Non-Lorentzian Geometry in Gravity and String Theory

Johburg Workshop

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based on work:

1504.0746 (JHEP) (Hartong,NO), 1604.08054 (PRD) ((Hartong,Lei,NO)

1607.01753, 1607.01926 (Festuccia, Hansen, Hartong NO)

in progress: Hartong,Lei,NO/Hartong,Lei,NO,Oling/

Harmark,Hartong,NO/Grosvenor,Hartong,Keeler,NO

de Boer,Hartong,NO,Sybesma,Vandoren

earlier work: 1409.1519 (PLB), 1409.1522 (PRD), 1502.00228 (JHEP)
(Hartong,Kiritsis,NO)

1311.4794 (PRD) & 1311.6471 (JHEP) (Christensen,Hartong,NO,Rollier

Introduction

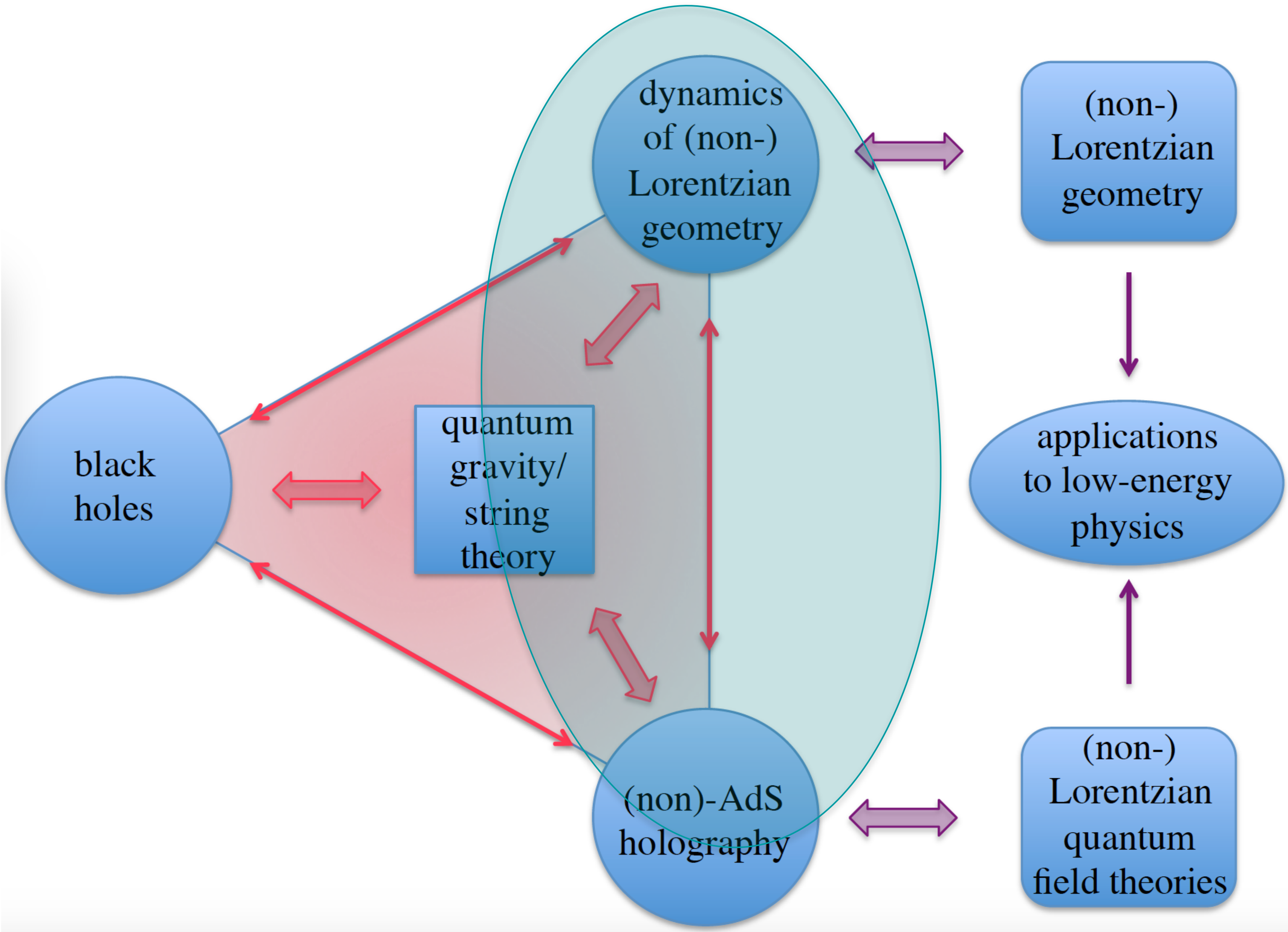
many branches of physics are not controlled by Lorentz symmetries, but by limits thereof or other effective symmetries:

- field theory (condensed matter/stat phys)
- gravity (approximations to GR/physics on null hypersurfaces)
- holography (boundaries may appear singular from pseudo-Riem pov)
- string theory (limits)

non-Lorentzian geometry has in recent years seen revival and appearance in these contexts

