Logarithmic Corrections to Black Hole Entropy

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Corrections to Black Hole Entropy

- Most ***precision*** studies of BHs in string theory focus on BPS black holes (and their near BPS relatives).

- Conventional wisdom: far from BPS there are large and complicated corrections.

- This talk: ***explicitly compute quantum corrections*** to black hole entropy far from the BPS limit.

- Generally the corrections are found to be fairly complicated, as expected.

- But they greatly simplify in some ***environments.***

Environmental Dependence

- Black holes are often solutions to many different theories.
- For example, ***Kerr-Newman black holes*** are usually considered ***solutions to the Einstein-Maxwell theory***

$$\mathcal{L} = \frac{1}{16\pi G_N}\left(R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

- A simple variation: augment the theory by adding a field that appears only quadratically in the action (such as a fermion ψ.)
- The ***solution is "the same"*** because it is consistent to assume that the additional field vanishes $\psi = 0$.
- Environmental dependence: ***corrections depend on such additional fields*** (for example, these fields run in quantum loops).
- This talk: ***Kerr-Newman black holes simplify in an environment with $\mathcal{N} \geq 2$ supersymmetry***.

This Talk

- ***Embedding*** of Kerr-Newman black holes into theories with $\mathcal{N} \geq 2$ SUSY.

- ***Quantum corrections*** to black hole entropy: explicit computation.

- Discussion: a ***non-renormalization theorem.***

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Black Hole Solutions

- Starting point: consider ***any solution to Einstein-Maxwell theory***

$$\mathcal{L} = \frac{1}{16\pi G_N}\left(R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

- The general asymptotically flat stationary black holes: Kerr-Newman (quantum numbers: M, J, Q).

- Special cases:
  - Schwarzchild: M general but J=Q=0
  - Kerr: M and J general but Q=0
  - Reissner-Nordström: M and Q general but J=0
  - BPS M=Q and J=0

- We want to ***consider these as solutions to*** $\mathcal{N} \geq 2$ ***SUGRA.***

N=2 SUGRA

- The baseline: $\mathcal{N}=2$ ***minimal*** SUGRA has bosonic part identical to Einstein-Maxwell theory.
- Any bosonic solution remains a solution after the two gravitini are added because fermions can be consistently set to zero.
- ***Coupling to n_V vector multiplets is a challenge***:

$$\mathcal{L} = \frac{1}{2\kappa^2}R - g_{\alpha\bar{\beta}}\nabla^\mu z^\alpha \nabla_\mu z^{\bar{\beta}} + \frac{1}{2}\mathrm{Im}\left[\mathcal{N}_{IJ}F^{+I}_{\mu\nu}F^{+\mu\nu J}\right]$$

- Comments:
  - Complex scalar fields in vector multiplets: z^α, $\alpha = 1,\ldots,n_V$.
  - Vector fields A^I_μ include the graviphoton so $I = 0,\ldots,n_V$ (one more value).
  - Kähler metric $g_{\alpha\bar{\beta}}$ and vector couplings $\mathcal{N}_{IJ}$ depend on scalars as specified by special geometry.

Adding Scalars to Kerr-Newman

- Kerr-Newman does not have scalars so to maintain the "same" solution we take the $\mathcal{N} = 2$ ***scalars constant***.

- An obstacle: ***generally the vector fields source the scalars*** so they cannot be constant.

- Solution: first specify the projective coordinates X^I for the scalar, then specify the $\mathcal{N} = 2$ vectors in terms of the Einstein-Maxwell vector and the scalars as:

$$F^{+I}_{\mu\nu} = X^I F^{+}_{\mu\nu} \, .$$

- Then the sources on the scalars cancel so ***it is consistent to have constant scalars***.

- Interpretation: a ***non-BPS version of the BPS attractor mechanism***.

More General Embedding

- We consider ***all theories with $\mathcal{N} \geq 2$ SUSY***.

