Giant Graviton Interactions in the PP-wave Limit

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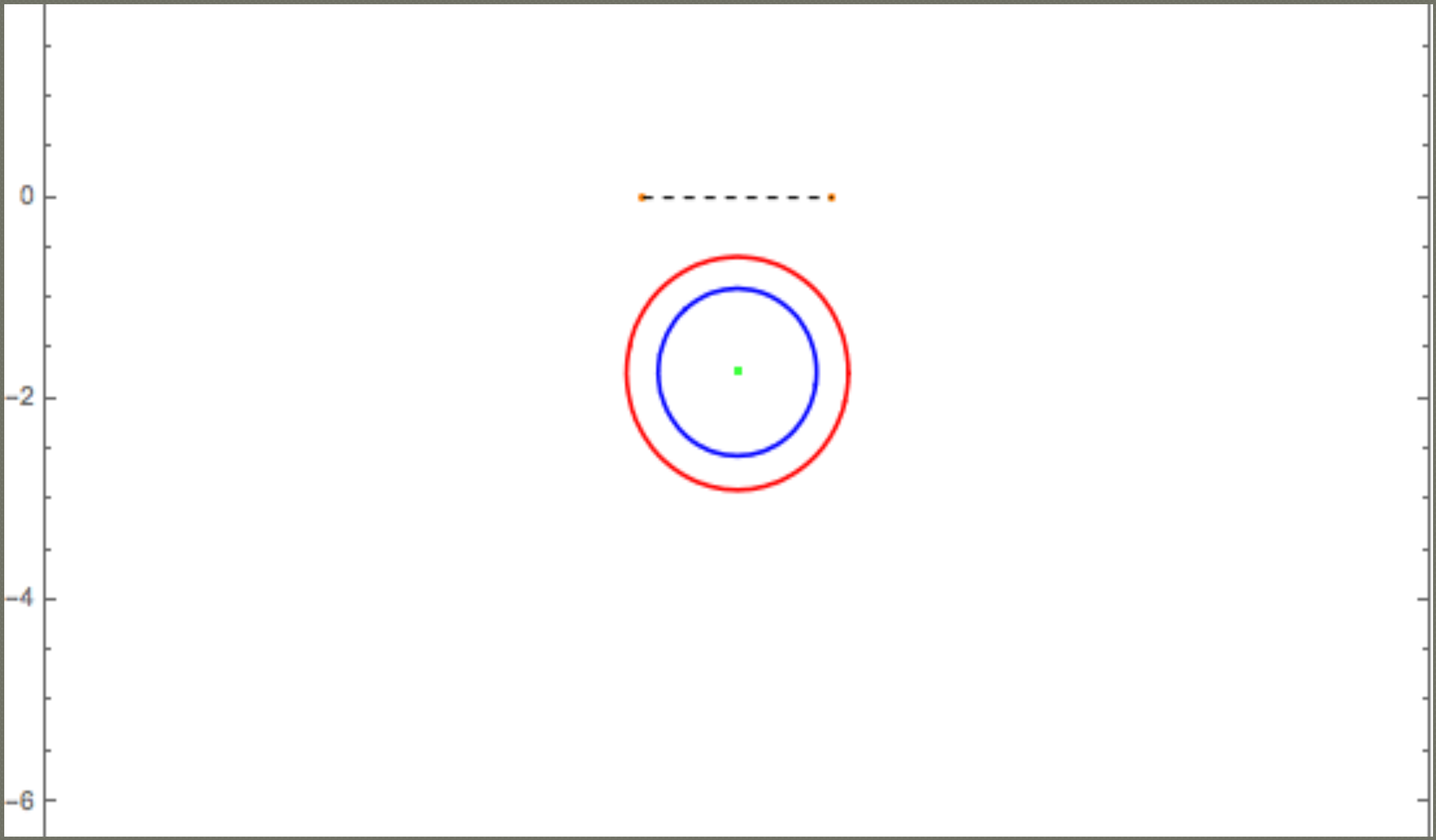
Shinji Hirano

(University of the Witwatersrand, MITP)

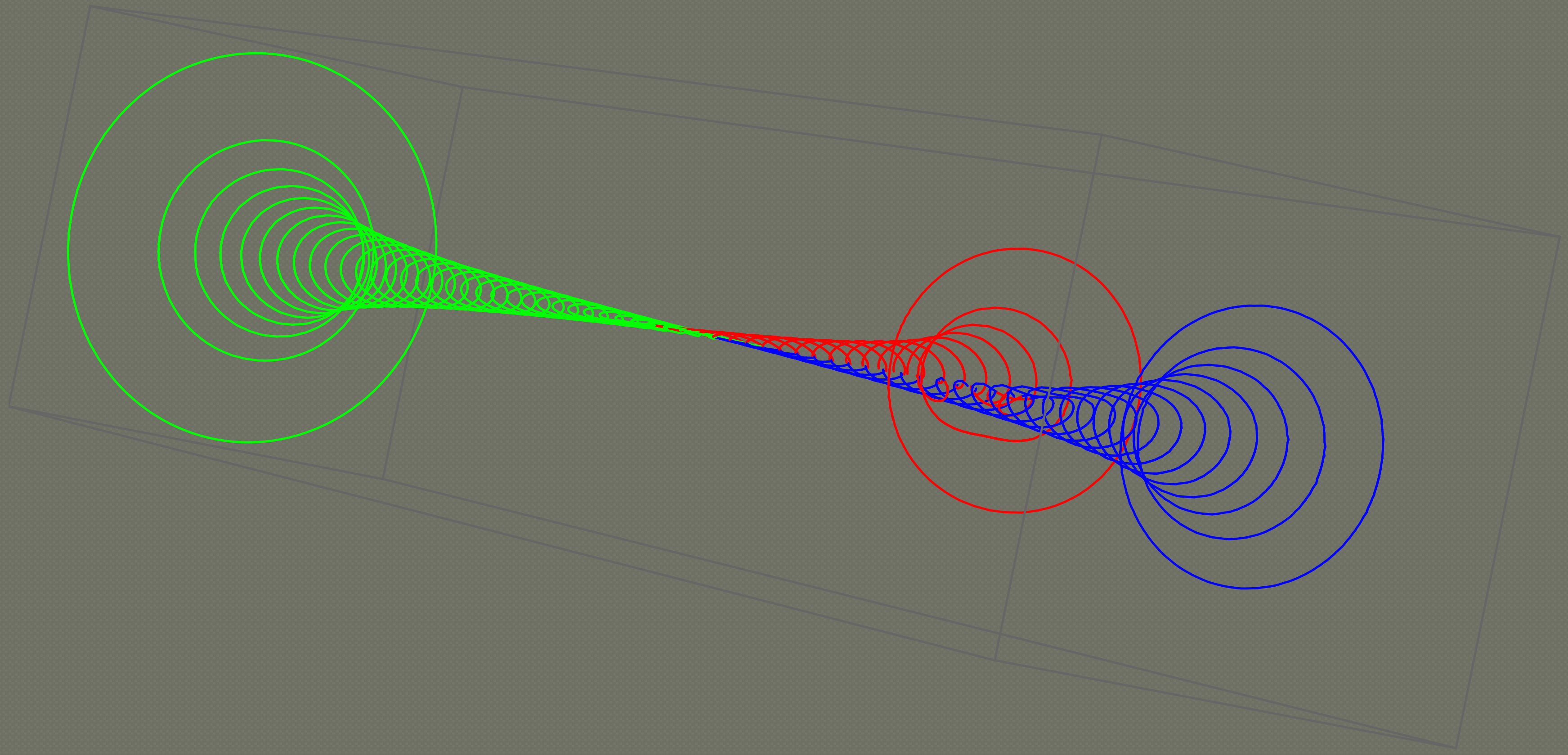
In preparation with Yuki Sato
(Chulalongkorn University)

Main Results

2 to 3 Giant Graviton Interaction



M2-brane Junction Ending on 3 M5-branes



This talk

- A study of interaction vertices of multi-graviton bound states, "giant gravitons", both in gravity and field theory in AdS/CFT
- Giant graviton interactions as instantons in type IIB pp-wave matrix model of Sheikh Jabbari (A gravity description)
- Precise agreements between gravity and CFT descriptions (CFT results by Corley-Jevicki-Ramgoolam, Takayanagi[2])
- ✓ As a technical byproduct solitons describing M2-branes ending on multiple M5-branes

An Overview

IIB on $AdS_5 \times S^5$

GG with J

m to n GG interactions

$N = 4$ SYM

Schur polynomials of dimension J

$$\langle \prod_{i=1}^{m} \mathcal{O}_{J_i}(Z) \prod_{k=1}^{n} \mathcal{O}_{J'_k}(\bar{Z}) \rangle$$