In this talk I will focus on non-Lorentzian geometry
and its relevance for new theories of gravity (holography)
and a novel appearance in string theory

The Non-Lorentzian "Universe"



Motivation

gravity

- interesting in own right to find new theories of gravity
  (w. other local symmetries and still diff inv.)
- applications in holography as new bulk theories
  (important for non-AdS holography)
- cosmology
- condensed matter
- toy models of quantum gravity, new insights into quantum behavior (BH ?)

string theory

- tractable limits of string theory
- non-relativistic dispersion relations seen in limits of AdS/CFT
- new string theories ? (non-relativistic sigma models on a non-rel WS)
- non-Lorentzian target spaces and
  low energy effective actions as novel theories of gravity

Outline

- part I: brief intro to non-Lorentzian geometry

- part II: CS theories on non-Lorentzian algebras
  (ext. Bargmann, Newton-Hooke, Schroedinger, scaling-Carroll)

punchlines:
- interesting new theories of 2+1 gravity
- novel types of FTs (w. anistropic scaling) on boundary/non-AdS holography
- potentially interesting asymptotic symmetry groups/parallels with AdS

- part III: non-Lorentzian geometry in non-relativistic string theory

punchlines:
- new non-relativistic limit of string theory involving non-Lorentzian WS
  and target space geometries,
- specific non-rel. WS limit connects to previously studied limits of AdS/CFT (SMT)
- LL model is a non-rel. ST with NC-like target space geometry

Part I: Non-Lorentzian geometry: first look

very generally: take some symmetry algebra that includes space and time translations (and spatial rotations, assume isotropic): "Aristotelian" symmetries

gauge the symmetry and turn space/time translations into local diffeomorphisms

Poincare -> Lorentzian(pseudo-Riemannian) geometry (relativistic)

Galilean/Bargmann -> torsional Newton-Cartan geometry (non-relativistic)

Carroll -> Carrollian geometry (ultra-relativistic)

crucial difference -> type of boosts		geometry
L: Lorentz	$t \to \gamma(t + \vec{v}\vec{x}/c^2) \quad , \quad \vec{x} \to \gamma(\vec{x} + \vec{v}t)$	$g_{\mu\nu}$
G/B: Galilean/Bargmann	$t \to t \quad , \quad \vec{x} \to \vec{x} + \vec{v}t$	$\tau_\mu \quad , \quad h^{\mu\nu} \quad , \quad m_\mu$
C: Carroll	$t \to t + \vec{v}\vec{x} \quad , \quad \vec{x} \to \vec{x}$	$v^\mu \quad , \quad h_{\mu\nu}$

Relevant non-relativistic algebras

Galilean

$H, P_a, J_{ab}, G_a \quad N$

Bargmann

(Galilean algebra is c to infinity limit of Poincare)

$$[H, G_a] = P_a \qquad [P_a, G_b] = 0$$

$$[P_a, G_b] = N\delta_{ab}$$

Lifshitz (aka "Aristotelian"+scaling)

$H, P_a, J_{ab}, D, \quad G_a, N, K(z=2)$

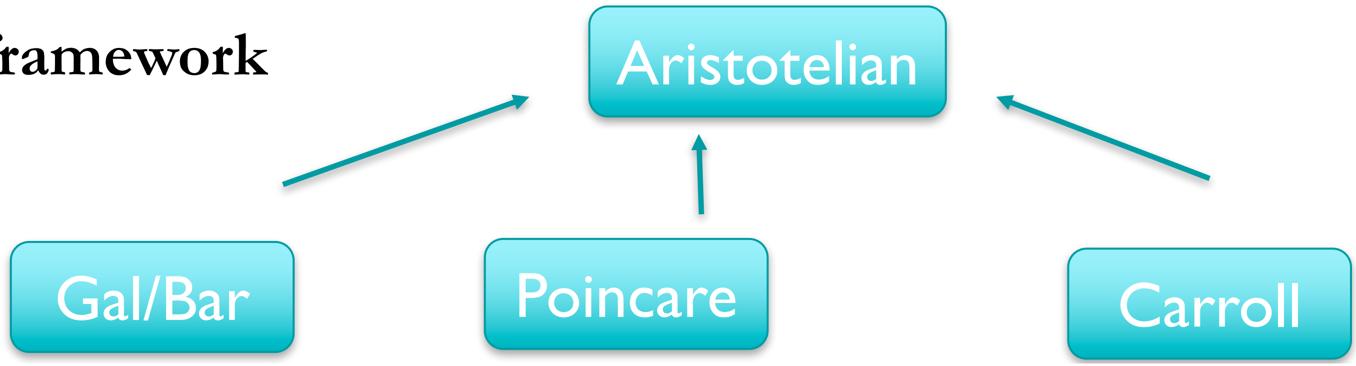
Schrödinger

$$[D, H] = zH \qquad [D, P_a] = P_a$$

$$[D, N] = (2-z)N$$

Schrödinger = Bargmann + dilatations (+ special conformal for z=2)

