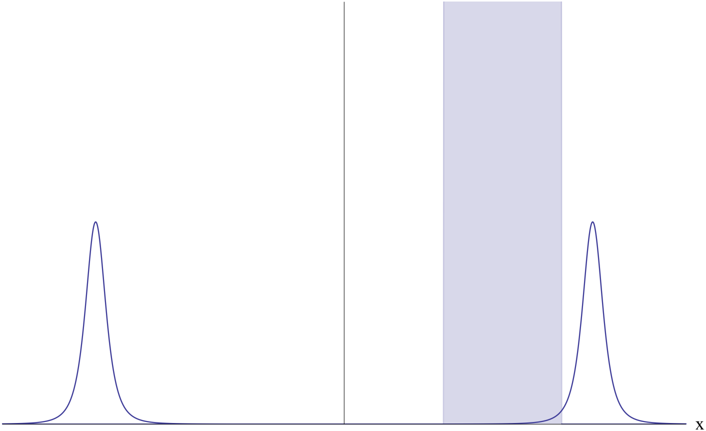
Energy time = 6.5
x


Energy time = 7
x

Energy time = 7.5



Energy time = 8



EPR primary

Primary operator $h = \bar{h} = \alpha^2$

$$\mathcal{O}_\alpha = \sqrt{a}\, e^{i\sqrt{2}\alpha\phi} \pm \sqrt{1-a}\, e^{-i\sqrt{2}\alpha\phi}$$

Left-right decomposition

$$\begin{aligned} \mathcal{O}_\alpha \left|0\right\rangle_L \otimes \left|0\right\rangle_R &= \sqrt{a} \left| e^{i\sqrt{2}\alpha\phi_L} \right\rangle_L \otimes \left| e^{i\sqrt{2}\alpha\phi_R} \right\rangle_R \\ &\pm \sqrt{1-a} \left| e^{-i\sqrt{2}\alpha\phi_L} \right\rangle_L \otimes \left| e^{-i\sqrt{2}\alpha\phi_R} \right\rangle_R \end{aligned}$$

Reduced (left) density matrix

$$\rho_L = \mathrm{diag}\{a, 1-a\}$$

Rényi entropies

$$S_L^{(n)} = \frac{1}{1-n} \log \mathrm{Tr}\rho_L^n = \frac{1}{1-n} \log\left[a^n + (1-a)^n\right]$$

EPR primary

Replica method ($n = 2$) matches

$$\Delta S_A^{(2)} \simeq \begin{cases} 0 & t \notin [l, L+l] \\ -\log\left[a^2 + (1-a)^2\right] & t \in [l, L+l] \end{cases}$$

Other measures

- Hartley entropy (max-entropy)

$$S_L^{(0)} = \log(2)$$

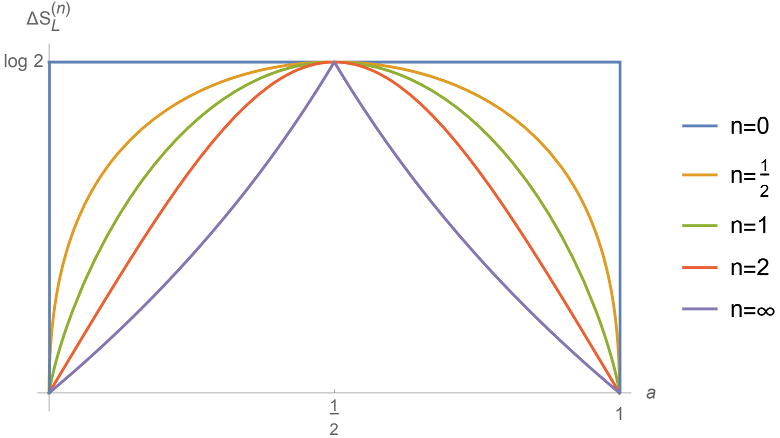
- Entanglement entropy

$$S_L^{(1)} = -a\log(a) - (1-a)\log(1-a)$$

- Min-entropy

$$S_L^{(\infty)} = -\log\max\left[a, 1-a\right].$$

EPR primary



Rényi's are equal for $a = \frac{1}{2}$, expected for RCFT (Ising model)

Change of purity $\Delta S_A^{(2)}$ (descendants)

Is this a property of conformal families?