- It is convenient to summarize the matter content in terms of $\mathcal{N} = 2$ fields: one SUGRA multiplet, $\mathcal{N} - 2$ (massive) gravitini, n_V vector multiplets, n_H hyper multiplets.

- This ***decomposition is useful for both BPS and non-BPS***.

- Our embedding takes the geometry unchanged, matter fields "minimal", and guarantees that all equations of motion of $\mathcal{N} \geq 2$ SUGRA are satisfied.

- We want to compute quantum corrections of the Kerr-Newman black hole ***as a solution to $\mathcal{N} \geq 2$ SUGRA.***

Quantum Corrections: Generalities

- The entropy of a large black hole allows the expansion:

$$S = \frac{A}{4G} + \frac{1}{2} D_0 \log A + \dots \; .$$

- ***Taking all parameters with dimension length large***: area $A \sim (2MG)^2$ by dimensional analysis ***up to a function of dimensionless ratios*** $J/M^2, Q/M$ that is nontrivial.

- In the same limit, the logarithmic correction is $\log A \sim \log 2MG$ up to the function D_0 of dimensionless ratios that is interesting.

- The area A and the coefficient D_0 can both be ***computed from the low energy theory***: only massless fields contribute.

- They each offer an ***infrared window into the ultraviolet theory***.

Quantum Fluctuations: Strategy

- All contributions from ***quadratic fluctuations*** around the classical geometry take the schematic form

$$e^{-W} = \int \mathcal{D}\phi \, e^{-\phi\Lambda\phi} = \frac{1}{\sqrt{\det\Lambda}} \, .$$

- The quantum corrections are encoded in the heat kernel

$$D(s) = \mathrm{Tr} \, e^{-s\Lambda} = \sum_i e^{-s\lambda_i} \, .$$

- The effective action becomes

$$W = -\frac{1}{2}\int_{\epsilon^2}^{\infty} \frac{ds}{s} D(s) = -\frac{1}{2}\int_{\epsilon^2}^{\infty} \frac{ds}{s} \int d^D x K(s) \, .$$

- The leading corrections are encoded in the ***the s-independent term in $D(s)$ denoted D_0***, *a.k.a.* the 2nd Seeley-deWitt coefficient, *a.k.a.* the ***integrated*** trace anomaly.

Interactions

- ***In principle***: computations are straightforward applications of techniques from the 70's.
- But: our embedding into SUGRA gives ***nonminimal couplings***.
- For example, for fermions in $\mathcal{N}=2$ hypermultiplets the background enters through Pauli couplings

$$\mathcal{L}_{\text{hyper}} = -2\overline{\zeta}_A \gamma^\mu D_\mu \zeta^A - \frac{1}{2}\left(\overline{\zeta}^A F_{\mu\nu}\Gamma^{\mu\nu}\zeta^B \epsilon_{AB} + \text{h.c.}\right) \; .$$

- Bosons in $\mathcal{N}=2$ vector multiplets (some effort to show)

$$\mathcal{L}_{\text{vector}} = -g_{\alpha\bar{\beta}}\left(\nabla_\mu z^\alpha \nabla^\mu \bar{z}^{\bar{\beta}} + \frac{1}{2} f^\alpha_{\mu\nu} f^{\mu\nu\bar{\beta}} - \frac{1}{2}(F^-_{\mu\nu} f^{\alpha\mu\nu}\bar{z}^{\bar{\beta}} + \text{h.c.})\right)$$

- Such nonminimal couplings force us to compute some new heat kernels.

Heat Kernel Technology

- Perturbative expansion of the (equal point) heat kernel density:

$$K(x, x; s) = \sum_{n=0}^{\infty} s^{n-2} a_{2n}(x)$$

- We need a_4 (the D_0 coefficient is the spacetime integral over a_4)

- Schematic for generalized kinetic operator Λ:

$$\Lambda^n_m = -I^n_m(\mathcal{D}^\mu \mathcal{D}_\mu) - (2\omega^\mu D_\mu)^n_m - P^n_m$$

- General result:

$$(4\pi)^2 a_4(x) = \text{Tr}\left[\frac{1}{2}E^2 + \frac{1}{12}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{180}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu})I\right] .$$