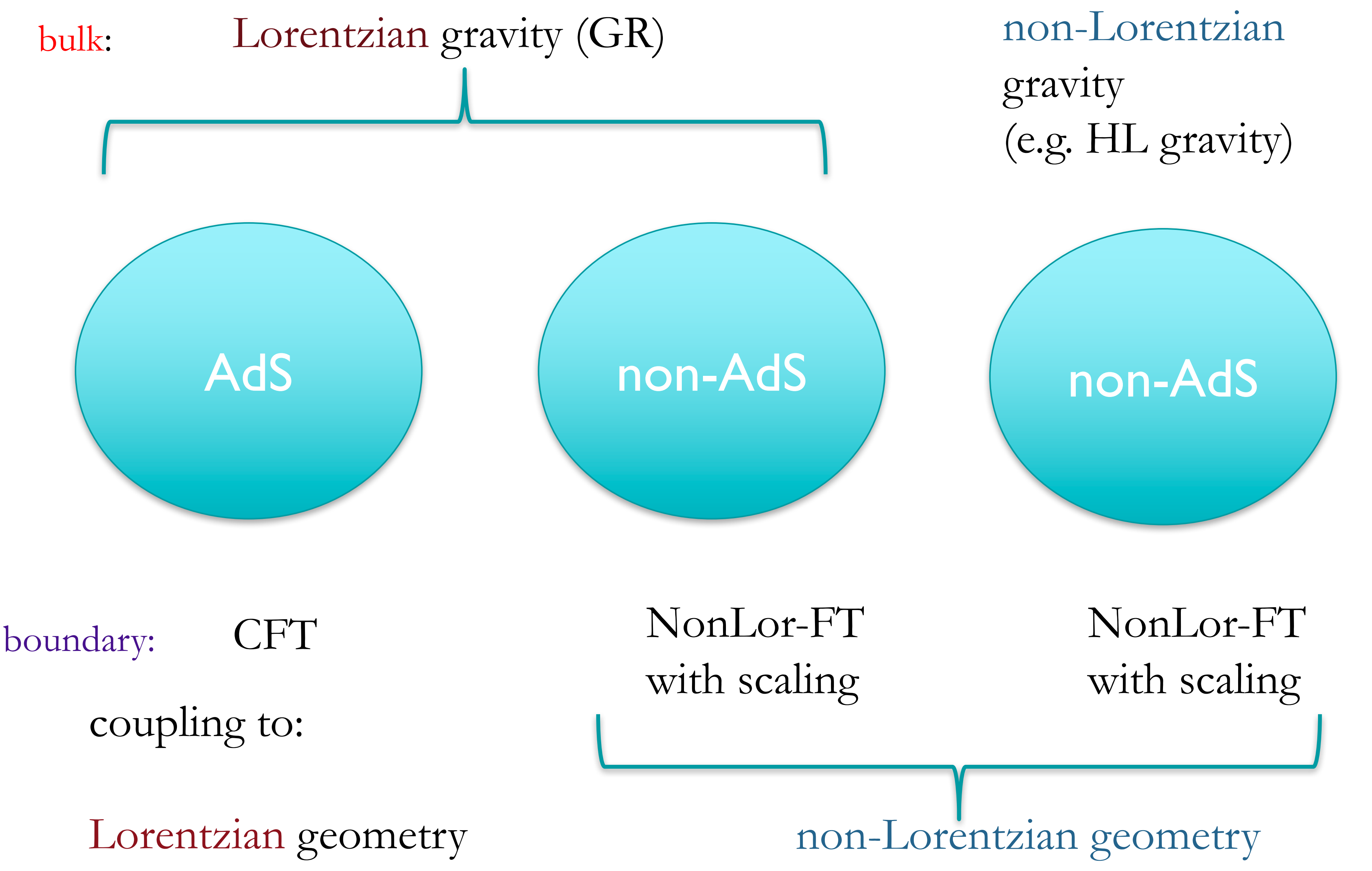
Classification framework



Symmetry	Galilean/Bargmann	Poincaré	Carroll
geometry	Newton–Cartan $\tau_\mu, h^{\mu\nu}, m_\mu$	pseudo-Riemannian. $g_{\mu\nu}$	Carrollian $v^\mu, h_{\mu\nu}$
causal structure	non-rel.	Minkowski	ultra-rel.
response	energy current momentum current symmetric stress	symmetric EM tensor	energy density momentum current symmetric stress
boost Ward-identity	momentum flux = mass flux	momentum current =energy current	energy flux = 0
scaling	$\forall z$	$z = 1$	$\forall z$
dynamical geometry	HL-gravity (w. $U(1)$) CS on non-rel. algebra + dyn. non-rel. sources	GR + dyn. rel. sources	ultra-relativistic gravity + dyn. ultra-rel. sources
holographic realization	EMD-model, . . . HL/CS gravity	AdS/CFT	flat space

hydro: universal framework in upcoming work (de Boer,Hartong,NO,Sybesma,Vandoren)

More General Holographic Setups



Newton-Cartan makes Galilean local

- NC geometry originally introduced by Cartan to geometrize Newtonian gravity

  Eisenhart,Trautman,Dautcourt,Kuenzle,Duval,Burdet,Perrin,Gibbons,Horvathy,Nicolai,Julia…..