Start with a famous descendant $T(z) = \hat{L}_{-2}\mathbb{I}$

$$\exp\left(-\Delta S_A^{(2)}(z)\right) = \frac{\langle T(w_1)T(w_2)T(w_3)T(w_4)\rangle_{\Sigma_2}}{\left(\langle T(w_1)T(w_2)\rangle_{\Sigma_1}\right)^2}$$

$\exp\left(-\Delta S_A^{(2)}\right)$ is given by

$$\frac{4}{c^2}\left[\mathcal{F}_T(z) + z^4(1-z)^4\left(\frac{c^4}{16} + \frac{c^3}{4}\frac{[1-z(1-z)]^2}{z^2(1-z)^2} - \frac{2c^2}{z(1-z)}\right)\right]$$

Recall

$$\mathcal{G}_T(z,\bar{z}) = (z(1-z))^{-4}\mathcal{F}_T(z,\bar{z})\,. \tag{1}$$

Change of purity $\Delta S_A^{(2)}$ (descendants)

In the limit $\epsilon \to 0$

$$\exp\left(-\Delta S_A^{(2)}\right) \simeq \begin{cases} \frac{4}{c^2}\mathcal{F}_T(0) & t \notin [l, l+L] \\ \frac{4}{c^2}\mathcal{F}_T(1) & t \in [l, l+L] \end{cases}$$

Where

$$\mathcal{F}_T(z) = \frac{c^2}{4}\left[z^4(1-z)^4 + (1-z)^4 + z^4\right] + 2c\, z^2(1-z)^2\left[1 - z(1-z)\right]$$

We thus find

$$\mathcal{F}(z \to 0) = \mathcal{F}(z \to 1) = \frac{c^2}{4} \quad \longrightarrow \quad \Delta S_A^{(2)} = 0$$

Same as the jump for the identity (Is this a coincidence?)

Change of purity $\Delta S_A^{(2)}$ (descendants)

Consider the descendant

$$\partial\mathcal{O}(z,\bar{z}) = L_{-1}\mathcal{O}(z,\bar{z}) = \partial_z\mathcal{O}(z,\bar{z})$$

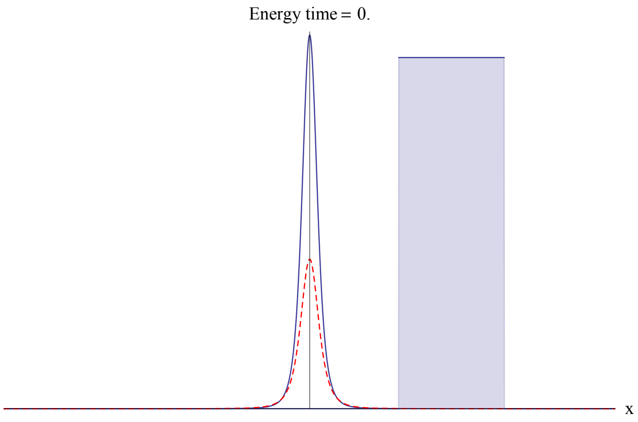
OPE for descendants

$$T(x)\partial\mathcal{O}(z_i,\bar{z}_i) \sim \frac{2h\mathcal{O}(z_i,\bar{z}_i)}{(x-z_i)^3} + \frac{(h+1)\partial_{z_i}\mathcal{O}(z_i,\bar{z}_i)}{(x-z_i)^2} + \frac{\partial_{z_i}^2\mathcal{O}(z_i,\bar{z}_i)}{x-z_i}$$

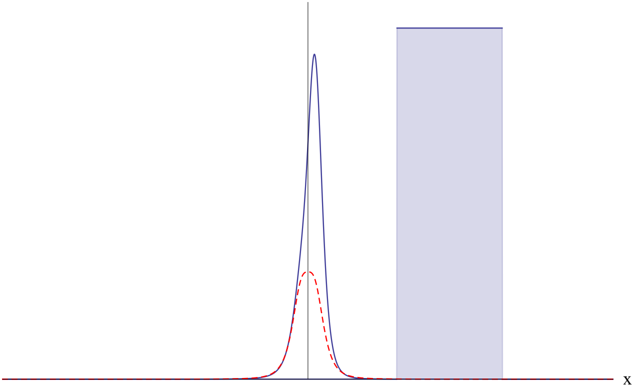
Energy density

$$\langle T_{tt}(x,x)\rangle_{\partial O} = \langle T_{tt}(x,x)\rangle_{O} + \frac{4\epsilon^2\left[(1-4h)(x+l-t)^2+(1+4h)\epsilon^2\right]}{(2h+1)\left[(x+l-t)^2+\epsilon^2\right]^3}$$