Notation:

$$E = P - \omega^\mu \omega_\mu - (D^\mu \omega_\mu) \ , \quad \mathcal{D}_\mu = D_\mu + \omega_\mu \ , \quad \Omega_{\mu\nu} \equiv [\mathcal{D}_\mu, \mathcal{D}_\nu] \ .$$

Example

- Lagrangian for gravitino in the $\mathcal{N}=2$ SUGRA multiplet:

$$\mathcal{L}_{\text{gravitini}} = -\frac{1}{2\kappa^2}\bar{\Psi}_{A\mu}\gamma^{\mu\nu\rho}D_\nu\Psi_{A\rho} + \frac{1}{4\kappa^2}\bar{\Psi}_{A\mu}\left(F^{\mu\nu} + \gamma_5\widetilde{F}^{\mu\nu}\right)\epsilon_{AB}\Psi_{B\nu}$$

  Note: Pauli coupling involving field strength $F_{\mu\nu}$.

- Heat kernel coefficient:

$$\begin{aligned}(4\pi)^2 a_4^{\text{gravitino}}(x) = -\frac{1}{360}\Big(& 212R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 32R_{\mu\nu}R^{\mu\nu} \\ & -360R_{\mu\nu}(F^{\mu\rho}F^{\nu}_{\ \rho} - \widetilde{F}^{\mu\rho}\widetilde{F}^{\nu}_{\ \rho}) + 180R_{\mu\nu\rho\sigma}(F^{\mu\nu}F^{\rho\sigma} - \widetilde{F}^{\mu\nu}\widetilde{F}^{\rho\sigma}) \\ & +45(F^{\mu\rho}F_{\nu\rho} - \widetilde{F}^{\mu\rho}\widetilde{F}_{\nu\rho})(F_{\mu\sigma}F^{\nu\sigma} - \widetilde{F}_{\mu\sigma}\widetilde{F}^{\nu\sigma})\Big) \ .\end{aligned}$$

- Note: terms of schematic form R^2, RF^2, F^4.

  Schematically: $[D+F, D+F] \sim R + F^2$.

Simplifications

- General form of 2nd Seeley-deWitt coefficient:

$$a_4(x) = \alpha_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \alpha_2 R_{\mu\nu} R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \ldots$$

- After simplifications using Einstein equation, Bianchi identities, ...

$$a_4(x) = \frac{c}{16\pi^2} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} - \frac{a}{16\pi^2} E_4 \ ,$$

  Euler density

$$E_4 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \ .$$

- Note: ***all dependence on field strength is traded for curvature terms***.

- Final results can be expressed in terms of c, a only!

Duality: Einstein-Maxwell Theory

- ***Why*** can all explicit dependence on field strength be eliminated?

- Electromagnetic duality requires that ***four derivative terms are duality invariant*** (even though two derivative terms are not).

- A unique duality invariant tensor: $\mathcal{I}_{\mu\nu\rho\sigma} = F^+_{\mu\nu} F^-_{\rho\sigma}$

- All Lorentz invariants (eg $\mathcal{I}_{\mu\nu\rho\sigma}\mathcal{I}^{\mu\nu\rho\sigma}$) can be recast in terms of:

$$\mathcal{I}_{(\mu\ \nu)\rho}^{\ \ \rho} = F^{+\rho}_{\mu} F^-_{\rho\nu} = R_{\mu\nu}$$

- Upshot: duality precludes explicit dependence on $F_{\mu\nu}$ so ***anomaly coefficients expressible in terms of geometry alone.***

Duality: $\mathcal{N} = 2$ Supergravity

- Duality in $\mathcal{N} = 2$ supergravity: symplectic invariance

- Embedding shows that the ***Maxwell field $F^+_{\mu\nu}$ is duality invariant***

$$F^{+I}_{\mu\nu} = X^I F^+_{\mu\nu} \, .$$

- $U(1)_R$ symmetry: $F^{+I}_{\mu\nu}$ neutral, X^I charged, so ***$F^+_{\mu\nu}$ is (negatively) charged***.