→ both Einstein's and Newton's theories of gravity admit geometrical formulations which are diffeomorphism invariant

- NC originally formulated in "metric" formulation
  more recently: vielbein formulation (shows underlying sym. principle better)

  Andringa,Bergshoeff,Panda,de Roo

Riemannian geometry: tangent space is Poincare invariant

Newton-Cartan geometry: tangent space is Bargmann (central ext. Gal.) invariant

- gives geometrical framework and extension to include torsion
i.e. as geometry to which non-relativistic field theories can couple
(boundary geometry in holographic setup is non-dynamical)

* will consider dynamical (torsional) Newton-Cartan

Equivalance principles

Manifestations of Einstein's equivalence principle

- gauging Poincare
- cosets (Minkowski = Pioncare/Lorentz)
- Noether procedure in field theory

- Non-Lorentzian geometry:
apply equivalence principle to any of other boost structures (or no boost at all)

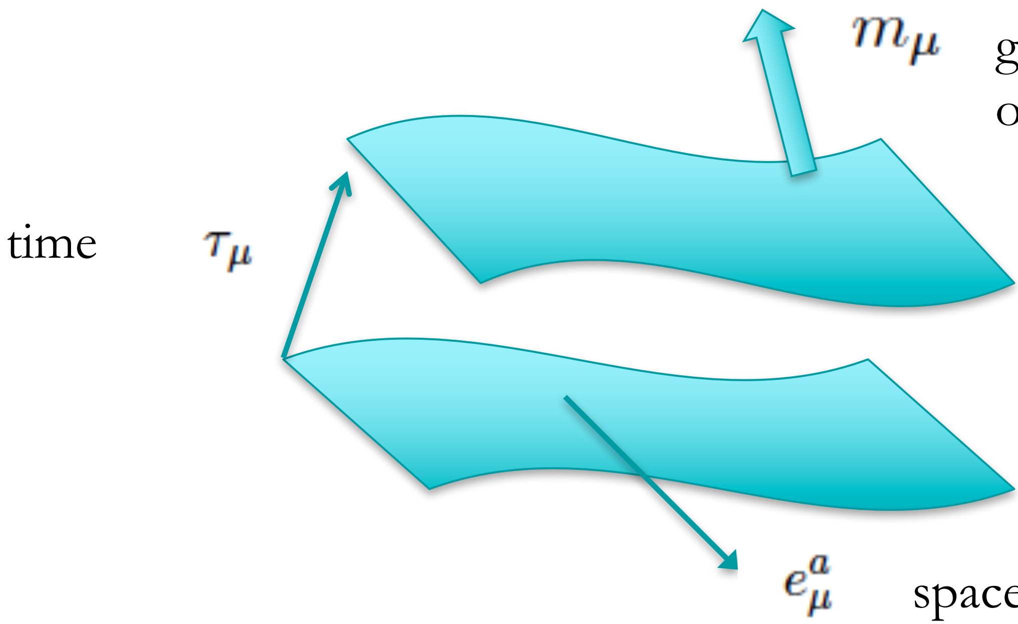
- gauging Bargmann

  Andringa,Bergshoeff,Panda,de Roo
- cosets

  Grosvenor,Hartong,Keeler,NO
- Noether procedure in Galilean/Bargmann field theory

  Festuccia,Hansen,Hartong,NO

Newton-Cartan geometry



gauge field of central extension
of Galilean algebra (Bargmann)

(inverse) spatial metric:

$$h^{\mu\nu} = e^\mu_a e^\nu_b \delta^{ab}$$

NC geometry = no torsion → $\tau_\mu = \partial_\mu t$ notion of absolute time

TTNC geometry = twistless torsion → $\tau_\mu = \mathrm{HSO}$ preferred foliation in equal time slices

TNC geometry no condition on τ_μ

- in TTNC: torsion measured by $a_\mu = \mathcal{L}_{\hat{v}} \tau_\mu$
  geometry on spatial slices is Riemannian

Adding torsion to NC

Christensen,Hartong,Rollier,NO
Hartong,Kiritsis,NO/Hartong,NO
Bergshoeff,Hartong,Rosseel

- inverse vielbeins (v^μ, e^μ_a)

$$v^\mu \tau_\mu = -1\,, \qquad v^\mu e^a_\mu = 0\,, \qquad e^\mu_a \tau_\mu = 0\,, \qquad e^\mu_a e^b_\mu = \delta^b_a$$

can build Galilean boost-invariants

$$\hat{v}^\mu = v^\mu - h^{\mu\nu} M_\nu\,,$$
$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \tau_\mu M_\nu - \tau_\nu M_\mu\,,$$
$$\tilde{\Phi} = -v^\mu M_\mu + \tfrac{1}{2} h^{\mu\nu} M_\mu M_\nu\,,$$