PP-wave limit

$$J \sim \mathcal{O}(\sqrt{N}) \ \& \ J^2/N \gg 1$$

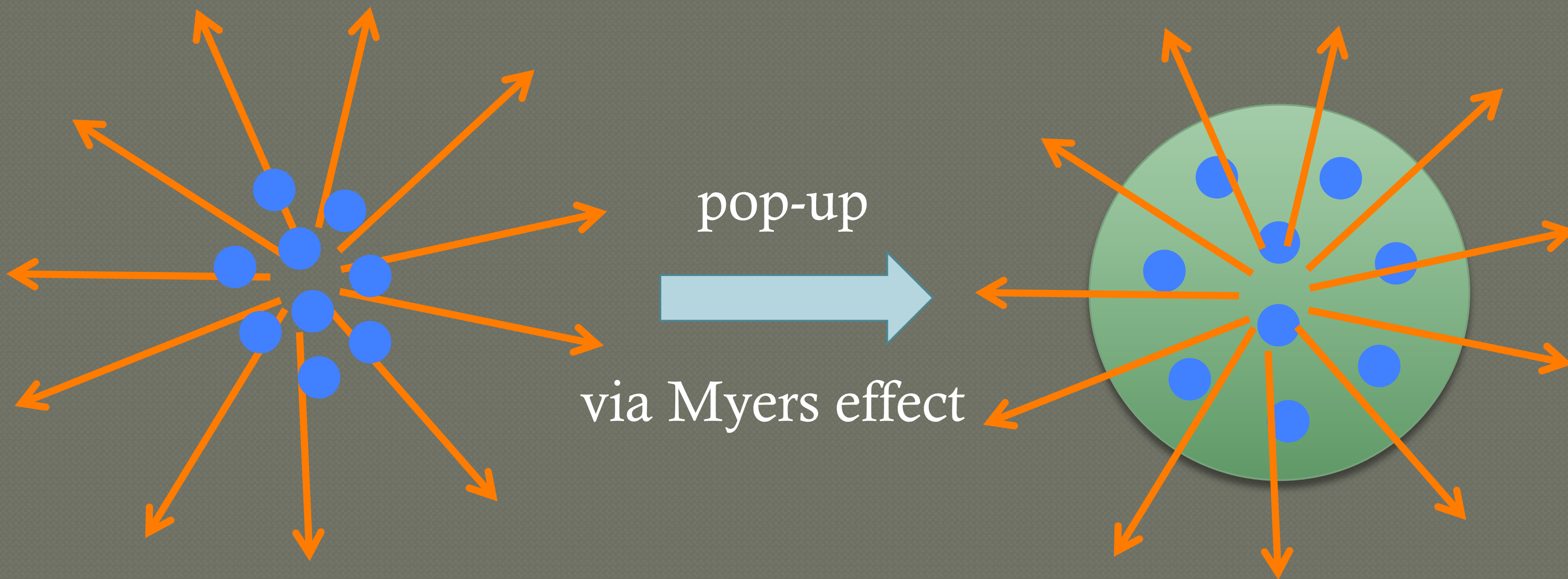
Instantons in IIB pp-wave matrix model (SJ)
(An alternative to BMN IIB strings on pp-wave)

Correlators ≈ exp(- S)

$$S = \frac{1}{4N}\left(\sum_{k=1}^{n} J_k'^2 - \sum_{i=1}^{m} J_i^2\right)$$

Giant Gravitons

- Giant gravitons are multi-graviton states with angular momentum J in S^5 and best described by D3-branes.
- Come in two varieties: (1) sphere giants, S^3 D3-branes in S^5 and (2) dual (AdS) giants, S^3 D3-branes in AdS_5.
- Arguably, the most natural description of gravitons in $AdS_5 \times S^5$ forming an orthogonal basis of multi-graviton states.
- Dual to Schur polynomial (multi-trace) operators in CFT. (Schur poly = orthogonal basis for multi-trace operators)



Gravitons with total J immersed in F_5

Gravitons reorganize themselves into a spherical D3-branes of size J/N

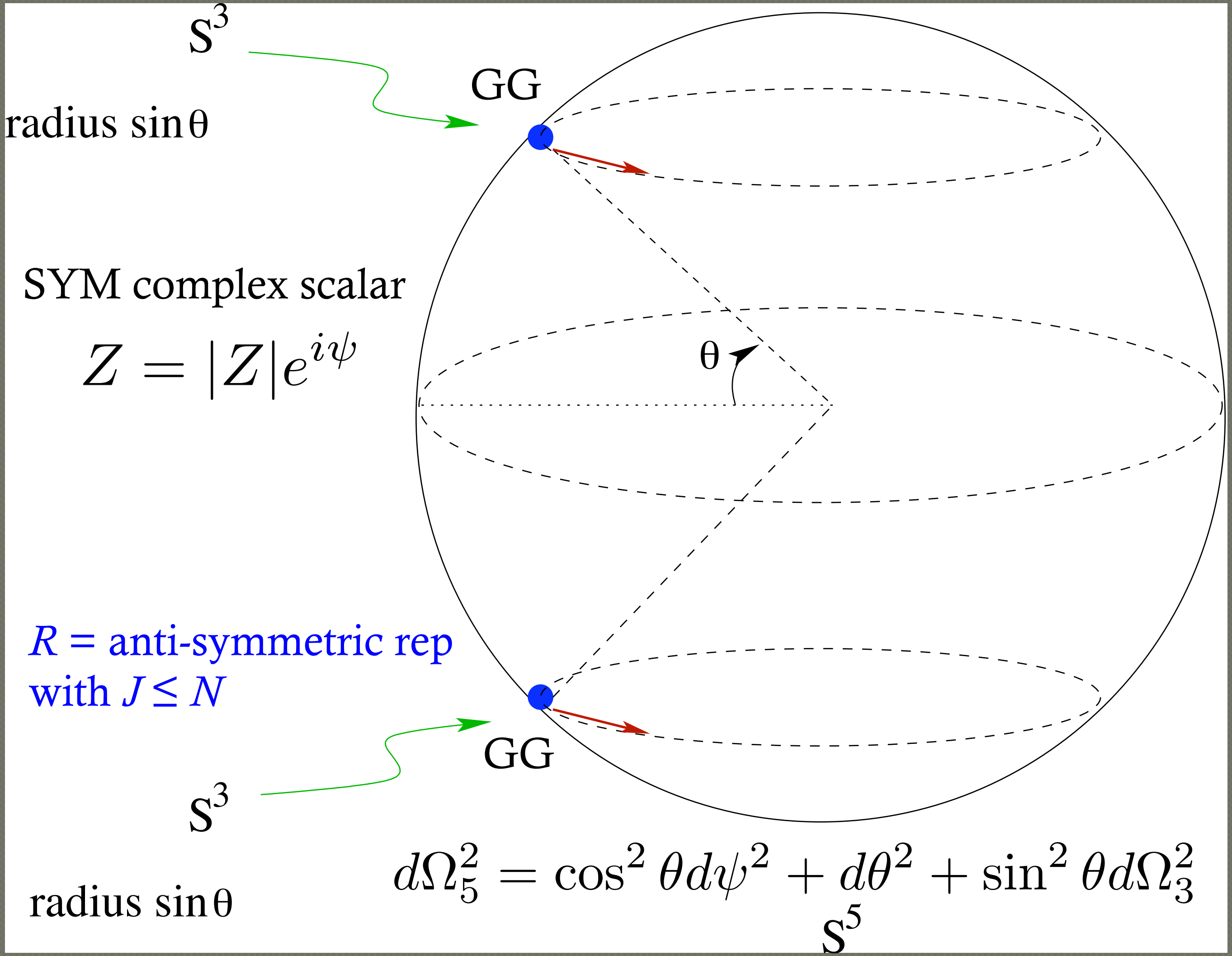
$$\mathcal{O}'_J \sim \mathcal{O}_J + \mathcal{O}_{J-J'}\mathcal{O}_{J'} + \cdots + (\mathcal{O}_1)^J$$

$$\mathcal{O}_J \equiv \mathrm{Tr} Z^J$$

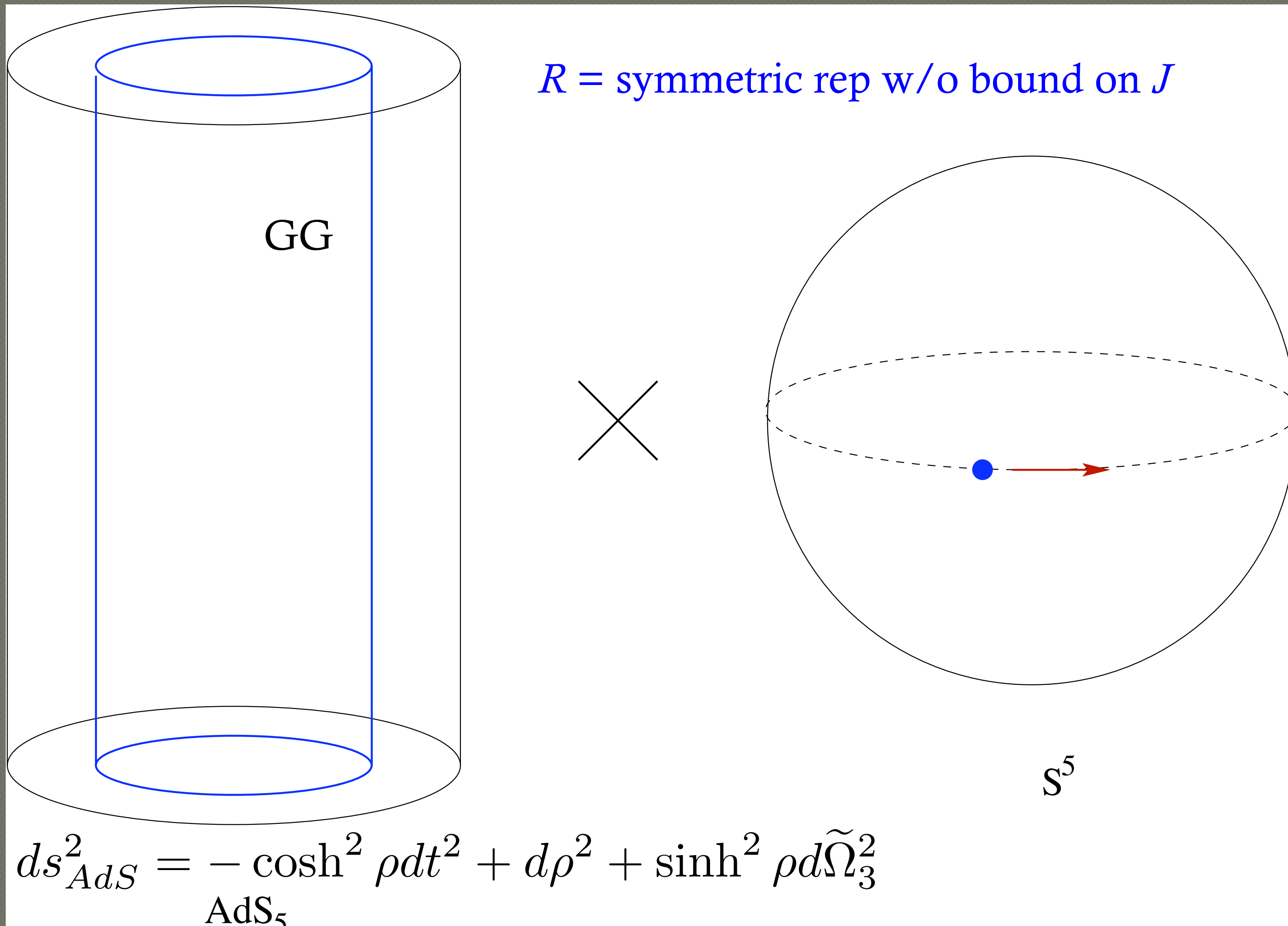
$$\mathcal{O}_J^R = \frac{1}{J!} \sum_{\sigma \in S_J} \chi_R(\sigma) \sum_{i=1}^{N} Z^{i_1}_{i_{\sigma(1)}} \cdots Z^{i_J}_{i_{\sigma(J)}}$$