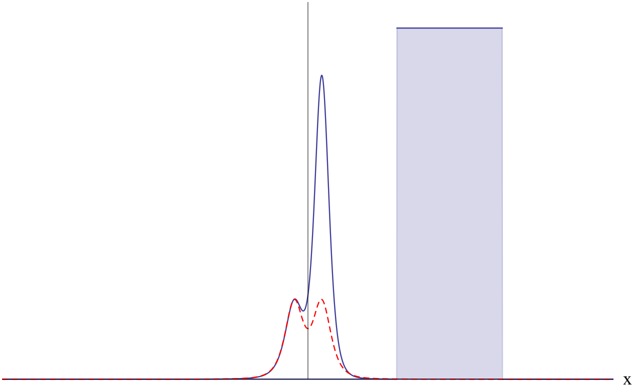
How does this quantity evolve in time?


Energy time = 0.
x

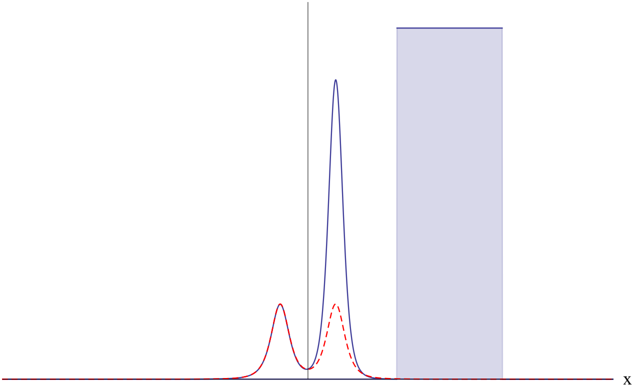
Energy time = 0.25

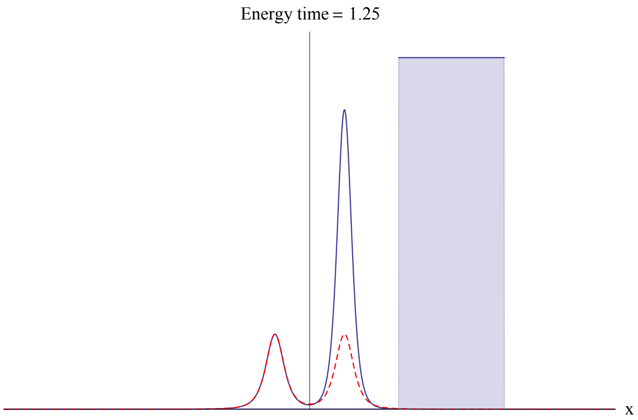


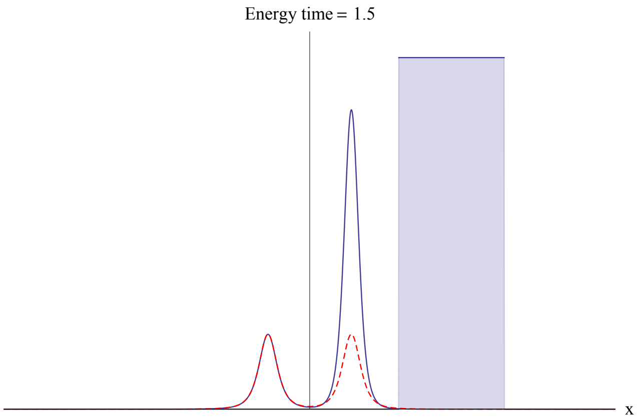
Energy time = 0.5

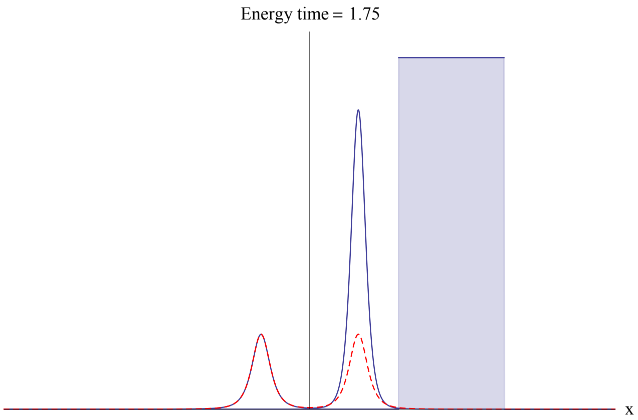


Energy time = 1



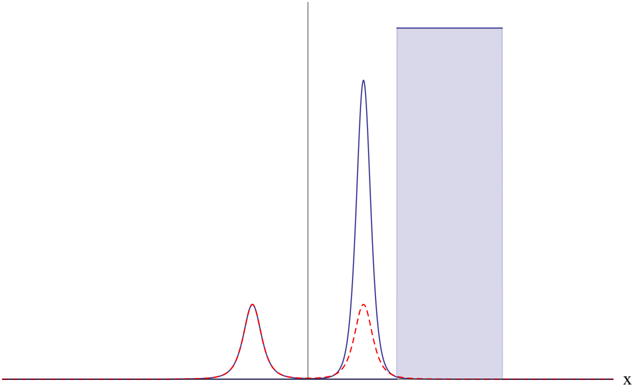

Energy time = 1.25
x


Energy time = 1.5
x

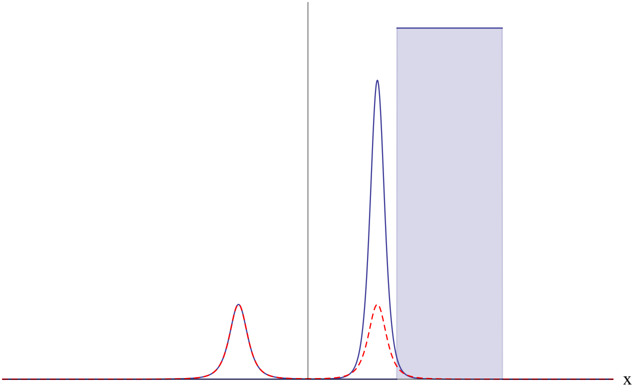


Energy time = 1.75
x

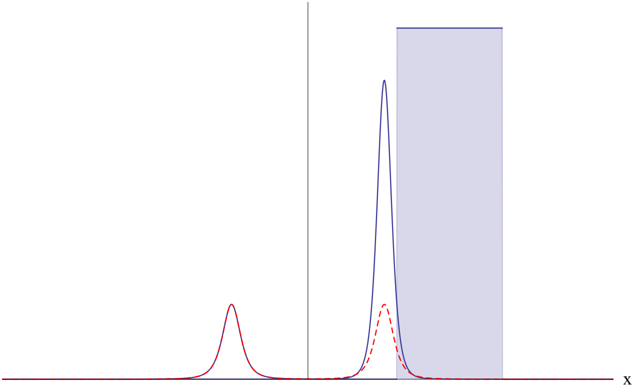
Energy time = 2



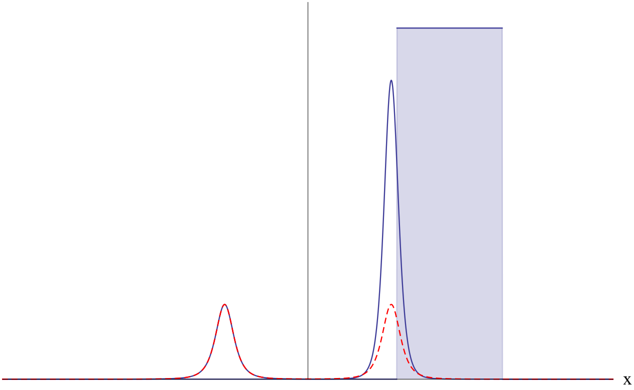