-introduce Stueckelberg scalar chi:
(to deal with broken/unbroken N-sym)

$$M_\mu = m_\mu - \partial_\mu \chi.$$

affine connection of TNC (inert under G,J,N)

$$\Gamma^\rho_{\mu\nu} = -\hat{v}^\rho \partial_\mu \tau_\nu + \frac{1}{2} h^{\rho\sigma} \left(\partial_\mu \bar{h}_{\nu\sigma} + \partial_\nu \bar{h}_{\mu\sigma} - \partial_\sigma \bar{h}_{\mu\nu} \right)$$

with torsion $\Gamma^\rho_{[\mu\nu]} = -\tfrac{1}{2} \hat{v}^\rho (\partial_\mu \tau_\nu - \partial_\nu \tau_\mu)$

$$\nabla_\mu \tau_\nu = 0\,, \qquad \nabla_\mu h^{\nu\rho} = 0\,,$$

analogue of metric compatibility

connection obtained via Noether: [Festuccia,Hansen,Hartong,NO](1607)

Further remarks on TNC

- can also get TNC structures from null reduction

$$ds^2 = \gamma_{AB} dx^A dx^B = 2\tau_\mu dx^\mu \left(du - m_\nu dx^\nu\right) + h_{\mu\nu} dx^\mu dx^\nu$$

$$\gamma^{uu} = 2\tilde{\Phi}\,, \qquad \gamma^{u\mu} = -\hat{v}^\mu\,, \qquad \gamma^{\mu\nu} = h^{\mu\nu}$$

- related to non-relativistic limit of GR

$$\begin{aligned} g_{\mu\nu} &= -c^2\tau_\mu\tau_\nu + h_{\mu\nu} - \tau_\mu m_\nu - \tau_\nu m_\mu + \mathcal{O}(c^{-1}) \\ g^{\mu\nu} &= h^{\mu\nu} + c^{-2}\left(-v^\mu v^\nu + \beta^{\mu\nu}\right) + \mathcal{O}(c^{-3}) \end{aligned}$$

for tau closed: NC gravity (Newtonian gravity in covariant form)

generalization of 1/c expansion for tau HSO:

van Bleeken(2017)

Non-Lorentzian Geometry in Gravity

- interesting to make non-Lorentzian geometry dynamical
  -> "new" theories of gravity

have shown: dynamical Newton-Cartan (NC) = Horava-Lifshitz (HL) gravity

Hartong,NO (1504)

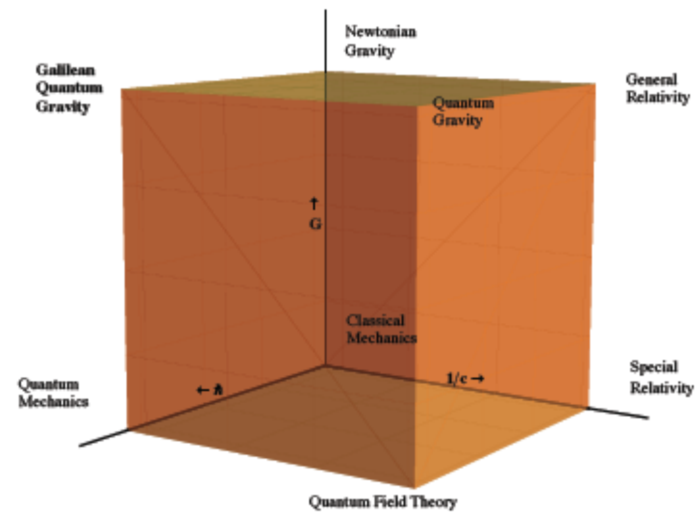
Horava (0812,0901);
Horava,Melby-Thompson (2010)

natural geometric framework with full diffeomorphism invariance

such theories of gravity interesting as
- other bulk theories of gravity in holographic setups
- effective theories (cond mat, cosmology)

Griffin,Horava,Melby-Thompson (2012)
Janiszweski,Karch (2012)

- Galilean quantum gravity
  infinite c limit of $(\hbar, G_N, 1/c)$ cube



- opposite case: Carollian gravity

Hartong (1505)

Part II: Non-Lorentzian Chern-Simons theories

3D Einstein gravity = CS gauge theory
-> insights into classical and quantum properties of theory
holographic dualities with 2D CFTs
black holes

can 3D non-relativistic gravity theories be reformulated as CS ?

Hartong,Lei,NO(1604)

CS on extended Bargmann/Newton-Hooke algebra
= 3D U(1)-invariant projectable HL gravity = 3D dynamical NC gravity
(with/without cosmo constant)

CS on extended Schroedinger algebra
= novel conformal non-projectable U(1)-invariant HL gravity
= novel dynamical TTNC gravity -> CS Schroedinger gravity

CS on ext. Bargmann: Papageorgiou,Schroers (0907)
Bergshoeff,Rosseel (1604): CS on ext. Bargmann from non-rel limit of Einstein gravity & 2 vectors
& extended Bargmann supergravity

Chern-Simons on non-relativistic algebras

Hartong,Lei,NO

CS Lagrangian $\mathcal{L} = \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$

need invariant bilinear form -> non-trivial requirement for non. rel algebras (non-semi simple Lie algebras)

Galilean algebra $[J\,, P_a] = \epsilon_{ab} P_b\,,\quad [J\,, G_a] = \epsilon_{ab} G_b\,,\quad [H\,, G_a] = P_a$

Bargmann algebra
(central extension = mass generator)