R = Young diagrams w/ J boxes = types of GG

Sphere giants – S^3 D3-branes in S^5



Dual (AdS) giants – S^3 D3-branes in AdS^5



$$ds^2_{AdS} = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\widetilde{\Omega}^2_3$$

Giant graviton interaction vertices in CFT

(CFT Schur correlators)

GG = Schur Polynomial operator

- Schur polynomial is an orthogonal polynomial.
- Rotation in S^5 with angular momentum J
  ➔ order J poly of $Z = |Z|e^{it}$ where $\psi = t$
- Schur poly of Z is multi-trace:

$$\chi_R(Z) = \frac{1}{J!} \sum_{\sigma \in S_J} \chi_R(\sigma) \sum_{i_1, i_2, \cdots, i_J = 1}^{N} Z^{i_1}_{i_{\sigma(1)}} Z^{i_2}_{i_{\sigma(2)}} \cdots Z^{i_J}_{i_{\sigma(J)}}$$

$\chi_R(\sigma) = \mathrm{Tr}\,\sigma$ = character of sym group S_J
R = rep of S_J classified by Young diagrams w/ J boxes

Correlators of Schur polynomials

- Tree-level exact w/o any λ corrections

- The 1 to n (normalized) correlator with $R = A$ (all sphere giants) is as simple as (suppressing spacetime dependence)

$$\frac{\langle \prod_{i=1}^{n} \chi_{A_{J_i}}(Z)\chi_{A_J}(\bar{Z})\rangle}{\prod_{i=1}^{n+1} ||\chi_{A_{J_i}}(Z)||} = \sqrt{\frac{(N-J_1)!(N-J_2)!\cdots(N-J_n)!}{(N-J)!(N!)^{n-1}}}$$

where $J_1 + \ldots + J_n = J$ and $J_{n+1} = J$

- In the pp-wave limit

$$\text{RHS} \to e^{-\frac{1}{4N}(J^2 - \sum_{i=1}^{n} J_i^2)}$$

Instanton action in a gravity description

Correlators of Schur polynomials –cont'd

- The m to n (normalized) correlator with $R = A$ (all sphere giants) at large N

$$\frac{\langle \prod_{i=1}^{n} \chi_{A_{J_i}}(Z) \prod_{k=1}^{m} \chi_{A_{J'_k}}(\bar{Z}) \rangle}{\prod_{i=1}^{n} ||\chi_{A_{J_i}}(Z)|| \prod_{k=1}^{m} ||\chi_{A_{J'_k}}(\bar{Z})||}$$

$$\simeq \sqrt{\frac{(N-J'_1)!(N-J'_2)!\cdots(N-J_n)!}{(N!)^{n+m}}} \prod_{k=1}^{n} \frac{(N+k-1)!}{(N-J_k+k-1)!}$$

where $J'_1 + \ldots + J'_m = J_1 + \ldots + J_n$

- In the pp-wave limit

$$\mathrm{RHS} \to e^{-\frac{1}{4N}\left(\sum_{k=1}^{n} J_k^2 - \sum_{i=1}^{m} (J'_i)^2\right)}$$

Instanton action in a gravity description

Giant graviton interaction vertices in gravity

(Instantons in type IIB pp-wave MM)

Type IIB pp-wave matrix model

- The pp-wave limit is the geometry zooming in the vicinity of the trajectory of a particle rotating very fast along the equator of S^5. The angular momentum $J \approx N^{1/2}$ for a finite lightcone momentum p^+.

- Type IIB string theory on the pp-wave reduces to a 2d free massive theory (BMN).

- An alternative gravity/stringy description was proposed by Sheikh-Jabbari and dubbed the “tiny graviton” matrix model.

Type IIB pp-wave matrix model – cont'd

- The "tiny graviton" matrix model is the low energy effective theory of a D3-brane on the pp-wave:

$$L_B = \frac{1}{R}\mathrm{Tr}\left[\frac{1}{2}\left(\frac{dX^I}{dt}\right)^2 - \frac{1}{2}\mu^2 X_I^2 - \frac{R^2}{12g_s^2}\left[X^I, X^J, X^K, \Upsilon_5\right]^2 + \frac{\mu R}{6g_s}\left(\epsilon^{ijkl}X^i\left[X^j, X^k, X^l, \Upsilon_5\right] + \epsilon^{abcd}X^a\left[X^b, X^c, X^d, \Upsilon_5\right]\right)\right]$$

(in $A_0 = 0$ gauge & only the bosonic part shown)

$I = 1,2, \ldots, 8$; $i = 1,2,3,4$ (AdS_5 part), $a = 5,6,7,8$ (S^5 part)
μ = mass parameter = 5-form flux
$X^- \approx X^- + 2\pi R$
$\Upsilon_5 \approx$ "γ_5"

✧ "Quantization"

$$\left\{X^I, X^J, X^K\right\}_{\mathrm{NB}} \equiv \epsilon^{\alpha\beta\gamma}\frac{\partial X^I}{\partial\sigma^\alpha}\frac{\partial X^J}{\partial\sigma^\beta}\frac{\partial X^K}{\partial\sigma^\gamma} \to -iJ\left[X^I, X^J, X^K, \Upsilon_5\right]$$

Nambu bracket

4-Lie bracket of $U(J)$ matrices

The BPS-like equation

- The advertized instantons are solutions to the BPS-like equation:

$$\frac{dX^i}{dt} \pm \mu X_i \pm \frac{iR}{3! g_s} \epsilon^{ijkl} \left[X^j, X^k, X^l, \Upsilon_5 \right] = 0$$

(i = 5,6,7,8 & Gauss constraint automatically satisfied)

- The vacua correspond to a cluster of fuzzy S^3's:

$$X_i = \mathrm{diag}(\text{“}\gamma_i^{(1)}\text{”}, \cdots, \text{“}\gamma_i^{(k)}\text{”}) \qquad \dot{X}_i = 0$$

- The on-shell action:

$$S_E = \mp \frac{1}{2R} \int_{-\infty}^{+\infty} dt \mathrm{Tr} \frac{d}{dt} \left(\mu X_i^2 + \frac{2iR}{4! g_s} \epsilon^{ijkl} X^i \left[X^j, X^k, X^l, \Upsilon_5 \right] \right)$$

The BPS-like equation – cont'd

- The instanton action with *m* initial S^3's and *n* final S^3's corresponding to the *m* to *n* interaction :

$$S_E = -\frac{\mu}{4R}\sum_{i=1}^{4}\mathrm{Tr}_{J\times J}\left[X_i(+\infty)^2 - X_i(-\infty)^2\right] = \frac{1}{4N}\left(\sum_{k=1}^{n}J_k^2 - \sum_{i=1}^{m}(J_i')^2\right)$$

This is the sought-after instanton action which precisely agrees with the CFT (antisymmetric) Schur correlators in the pp-wave limit!

Construction of instantons

BPS equation = 4d Laplace equation

(Poisson's equation)

- Remarkably, explict instanton solutions can be constructed in the "classical/continuum limit" (large *J*).