- Electromagnetic ***duality symmetry of Einstein-Maxwell descends from $U(1)_R$ symmetry of $\mathcal{N} = 2$ supergravity***.

- Upshot: $U(1)_R$ symmetry precludes explicit dependence on $F_{\mu\nu}$, ***anomaly coefficients expressible in terms of geometry alone.***

Integrals

- Form of quantum corrections to the entropy:

$$\delta S = \frac{1}{2} D_0\left(\frac{Q}{M}, \frac{J}{M^2}\right) \log A_H$$

- The a-term gives a ***universal*** (independent of BH parameters) contribution to D_0 because

$$\chi = \frac{1}{32\pi^2} \int d^4x \sqrt{-g}\, E_4 = 2\,.$$

- The c-term gives a ***complicated contribution*** to D_0:

$$\int d^4x \sqrt{-g}\, W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} = 64\pi^2 + \frac{\pi\beta Q^4}{b^5 r_H^4 (b^2 + r_H^2)} \Big[4b^5 r_H + 2b^3 r_H^3 + 3(b^2 - r_H^2)(b^2 + r_H^2)^2 \tan^{-1}\left(\frac{b}{r_H}\right) + 3b r_H^5 \Big]\,.$$

$b = J/M$, $r_H = M + \sqrt{M^2 - b^2}$, $\beta = 1/T$.

Results: Logarithmic Corrections

- Contributions from bosons in $\mathcal{N} \geq 2$ theory:

$$c^{\text{boson}} = \frac{1}{60}(137 + 12(\mathcal{N} - 2) - 3n_V + 2n_H)$$
$$a^{\text{boson}} = \frac{1}{90}(106 + 31(\mathcal{N} - 2) + n_V + n_H)$$

- The bosons in the n_H hyper multiplets and $\mathcal{N} - 2$ gravitino multiplets are minimally coupled so these values for a, c are standard.

- The bosons in the n_V vector multiplets and the supergravity multiplet couple to the field strength so, after eliminating F^2 in favor of R, these values of a, c are nonstandard.

- For fermions the situation is reversed.

Results: Logarithmic Corrections

- Contributions from bosons in $\mathcal{N} \geq 2$ theory:

$$c^{\text{boson}} = \frac{1}{60}\left(137 + 12(\mathcal{N}-2) - 3n_V + 2n_H\right)$$
$$a^{\text{boson}} = \frac{1}{90}\left(106 + 31(\mathcal{N}-2) + n_V + n_H\right)$$

- Contributions from fermions in $\mathcal{N} \geq 2$ theory:

$$c^{\text{fermion}} = \frac{1}{60}\left(-137 - 12(\mathcal{N}-2) + 3n_V - 2n_H\right)$$
$$a^{\text{fermion}} = \frac{1}{360}\left(-589 + 41(\mathcal{N}-2) + 11n_V - 19n_H\right)$$

- The c ***coefficent vanishes in*** $\mathcal{N} \geq 2$ ***theory!***

- A ***huge simplification***: Weyl^2 terms are complicated in general backgrounds.

- It is ***a surprise***: SUSY of the background $\Rightarrow$ AdS$_2 \times S^2 \Rightarrow$ $\text{Weyl}^2 = 0 \Rightarrow$ vanishing ***coefficient*** of Weyl^2 not noticed.

Summary: Quantum Corrections

- Logarithmic corrections to black hole entropy in $\mathcal{N} \geq 2$ SUGRA are determined by the coefficient of the Euler invariant.

- This coefficient is universal: it ***depends only on the theory*** (not on parameters of the black hole)

$$\delta S = \frac{1}{12}\left(23 - 11(\mathcal{N} - 2) - n_V + n_H\right) \log A_H \ .$$

- These corrections can be reproduced from microscopic theory in some BPS cases.