$[P_a\,, G_b] = N \delta_{ab}$

2+1 dim special:
can further extend
(S,Y,Z central wrt Gal
but not nec. rest)

$$[G_a\,, G_b] = S \epsilon_{ab}\,,\quad [P_a\,, P_b] = Z \epsilon_{ab}$$
$$[P_a\,, G_b] = N \delta_{ab} - Y \epsilon_{ab}\,.$$

can add a SL(2,R) generated by H,D,K to this

From 1st order formulation to 3D HL/NC gravity

CS action on extended Bargmann includes a term:

$$\mathcal{L}_{\text{kin}} = R^2(G) \wedge e^1 - R^1(G) \wedge e^2 + \tau \wedge \Omega^1 \wedge \Omega^2 - m \wedge d\omega + \zeta \wedge d\tau$$

write in metric form by integrating out:

zeta = Lagrange multiplier for torsionless constraint
Omega1,2 = boost connections
omega = (2dim) rotation connection

leaves as indep variables: NC variables τ_μ, e^a_μ and m_μ

-> gives U(1)-inv. 3D HL in NC covariant form

[Hartong,Lei,NO]

$$S = \int d^3x e \left[C \left(h^{\mu\rho} h^{\nu\sigma} K_{\mu\nu} K_{\rho\sigma} - (h^{\mu\nu} K_{\mu\nu})^2 - \tilde{\Phi} \mathcal{R} \right) - \mathcal{V} \right]$$

Horava,Melby-Thompson(2010)
Hartong,NO(1504)

Chern-Simons on extended Newton-Hooke

Hartong,Lei,NO (to app.)

$$[G_a\,, G_b] = S\epsilon_{ab}\,, \quad [H\,, P_a] = -\Lambda_c G_a\,,$$
$$[P_a\,, P_b] = \Lambda_c S\epsilon_{ab}\,.$$

has four parameter non-degenerate bilinear form

$$\mathcal{J} = \frac{J - iH}{2}; \quad \mathcal{P}_1 = \frac{P_1 + iG_2}{2}; \quad \mathcal{P}_2 = \frac{P_2 - iG_1}{2}; \quad T = \frac{1}{2}(-S - iN)$$

algebra is 2 copies of the central extended 2D Poincare algebra E_2^c

$$[\mathcal{J}, \mathcal{P}_a] = \epsilon_{ab}\mathcal{P}_b, \qquad [\mathcal{P}_a, \mathcal{P}_b] = T\epsilon_{ab}$$

so we get CS theory on the product of two E_2^c

CS gauge field on one copy $\quad \mathcal{A} = \mathcal{J}E + \mathcal{P}_1 E_1 + \mathcal{P}_2 E_2 + T\eta$

CS action $\quad \mathfrak{L} = 2E \wedge d\eta + E^a \wedge dE^a + 2E \wedge E^1 \wedge E^2$

Combining the E2_c copies

gauge field in original Newton-Hooke basis

$$A = H\tau + P_a e^a + G_a \omega^a + J\omega + Nm + \zeta S$$

$$\begin{aligned}
\mathfrak{L}[c_1] &= \tau \wedge d\zeta - \omega \wedge dm + \omega^2 \wedge de^1 - e^2 \wedge d\omega^1 - \omega \wedge e^a \wedge \omega^a - \tau \wedge e^1 \wedge e^2 + \tau \wedge \omega^1 \wedge \omega^2 \\
\mathfrak{L}[c_2] &= -2\omega \wedge d\zeta - 2\tau \wedge dm + e^a \wedge de^a - \omega^a \wedge d\omega^a + 2\omega \wedge e^1 \wedge e^2 - 2\omega \wedge \omega^1 \wedge \omega^2 \\
&\quad -2\tau \wedge e^1 \wedge \omega^1 - 2\tau \wedge e^2 \wedge \omega^2
\end{aligned}$$

then

$$\begin{aligned}
\mathfrak{L}[\mathcal{A}] + \mathfrak{L}[\bar{\mathcal{A}}] &= \mathfrak{L}[c_2] \\
\mathfrak{L}[\mathcal{A}] - \mathfrak{L}[\bar{\mathcal{A}}] &= \mathfrak{L}[c_1]
\end{aligned}$$

expect propagating degrees of freedom when c_2 not zero
(cf. topological massive gravity with different CS levels in the two sectors)

Vacuum solution and relation to SO(2,2) CS

positive CC $\mathcal{A} = i\mathcal{J}dt + \mathcal{P}_1 e^t dz + \mathcal{P}_2(-ie^t)dz = i\mathcal{J}dt + \mathcal{P}\_e^t dz$

$\tau = dt; \quad e^a = e^t dx^a$ (dS space)

negative CC $\mathcal{A} = \mathcal{J}dt + \mathcal{P}_1 \sin t dz + \mathcal{P}_2 \cos t dz$

$\tau = dt, \quad e^1 = \sin t dx_1; \quad e^2 = \cos t dx_2$ (AdS space)

CS on extended NH can be obtained as c -> infinity limit of relativistic SO(2,2) CS + two U(1)'s (Bergshoeff,Rosseel)

acting on AdS3 we (probably) get a time-independent (non-metric) solution with m_mu turned on
-> examine this is also for the BTZ black hole

Infinite-dimensional extension

E_2^c has infinite dim extension:

$$[L_m, L_n] = (m-n)L_{m+n} + cm^3\delta_{m+n,0}$$

$$[L_m, N_n] = -nN_{m+n} + kn^2\delta_{m+n,0}$$

Virasoro $\times \breve{\hat{u}(1)}$.