[generalization of Kovacs- Sato-Shimada]

$$-iJ\left[X^I, X^J, X^K, \Upsilon_5\right] \to \left\{X^I, X^J, X^K\right\}_{\mathrm{NB}}$$

- After change of variables

$$\boxed{\frac{\partial z^i}{\partial s} = -\frac{2\pi^2}{J}\epsilon^{ijkl}\frac{\partial z^j}{\partial \sigma_1}\frac{\partial z^k}{\partial \sigma_2}\frac{\partial z^l}{\partial \sigma_3}}$$

where $\quad z^i = \sqrt{\frac{R}{4\pi^2\mu g_s}}e^{\mu t}X^i \quad$ and $\quad s = e^{-2\mu t}$

BPS equation = 4d Laplace equation – cont'd

(Poisson's equation)

- The solutions to the continuum BPS equation represent 3d surfaces in 4d parametrized by σ_a evolving in time s.

- The surfaces at a constant time slice

$$s = s(z_1, z_2, z_3, z_4)$$

which can be interpreted as an electrostatic potential with a uniform electric flux density

$$d\sigma_1 \wedge d\sigma_2 \wedge \sigma_3 = -\frac{2\pi^2}{J}\frac{\partial s}{\partial z_i}\epsilon^{ijkl}dz^j \wedge dz^k \wedge dz^l = -\frac{2\pi^2}{J}\vec{\nabla}s \cdot d\vec{A}$$

$$\vec{\nabla}s = \vec{E}$$

$$\Downarrow$$

$$\triangle_4 s = \rho$$

Electrostatic potential and giants

- The Coulomb potential is obviously a solution:

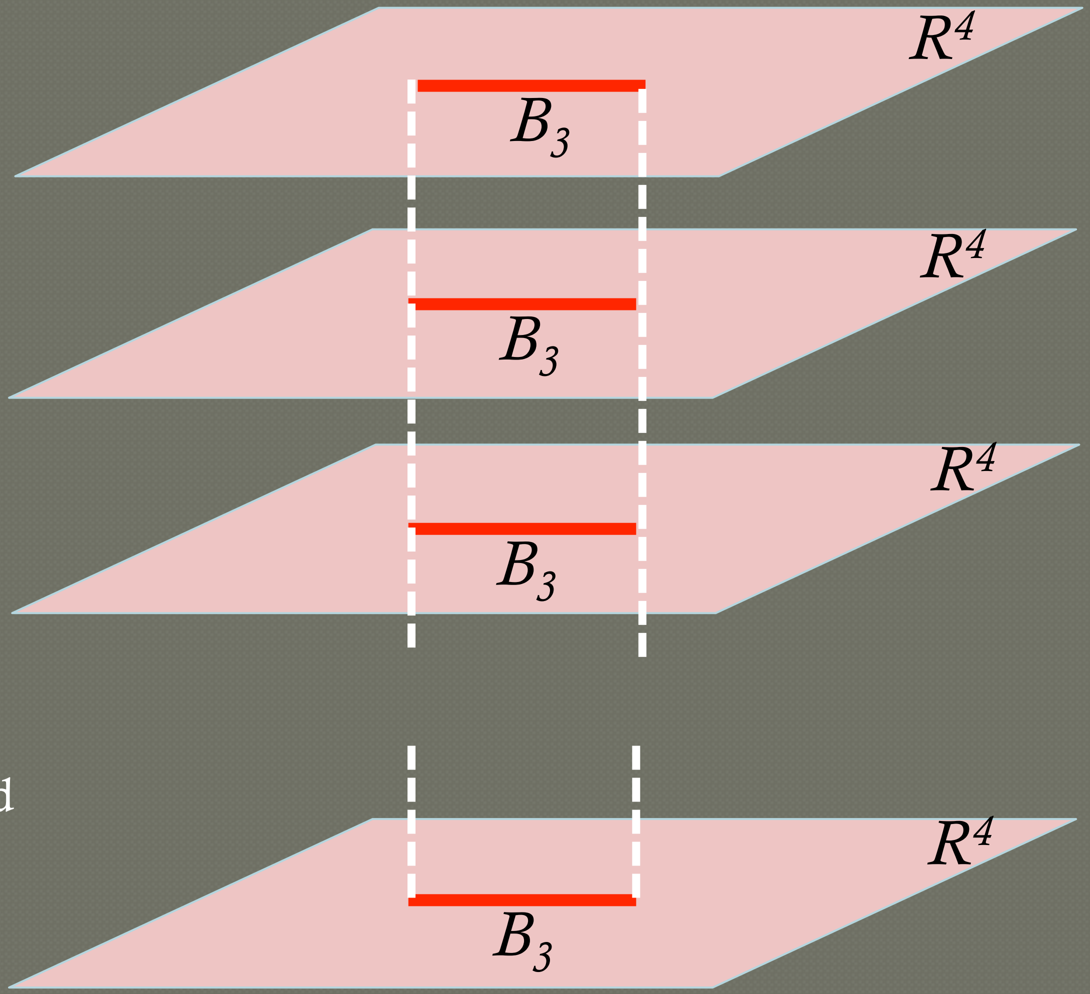
$$e^{-2\mu t} = s = \frac{Q}{|\vec{z} - \vec{z}_0|^2} = \frac{cQe^{-2\mu t}}{|\vec{X} - \vec{X}_0|^2}$$

which implies Q at $s = +\infty$ is the S^3 radius of the initial GGs whereas Q at $s = 0$ is the S^3 radius of the final GGs.

- ✓ The charge source = giant at $t = -\infty$
- ✓ The asymptotic infinity = giant at $t = +\infty$

This sets the boundary conditions

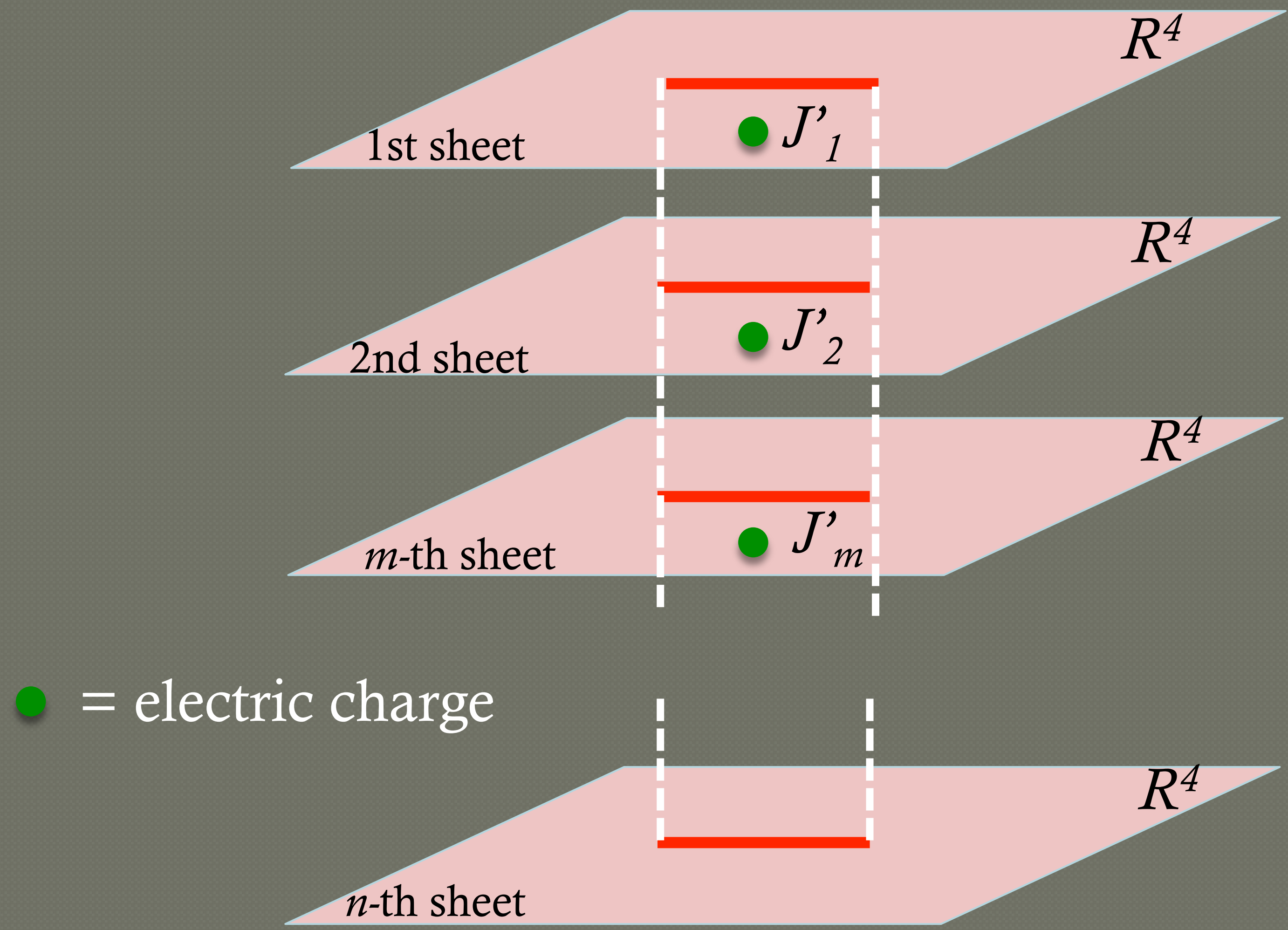
Riemann space
(higher dim generalization of Riemann surface)