Energy time = 2.5



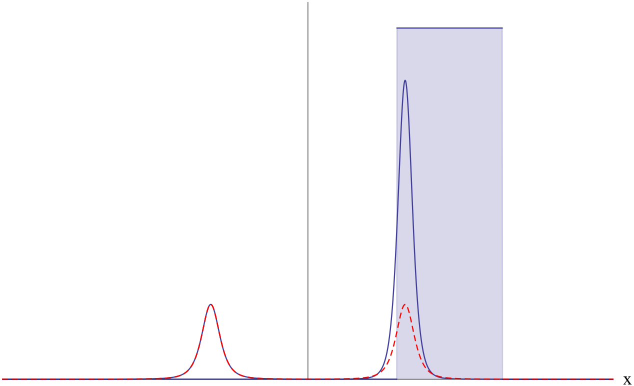
Energy time = 2.75



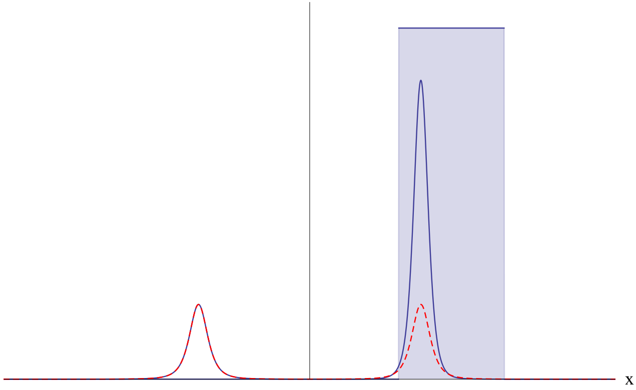
Energy time = 3



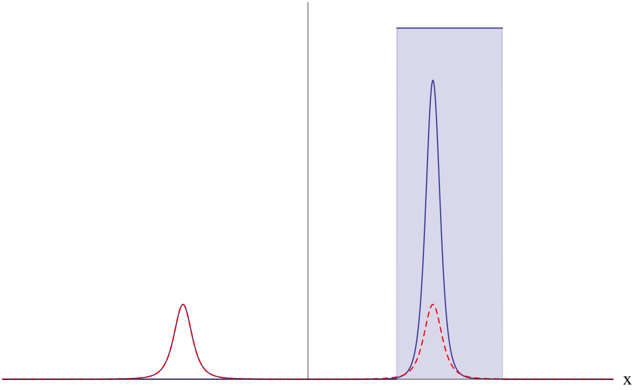
Energy time = 3.5

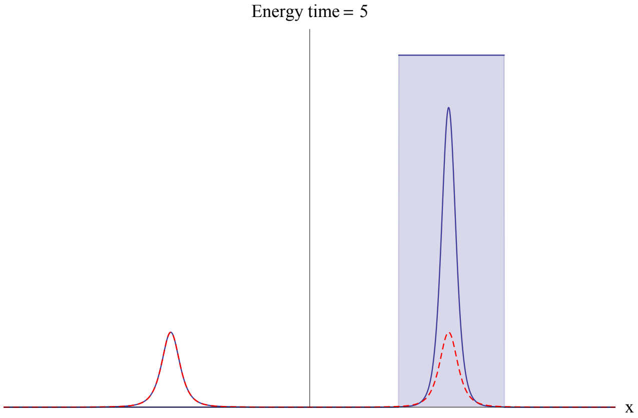


Energy time = 4

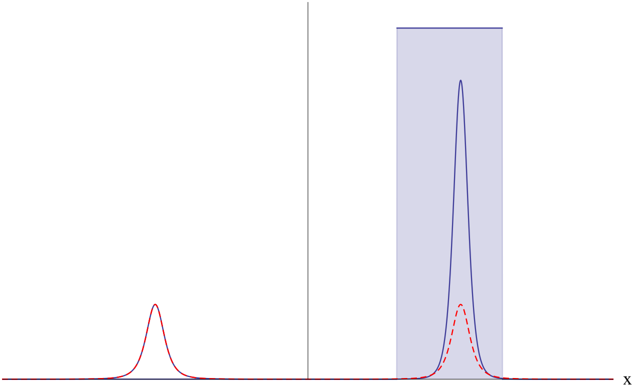


Energy time = 4.5

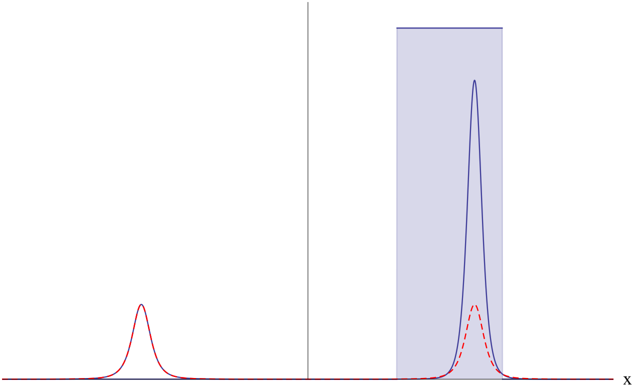



Energy time = 5
x

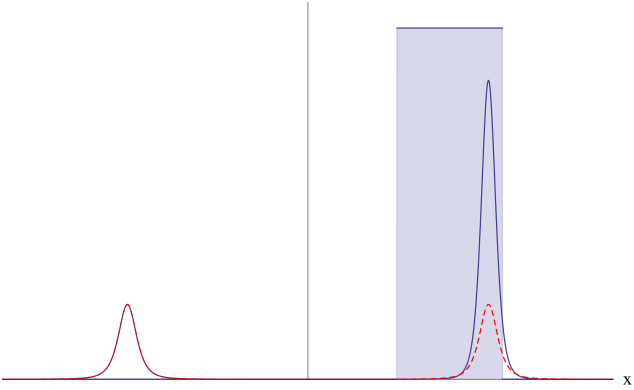
Energy time = 5.5



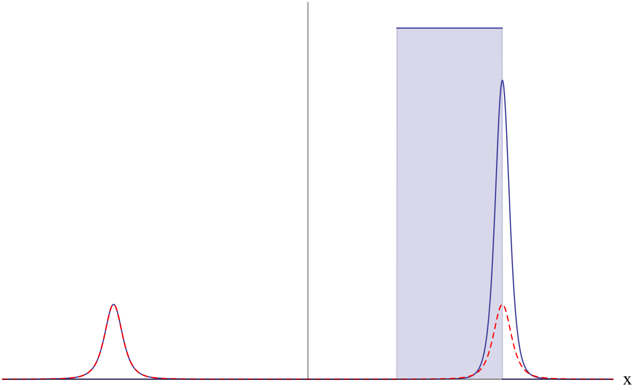
Energy time = 6



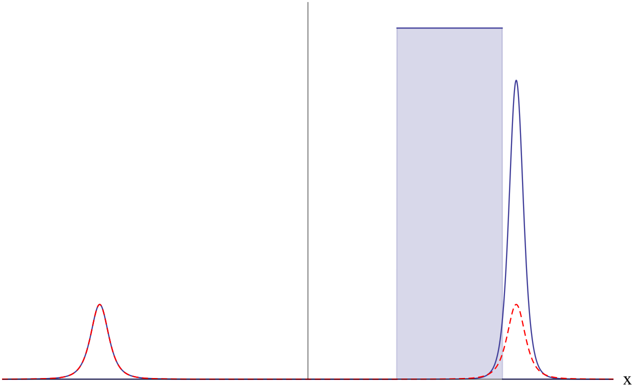