2 central extensions

finite subgroup: L0, L1, N0, N-1 = E2^c

suggestive for asymptotic sym. group and connection to WCFT

[Hofman, Rollier](1411)

[Afshar,Detournay,Grumiller,Oblak](1512)

algebra is twisted U(1) current algebra also seen in

(note twist cannot be removed unless U(1) has central term)

Chern-Simons Schroedinger Gravity

Hartong,Lei,NO(1604)

CS with gauge connection on "triply-extended" Schroedinger algebra

$$\begin{aligned} A &= H\tau + P_a e^a + G_a \omega^a + J\omega + Nm + Db + Kf \\ &\quad + S\zeta + Y\alpha + Z\beta\,. \end{aligned} \tag{17}$$

(gives action with 3 distinct invariants -> 3 central charges)

-> new way of constructing conformal actions for non-projectable HL gravity (with U(1) sym.)

- useful starting point to do holography with HL (in CS form)
- theory has Lifshitz vacua ("minimal" setup to do Lif holography)
- new dualities involving novel class of 2D non-rel field theories on bdry featuring:
- novel (infinite dimensional) conformal non-relativistic algebras

[Hartong,Lei,NO (to appear)

- analogue of Lorentz CS terms (Galilean boost, rotation anomalies ?)

New infinite dimensional NR conformal algebra

[Hartong,NO,Oling,Yang](in progress)

extended Schroedinger algebra is combo of: (contraction of rel. conf. algebra)

-2D GCA (Galilean conformal algebra) H, D, K, S, Y, Z

-3D z=2 Schroedinger algebra H, D, K, P_a, G_a, J, N

(H, D, K: SL(2,R))

This finite algebra can be embedded into a novel NR infinite-dim conformal algebra

$$H, D, K \rightarrow L_n \quad , \quad S, Y, Z \rightarrow M_n$$

$$N \rightarrow N_n \quad , \quad (P_a, G_a) \rightarrow Y^a_r \quad , \quad J \rightarrow J_n$$

$$[\hat{L}_m, \hat{L}_n] = (m-n)\hat{L}_{m+n} + \frac{c_L}{2}(m^3 - m)\delta_{m+n,0} \qquad [\hat{L}_m, \hat{M}_n] = (m-n)\hat{M}_{m+n} + \frac{c}{2}(m^3 - m)\delta_{m+n,0}$$

$$[\hat{L}_m, Y^i_r] = (\frac{m}{2} - r)Y^i_{m+r}, \qquad [\hat{L}_m, J_n] = -nJ_{m+n}, \qquad [\hat{L}_m, N_n] = -nN_{m+n}$$

$$[Y^i_r, Y^i_s] = (r-s)N_{s+r}, \qquad [Y^1_r, Y^2_s] = -\hat{M}_{r+s} + c\left(s^2 - \frac{1}{4}\right)\delta_{r+s,0}, \qquad [J_n, Y^i_r] = Y^j_{r+n}\epsilon_{ij}$$

$$[J_m, N_n] = cn\delta_{m+n,0} \qquad [J_n, \hat{M}_m] = -2nN_{m+n}, \qquad [J_m, J_n] = c_J n\delta_{m+n,0}$$

three central charges

Remarks

- the infinite dim algebra admits three central extensions:
-> matches the number of constants that enter the action

- plays role in asymptotic symmetry algebra around vacuum solutions with appropriate boundary conditions (under investigation)

- bosonic version of centrally extended homogeneous super GCA

Carrollian (ultra-relativistic) CS theory

[Hartong,NO,Yang](to appear)

does there exist an analogue of the CS Schr. gravity for the ultra-relativistic case ?

starting point: Carroll algebra and add a scaling generator

$$[J, P_a] = \epsilon_{ab} P_b; \qquad [J, C_a] = \epsilon_{ab} C_b; \qquad [P_a, C_b] = \delta_{ab} H$$
$$[D, P_a] = -P_a; \qquad [D, C_a] = (1-z) C_a; \qquad [D, H] = -zH$$

for non-deg bilinear from, need z=0 and extra generator

$$[P_a, C_b] = \delta_{ab} H - \epsilon_{ab} Y$$

determinant = $(c_1^2 + c_2^2)^2$.