- The IR theory is a window into the UV theory: apparently the ***deformation (far!) off extremality is independent of coupling!***

- A minor caveat: fermionic zero modes (due to enhanced SUSY) gives a jump at extremality (in most ensembles).

Higher Derivative Corrections

- ***Why*** is the anomaly coefficient $c = 0$?

- Background is generally not supersymmetric so ***fluctuations are not in supermultiplets***.

- Background field formalism realizes symmetry explicitly: dependence on background fields respect $\mathcal{N} = 2$ supersymmetry.

- Schematic form of effective action

  $$\mathcal{L}_4 = g_W(\mathrm{Weyl}^2 + \mathrm{SUSY\ partners}) - g_E(\mathrm{Euler} + \mathrm{SUSY\ partners})$$

  Coefficients g_W, g_E are running couplings with β-function essentially c, a.

Higher Derivatives and $\mathcal{N}=2$ SUSY

- Details of the action: off-shell formalism from reduction of superconformal supersymmetry, a lot of auxiliary fields,...... (details involve hard work).

- SUSY partners to Weyl^2 were identified a long time ago.

- Schematic of on-shell structure:

$$\mathrm{Weyl}^2 + \mathrm{SUSY\ partners} = E_4$$

  Cartoon: there is an elaborate cancellation between gravitational terms (Weyl^2), their matter partners (F^4), and cross-terms (RF^2).

Higher Derivatives and $\mathcal{N}=2$ SUSY

- SUSY partners to E_4 were identified only in the last few years.

- Schematic of on-shell structure:

$$E_4 + \text{SUSY partners} = E_4$$

  Cartoon: the matter terms vanish on-shell.

- So: all matter terms can be eliminated in favor of geometry alone.

- And: both four-derivative $\mathcal{N}=2$ invariants reduce to the Euler invariant E_4.

- ***The anomaly $c=0$ because W^2 is inconsistent with $\mathcal{N}=2$ SUSY.***

SUSY Theory and Kerr-Newman

- Another application: string theory corrections can give a Weyl^2 term ***directly in the action***.

- Quantum result: the coefficient of this term does not receive quantum corrections, it is not renormalized in $\mathcal{N} \geq 2$ SUGRA.

- The full equations of motion are extremely elaborate due to all the terms required by SUSY but they ***are satisfied by Kerr-Newman***. (in pure $\mathcal{N} = 2$ SUGRA, no vector multiplets)

- Again: there is an elaborate cancellation between gravitational terms (Weyl^2), their matter partners (F^4), and cross-terms (RF^2).

- The simplifications are for a ***theory with $\mathcal{N} = 2$ SUSY*** but ***solutions that preserve no SUSY***.

Wald Entropy

- The geometry is the same, but the Wald entropy is changed.

- ***Corrections to Wald entropy simplify greatly***. Schematically:

$$\partial_{\text{Riem}}(\text{Weyl}^2 + \text{SUSY partners}) = \partial_{\text{Riem}}\left((\text{Riem}^2 - 2\text{Ric}^2 + \frac{1}{3}R^2) + \frac{1}{4}\text{Ric}\ F^2\right)$$
$$= \partial_{\text{Riem}}\left(\text{Riem}^2 - 4\text{Ric}^2 + R^2\right) = \partial_{\text{Riem}} E_4$$

- The correction to the Wald entropy due to higher order derivatives is a ***constant, independent of black hole parameters***:

$$\Delta S = 256\pi g_W$$

- The value of the constant can be related to microscopic theory for BPS black holes.

Summary

- We evaluated corrections to Kerr-Newman black holes motivated by string theory: quantum corrections and higher derivative corrections.

- Perspective: $\mathcal{N} \geq 2$ ***SUSY of the theory simplifies results greatly even when BHs preserve no SUSY.***

- Quantum corrections: independent of mass (so the same as for BPS black holes)

- Higher derivative corrections $(\mathrm{Weyl}^2 + \mathrm{SUSY})$ also independent of mass (so the same as for BPS black holes)

- Significance: evidence that ***black hole entropy far from extremality is accounted for by weakly coupled strings.***