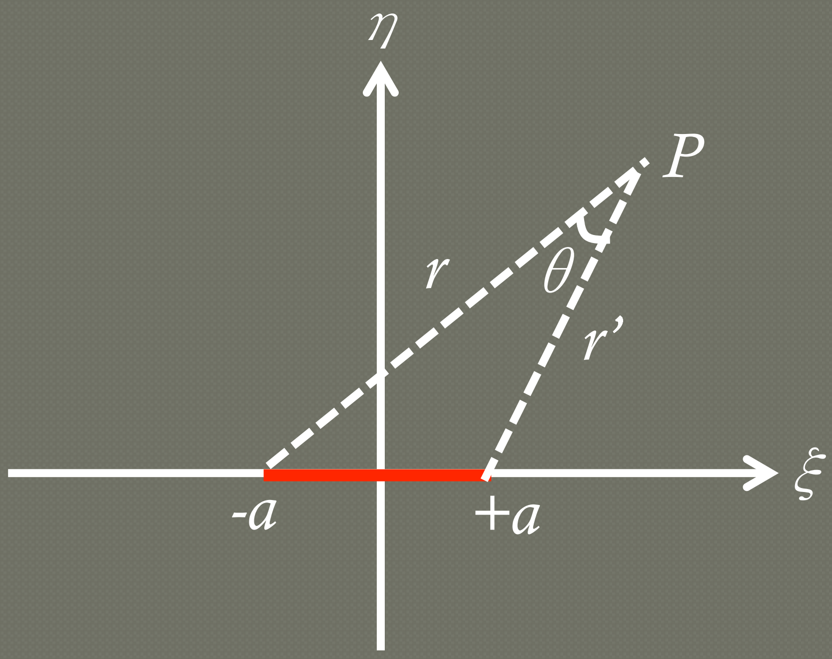
branch cut B_3

cf. Interval I in 2d
Disk D_2 in 3d

The m to n giant graviton interaction



The coordinate system (bipolar)



$$(\xi, \eta) = \frac{a}{\cosh\rho - \cos\theta}(\sinh\rho, \sin\theta)$$

where $\rho = \ln(r/r')$

$$\begin{cases} z_1 = \eta \\ z_2 = \xi\cos\phi \\ z_3 = \xi\sin\phi\cos\omega \\ z_4 = \xi\sin\phi\sin\omega \end{cases}$$

1st sheet $-\pi \le \theta \le \pi$

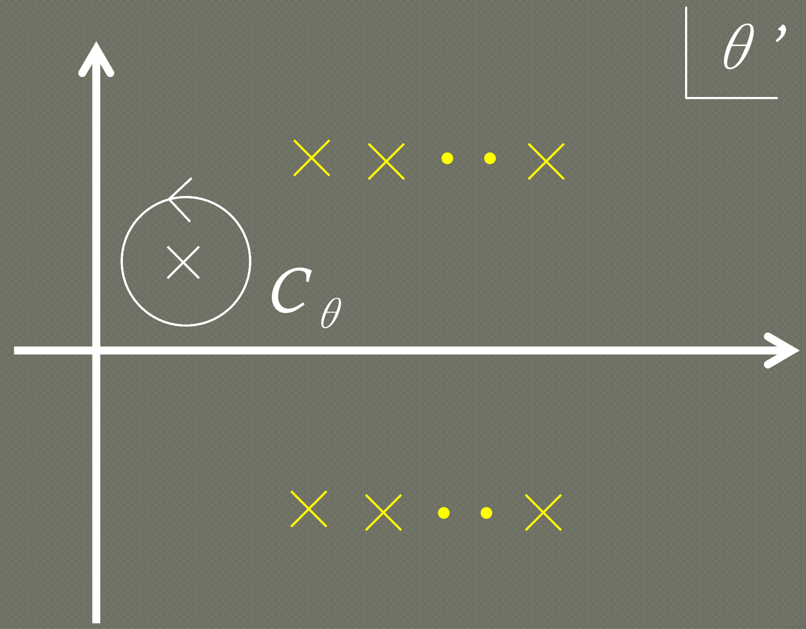
2nd sheet $\pi \le \theta \le 3\pi$

$\vdots$

Electrostatic potential in 4d Riemann space

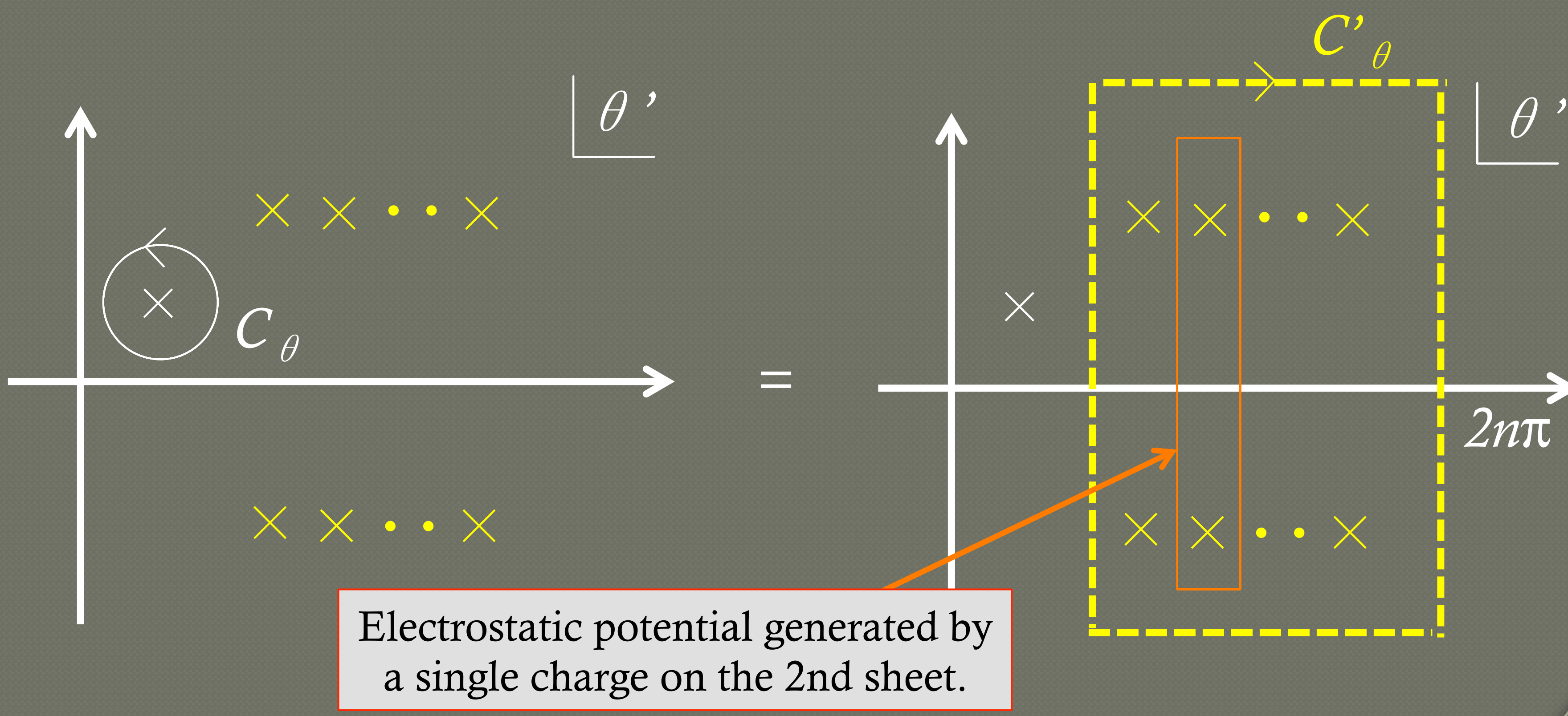
- The Coulomb potential corresponds to placing a point charge on every single Riemann sheet at the same location.
- A single charge potential can be distilled by a contour integral trick:

Coulomb potential $\frac{1}{R^2}$ =



Electrostatic potential in 4d Riemann space

- By deforming the contour the Coulomb potential is expressed as the sum of potentials in each sheet:



The *m* to *n* GG interaction in equation

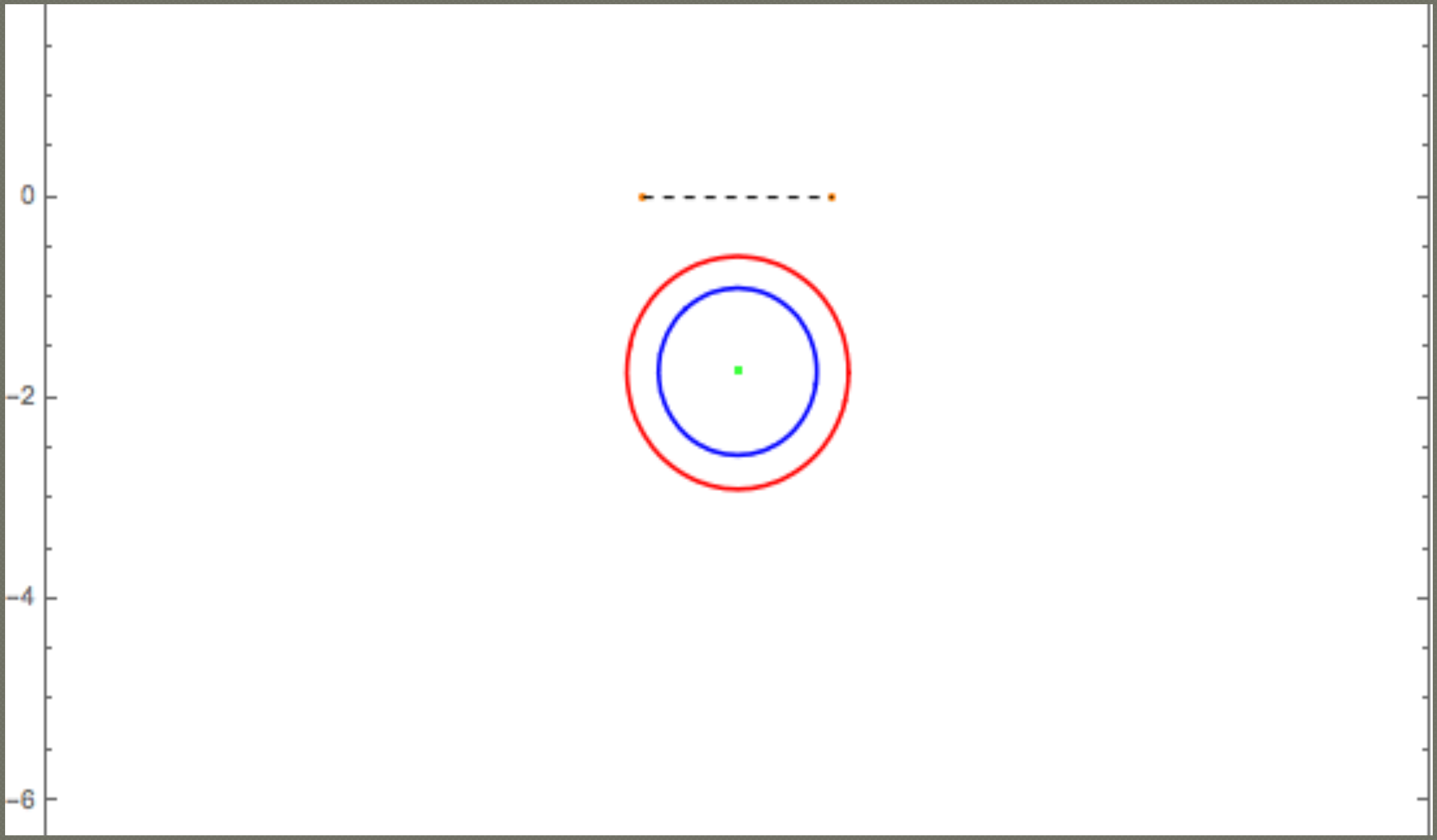
- The electrostatic potential of a single charge in the *n*-sheeted Riemann space

$$s_n^{(J)}(\vec{z}) = \frac{J}{4\pi^2 R^2} \frac{\sinh\frac{\alpha}{n}\left(\cosh^2\frac{\alpha}{2} - \cos^2\frac{\theta-\theta_0}{2}\right)}{n\sinh\alpha\left(\cosh^2\frac{\alpha}{2n} - \cos^2\frac{\theta-\theta_0}{2n}\right)}$$

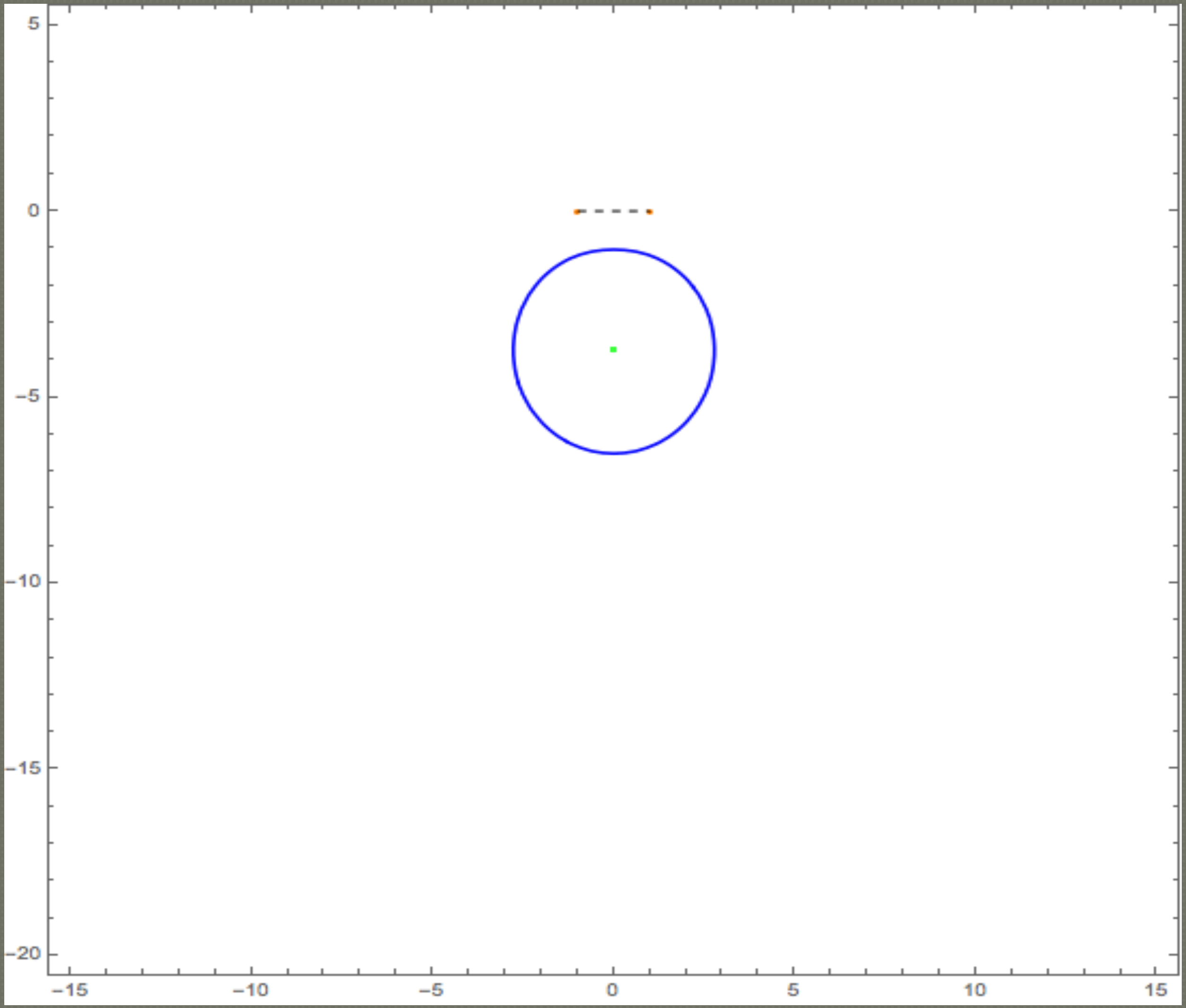
- The electrostatic potential corresponding to the *m* to *n* GG interaction