Energy time = 6.5



Energy time = 7



Energy time = 7.5



Change of purity $\Delta S_A^{(2)}$ (descendants)

Challenges in computing $\Delta S_A^{(2)}(z, \bar{z})$ for a descendant

- Descendants transform in a convoluted manner

$$\mathcal{O}^{(-2)} = (f')^{h+2}\left[\phi^{(-2)}(f) + \frac{3f''}{2f'^2}\phi^{(-1)}(f) + \right.$$
$$\left. + \left(\frac{3hf''^2}{4f'^4} + \left(4h + \frac{c}{2}\right)\frac{Sf}{6f'^2}\right)\phi(f)\right]$$

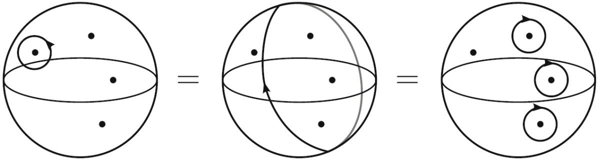
Gaberdiel '94

- Descendant four-point functions

$$\langle \mathcal{O}_1^{(-2)}\mathcal{O}_2^{(-2)}\mathcal{O}_3^{(-2)}\mathcal{O}_4^{(-2)}\rangle = \mathcal{D}\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle$$

we must find the differential operator $\mathcal{D}$

Change of purity $\Delta S_A^{(2)}$ (descendants)



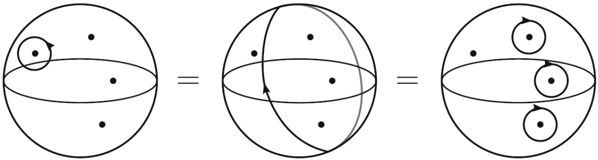
$$\langle \mathcal{O}_1^{(-2)} \mathcal{O}_2^{(-2)} \mathcal{O}_3^{(-2)} \mathcal{O}_4^{(-2)} \rangle = -\sum_{i=2}^{4} \oint_{\mathcal{C}(z_i)} \frac{dw}{2\pi i} (w - z_1)^{-1} \langle\, \mathcal{O}_1 \ldots \Big(T(w) \mathcal{O}_i^{(-2)} \Big) \ldots \mathcal{O}_4^{(-2)} \rangle$$