$$s_{m,n}(\vec{z}) = \sum_{l=1}^{m} s_n^{(J_l)}(\vec{z}; \theta_0 + 2\pi(l-1))$$

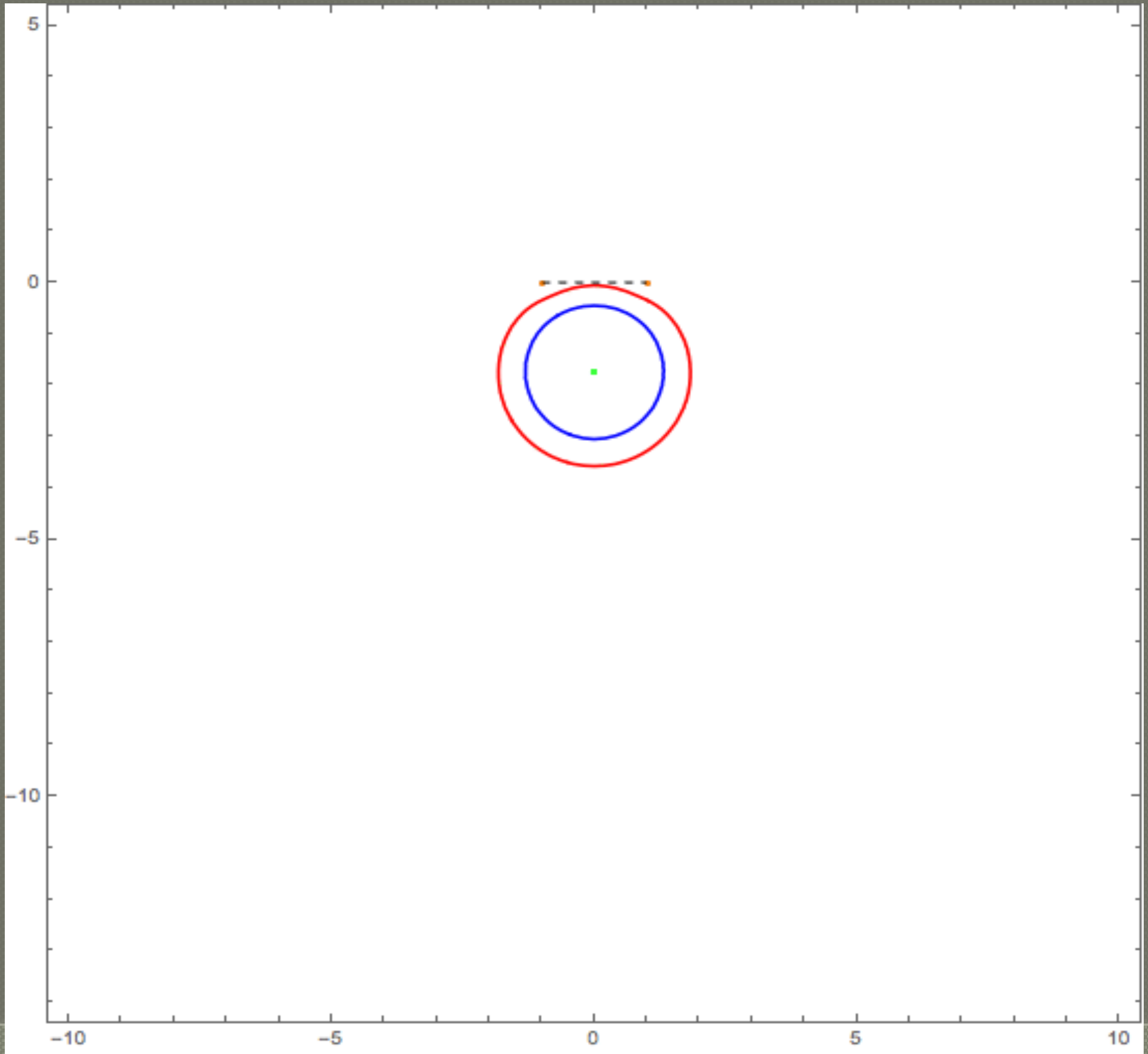
2 to 3 Giant Graviton Interaction



1 to 2 Giant Graviton Interaction



2 to 4 Giant Graviton Interaction



M2-branes ending on multiple M5-branes

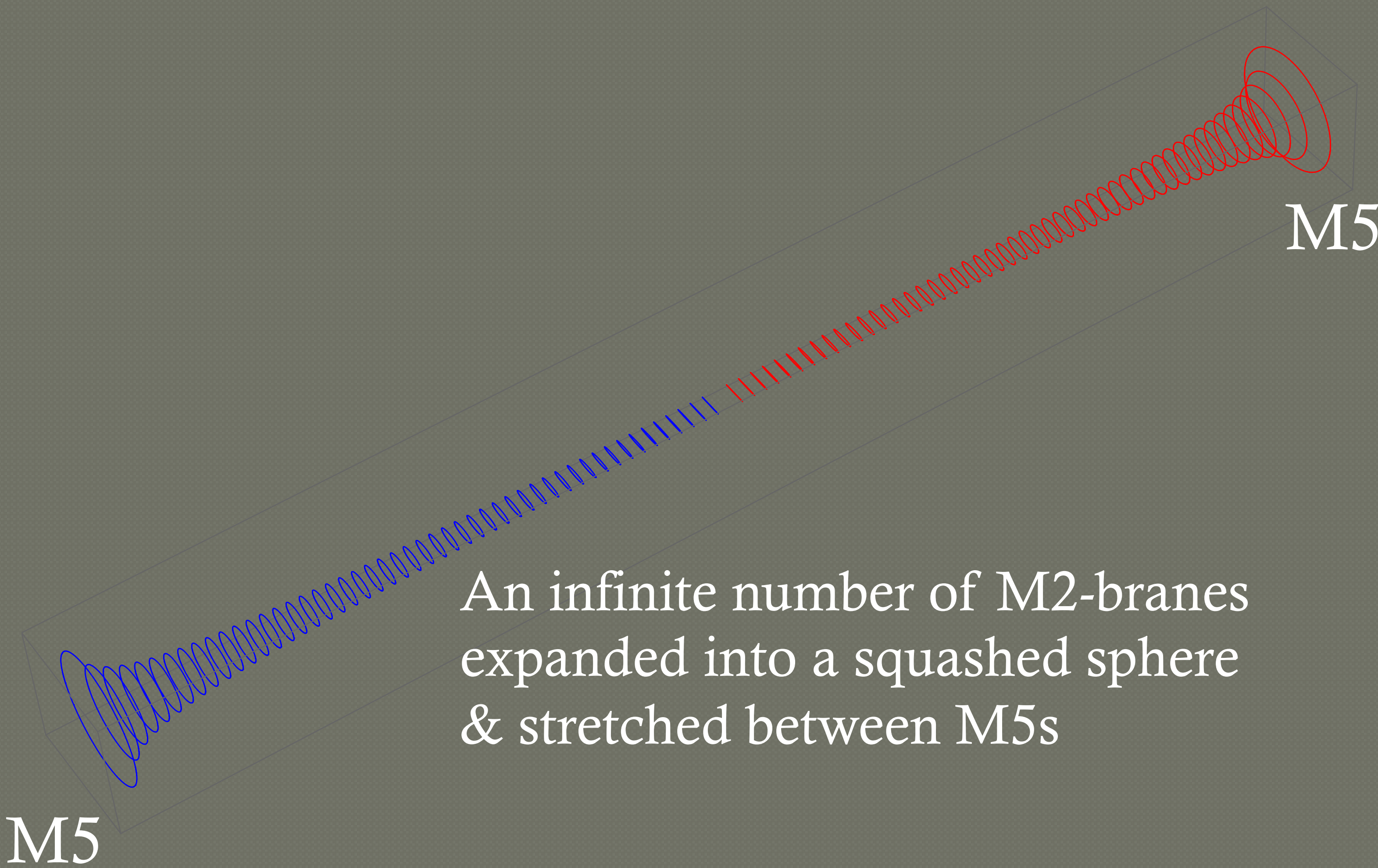
The Basu-Harvey equation

- The BPS equation is identical to the Basu-Harvey eq. which was proposed to describe M2-branes ending on M5-branes.
- Applying out technique in the continuum limit, we can construct generic M2 ending on M5 in a surprisingly simple manner.
- A key is the boundary conditions which are quite different from the GG case.

Near each M5-brane

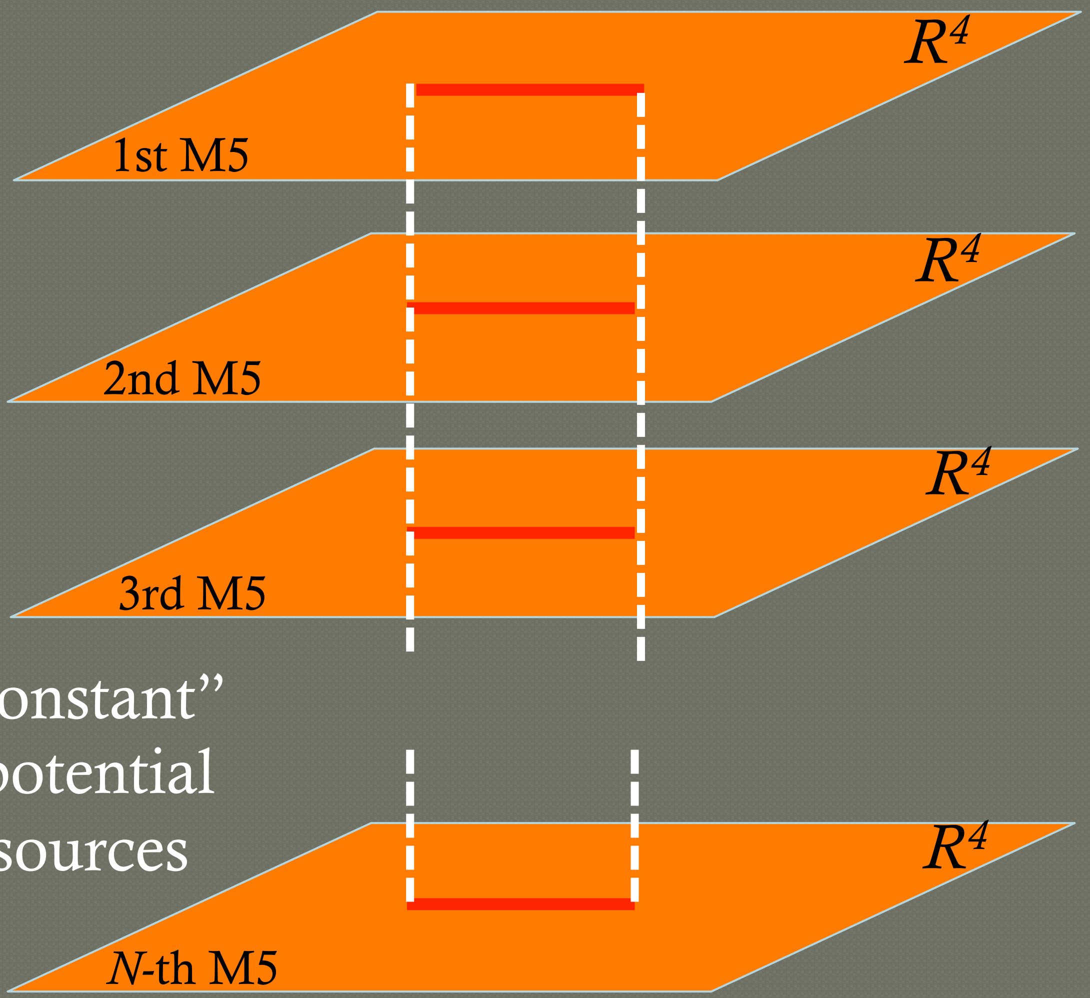
$$\sqrt{\sum_{i=1}^{4}(z_i - a_i)^2} = \frac{C}{\sqrt{s - s_0}}$$

The funnel
M2-branes stretched between 2 M5-branes



An infinite number of M2-branes expanded into a squashed sphere & stretched between M5s

M2-branes junctions ending on multiple M5-branes

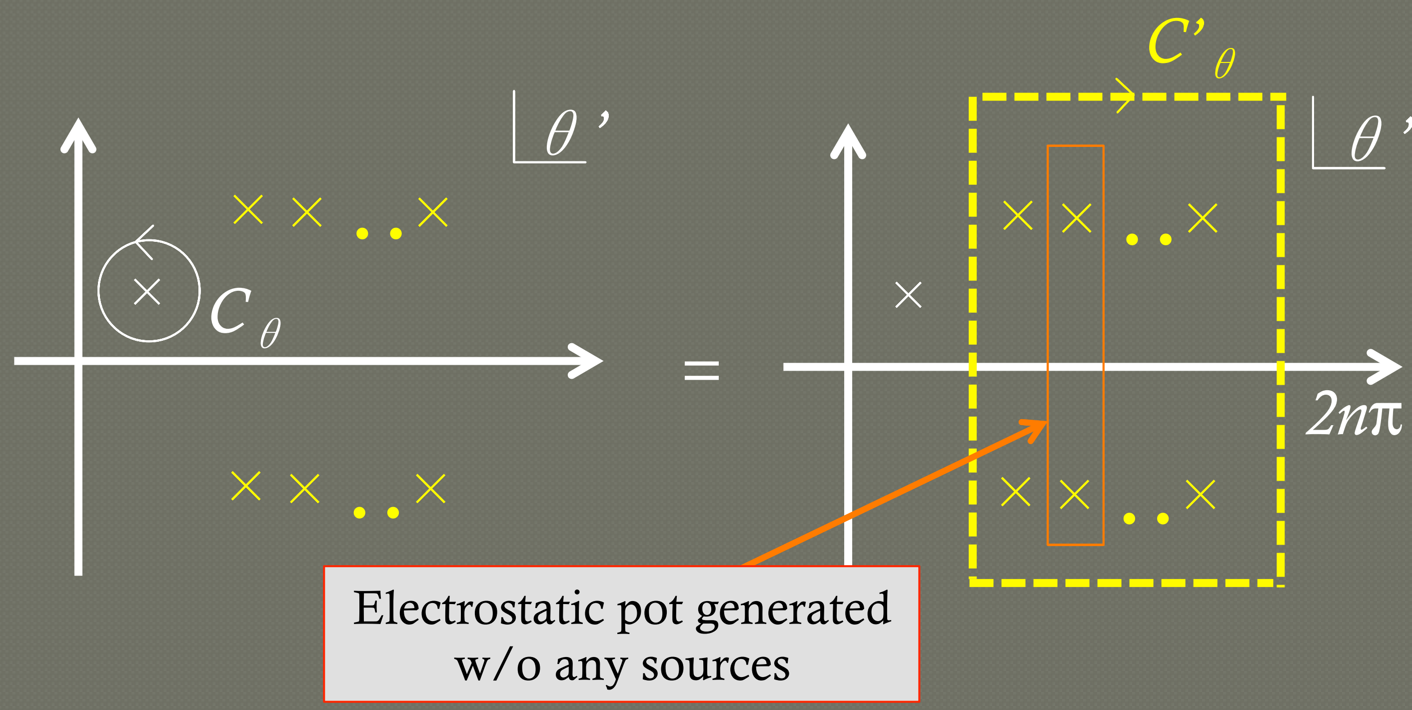


Filled with "constant" electrostatic potential w/o charge sources

M2's ending on M5's in equation

- Somewhat surprisingly, the constant electrostatic potential can be distilled to a sum of nontrivial potentials.

constant =

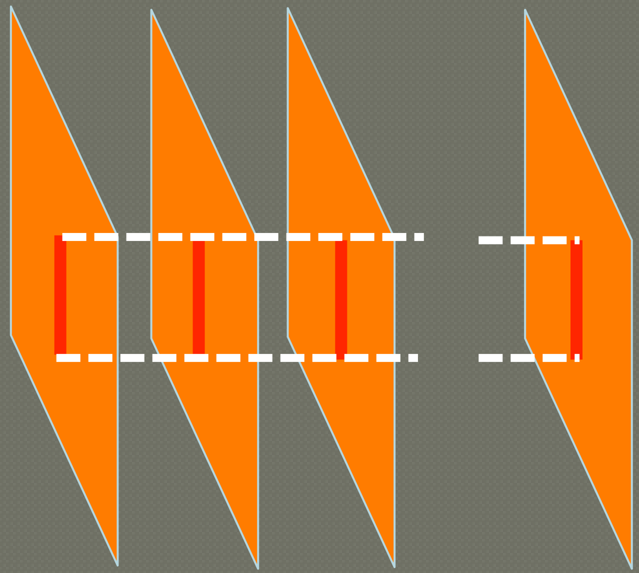


M2's ending on M5's in equation – cont'd

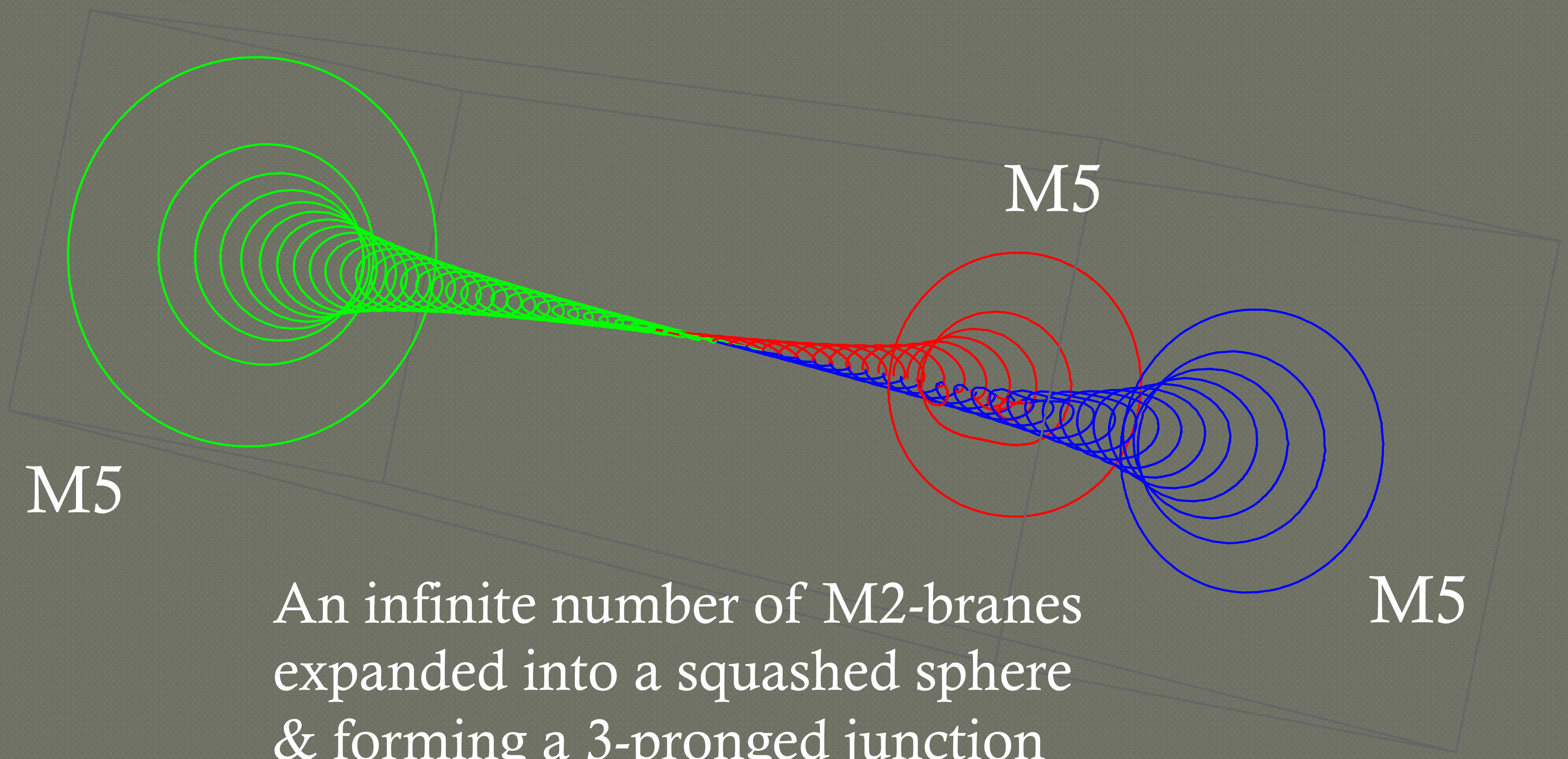
- Multiple M5-branes located at $s = s_k$ and connected by a bundle of M2-branes:

$$s(\vec{z}) = \sum_{k=0}^{n-1} \frac{s_k \sinh \frac{\rho}{n} (\cosh \rho - \cos \theta)}{2n \sinh \rho \left(\cosh^2 \frac{\rho}{2n} - \cos^2 \frac{(\theta + 2(n-k)\pi)}{2n} \right)}$$

which is the electrostatic potential corresponding to



M2-brane junction ending on 3 M5-branes



An infinite number of M2-branes expanded into a squashed sphere & forming a 3-pronged junction

- Good chance to make progress in the construction of the multiple M5-theory, the non-abelian (2,0) CFT_6, from the BLG theory

- Interesting to explore possible connection of 3 M2-brane junction to SYK-like tensor models?

Thank you!