Descendant OPE

$$T(z)\mathcal{O}^{(-n)}(w) \sim \frac{cn(n^2 - 1/12)}{(z-w)^{n+2}} \mathcal{O}(w) + \sum_{k=1}^{n} \frac{n+k}{(z-w)^{k+2}} \mathcal{O}^{(k-n)}(w)$$

$$\sum_{k \geq 0} (z-w)^{k-2} \mathcal{O}^{(-k,-n)}(w)$$

Change of purity $\Delta S_A^{(2)}$ (descendants)



$$\langle \mathcal{O}_1^{(-2)} \mathcal{O}_2^{(-2)} \mathcal{O}_3^{(-2)} \mathcal{O}_4^{(-2)} \rangle = -\sum_{i=2}^{4} \oint_{\mathcal{C}(z_i)} \frac{dw}{2\pi i} (w - z_1)^{-1}$$
$$\langle \, \mathcal{O}_1 \ldots \left(T(w) \mathcal{O}_i^{(-2)} \right) \ldots \mathcal{O}_4^{(-2)} \rangle$$

Descendant OPE

$$T(z)\mathcal{O}^{(-2)}(w) \sim \frac{\left(\frac{c}{2} + 4h\right) \mathcal{O}(w)}{(z-w)^4} + \frac{3\partial\mathcal{O}(w)}{(z-w)^3} + \frac{(h+2)\mathcal{O}^{(-2)}(w)}{(z-w)^2} + \frac{\partial\mathcal{O}^{(-2)}(w)}{z-w}$$

Change of purity $\Delta S_A^{(2)}$ (descendants)

...following recursively

$$\begin{aligned}\langle \mathcal{O}_1^{(-2)}\mathcal{O}_2^{(-2)}\mathcal{O}_3^{(-2)}\mathcal{O}_4^{(-2)}\rangle =& \Big[\mathcal{D}_{1,2}\left(\mathcal{D}_{3,4}+\mathcal{I}^{(3)}_{(1,2)}\mathcal{L}^{(4)}_{-2}\right)+\mathcal{D}_{1,3}\left(\mathcal{D}_{2,4}+\mathcal{I}^{(2)}_{(1,3)}\mathcal{L}^{(4)}_{-2}\right)\\ &+\mathcal{D}_{1,4}\left(\mathcal{D}_{2,3}+\mathcal{I}^{(2)}_{(1,4)}\mathcal{L}^{(3)}_{-2}\right)+\mathcal{H}^{(1)}_{(1)}\left(\mathcal{D}_{4,2}\mathcal{L}^{(3)}_{-2}+\mathcal{D}_{4,3}\mathcal{L}^{(2)}_{-2}\right)\\ &+\mathcal{H}^{(1)}_{(1)}\mathcal{I}^{(4)}_{(1,4)}\left(\mathcal{D}_{2,3}+\mathcal{I}^{(2)}_{(1,4)}\mathcal{L}^{(3)}_{-2}\right)\Big]\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle\end{aligned}$$

where

$$\mathcal{D}_{i,j}=\frac{\frac{c}{2}+4h}{w_{ij}^4}+\frac{3\partial_j}{w_{ij}^3},\qquad \mathcal{L}^{(i)}_{-2}=\sum_{j\neq i}\left(\frac{h}{w_{ji}^2}-\frac{\partial_j}{w_{ji}}\right)$$

$$\mathcal{I}^{(i)}_{(k,l)}=\sum_{j\neq i}\left(\frac{h_j}{w_{ij}^2}-\frac{\partial_j}{w_{ji}}\right),\qquad \mathcal{H}^{(i)}_{(k)}=\sum_{j\neq i}\left(\frac{h_j}{w_{ij}^2}-\frac{\partial_j}{w_{ji}}\right)$$

Observe that the full operator reduces powers of z by 8

Change of purity $\Delta S_A^{(2)}$ (descendants)

...finally, the jump in purity due to the insertion of $\mathcal{O}^{(-2)}$

$$\exp\left(-\Delta S_A^{(2)}(z,\bar{z})\right) = \mathcal{N}_{O^{(-2)}}\left[\,\mathcal{G}_{O^{(-2)}}(z,\bar{z}) + \dots\right]$$

the ... are subleading both at $z \to 0, 1$.

The prefactor reads

$$\mathcal{N}_{O^{(-2)}} = \frac{(4z_1 z_2)^{-2(h+2)}\,(4\bar{z}_1\bar{z}_2)^{-2\bar{h}}\prod_{i=1}^{4}\alpha_i\bar{\alpha}_i}{\left(\langle\mathcal{O}^{(-2)}(w_1,\bar{w}_1)\mathcal{O}^{(-2)}(w_2,\bar{w}_2)\rangle_{\Sigma_1}\right)^2}$$

Change of purity $\Delta S_A^{(2)}$ (descendants)

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the $\dots$ are subleading both at $z \to 0, 1$.

Which yields

$$\mathcal{N}_{O^{(-2)}} = \frac{4(z(1-z))^{2(h+2)}(\bar{z}(1-\bar{z}))^{2\bar{h}}}{(c+2h(9h+22))^2}$$

Change of purity $\Delta S_A^{(2)}$ (descendants)

...finally, the jump in purity due to the insertion of $\mathcal{O}^{(-2)}$

$$\exp\left(-\Delta S_A^{(2)}(z,\bar{z})\right) = \mathcal{N}_{O^{(-2)}} \left[\, \mathcal{G}_{O^{(-2)}}(z,\bar{z}) + \ldots \right]$$

Amazingly!!

$$\mathcal{F}_{O^{(-2)}}(z,\bar{z}) \simeq \frac{1}{4}\left(c + 2h(9h+22)\right)^2 \begin{cases} \mathcal{F}_{\mathcal{O}}(0,0) & t \notin [l, L+l] \\ \mathcal{F}_{\mathcal{O}}(1,0) & t \in [l, L+l] \end{cases}$$

Therefore

$$\boxed{\exp\left(-\Delta S_A^{(2)}(t)\right) \simeq \begin{cases} \mathcal{F}_{\mathcal{O}}(0,0) & t \notin [l, L+l] \\ \mathcal{F}_{\mathcal{O}}(1,0) & t \in [l, L+l] \end{cases}}$$

Change of purity $\Delta S_A^{(2)}$ (descendants)

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The procedure can be generalized, for any descendant we have

$$\boxed{\exp\left(-\Delta S_A^{(2)}(t)\right) \simeq \begin{cases} \mathcal{F}_{\mathcal{O}}(0,0) & t \notin [l, L+l] \\ \mathcal{F}_{\mathcal{O}}(1,0) & t \in [l, L+l] \end{cases}}$$

Caputa, AVO '15

Change of purity $\Delta S_A^{(2)}$ (descendants)

...finally, the jump in purity due to the insertion of $\mathcal{O}^{(-2)}$

$$\exp\left(-\Delta S_A^{(2)}(z,\bar{z})\right) = \mathcal{N}_{O^{(-2)}}\left[\, \mathcal{G}_{O^{(-2)}}(z,\bar{z}) + \ldots \right]$$

Amazingly!!

$$\mathcal{F}_{O^{(-2)}}(z,\bar{z}) \simeq \frac{1}{4}\left(c + 2h(9h+22)\right)^2 \begin{cases} \mathcal{F}_{\mathcal{O}}(0,0) & t \notin [l, L+l] \\ \mathcal{F}_{\mathcal{O}}(1,0) & t \in [l, L+l] \end{cases}$$

The procedure can be generalized, for RCFT

$$\boxed{\exp\left(-\Delta S_A^{(n)}(t)\right) \simeq \begin{cases} 0 & t \notin [l, L+l] \\ \log(d_{\mathcal{O}}) & t \in [l, L+l] \end{cases}}$$

Caputa, AVO '15

Epilogue

Epilogue

- **Large c and holography**
  Late time for $T(z)$ insertion

$$\Delta S^{(2)} \simeq 8 \log \frac{2t}{\epsilon} - 2 \log \frac{c}{2}$$

  The entanglement constant is non-perturbative.

- **Entropy for BTZ black holes**
  From Liouville theory

$$S_{BTZ} = S_{top}$$

McGough, Verlinde '13

- **Theories with Kač-Moody symmetry**
  - How close can two states be?
  - Microstates and typical shapes
  - Branching graphs and entanglement

Thank you for your attention!!