CS gauge field: $A = H\tau + P_a e^a + C_a \omega^a + J\omega + Db + \alpha Y$

$$\begin{aligned} \mathcal{L} \;=\; & 2c_1\big(d\omega \wedge \tau + \epsilon^{ab} e^b \wedge d\omega^a - \alpha \wedge db - \omega \wedge \omega^a \wedge e^a + \epsilon_{ab} b \wedge e^a \wedge \omega^b\big) \\ & + 2c_2\big(\tau \wedge db - e^a \wedge d\omega^a + \omega \wedge d\alpha + b \wedge e^a \wedge \omega^a - \omega \wedge e^a \wedge \omega^b \epsilon_{ab}\big) \\ & + c_3 \omega \wedge d\omega \end{aligned}$$

3 indep. parameters

- 2 gravitational curvature-like terms (HL type coupling with lambda neq 1)

rewrite in metric type fiels (2nd order formulation)

Factorized algebra involving E_2^c

existence of two copies of bilinear products suggest we can factorize algebra

$$\mathcal{S}_1 = \{\underbrace{D + iJ}_{2\mathcal{D}:}, \underbrace{P_1 - iP_2}_{\mathcal{P}}, \underbrace{C_1 + iC_2}_{\mathcal{C}}, \underbrace{H - iY}_{\mathcal{H}}\}$$

$$\mathcal{S}_2 = \{D - iJ, P_1 + iP_2, C_1 - iC_2, H + iY\}$$

algebra is 2 copies of the central extended 2D Poincare algebra E_2^c

so we get CS theory on the product of two E_2^c

"dual" realization of the Newton-Hook CS considered before

part III: Non-Relativistic string theories

non-relativistic geometries/field theories/gravity can also be approached by null-reduction -> apply also to string theory [Harmark,Hartong,NO](to appear)

warmup: null-reduction of relativistic particle $$S = \int d\lambda \frac{1}{e} G_{\mathcal{M}\mathcal{N}} \dot{X}^{\mathcal{M}} \dot{X}^{\mathcal{N}}$$

target space with null Killing vector

$$ds^2 = G_{\mathcal{M}\mathcal{N}} dx^{\mathcal{M}} dx^{\mathcal{N}} = 2\tau_M dx^M \left(du - m_N dx^N\right) + h_{MN} dx^M dx^N$$

can solve EOM for e for $\dot{X}^u$.

can. conjugate momentum $$P_u = \frac{\partial L}{\partial \dot{X}^u} = \frac{2}{e} \tau_M \dot{X}^M$$

Legendre transform $\tilde{L} = L - P_u \dot{X}^u$

(does not contain X^u. & P_u indep. variable)

$$\tilde{S} = \int d\lambda \tilde{L} = \int d\lambda \left(-P_u m_N \dot{X}^N + \frac{P_u}{2} \frac{h_{\mathcal{M}\mathcal{N}} \dot{X}^M \dot{X}^N}{\tau_M \dot{X}^M} \right)$$

[Kuchar], [Bergshoeff et al]

for $P_u = m = \text{cst}$ action has TNC local target space symmetries

Null-reduction of Polyakov action

perform similar steps for $$S = -\frac{T}{2}\int d^2\sigma\sqrt{-\gamma}\gamma^{\alpha\beta}g_{\alpha\beta}$$

momentum of string along u-direction $$P^\alpha_u = \frac{\partial\mathcal{L}}{\partial\left(\partial_\alpha X^u\right)} = -T\sqrt{-\gamma}\gamma^{\alpha\beta}\tau_\beta\,.$$

solve for $\sqrt{-\gamma}\gamma^{\alpha\beta}$ in terms of the momenta.

τ_α, m_α and $h_{\alpha\beta}$ are the pullbacks of τ_M, m_M and h_{MN}

Legendre transform $$\tilde{\mathcal{L}} = \mathcal{L} - P^\alpha_u\partial_\alpha X^u$$

define furthermore vielbeins (to elucidate WS geometry)

$$e_\alpha = \frac{1}{T}e_{\alpha\beta}P^\beta_u\,, \qquad v^\alpha = -\frac{P^\alpha_u}{P^\gamma_u\tau_\gamma}\,, \qquad e^\alpha = -T\frac{e^{\alpha\beta}\tau_\beta}{P^\gamma_u\tau_\gamma}\,,$$

$$v^\alpha\tau_\alpha = -1\,, \qquad v^\alpha e_\alpha = 0\,, \qquad e^\alpha\tau_\alpha = 0\,, \qquad e^\alpha e_\alpha = 1$$

conservation of P_u: $e_\beta = \partial_\beta\eta$ (eta is worldsheet scalar)

Non-Relativistic string action

$$\tilde{S} \;=\; \int d^2\sigma \left(-P_u^\alpha m_\alpha + \frac{1}{2}P_u^\gamma \tau_\gamma \left(v^\alpha v^\beta - e^\alpha e^\beta\right) h_{\alpha\beta}\right)$$

- stringy counterpart of non-rel. particle action coupling to TNC
- has all local TNC symmetries for P_u = constant
- for $P_u^\sigma = 0,\; P_u^\tau = P = \text{cst}$ and use static gauge $\tau = t$,

action on flat NC background becomes standard non-rel string which has 1+1 dimensional world volume Poincare sym.

$$S = T\int dt dx \left(\frac{1}{2}\left(\partial_t \vec{Y}\right)^2 - \frac{1}{2}\left(\partial_x \vec{Y}\right)^2\right)$$

- coupling to target space different from: [Andringa,Bergshoeff,Gomis,de Roo (1206)]

- Vir constraints X^u decouples from action)
- remnant of Virasoro constraint in reduced theory: $\partial_\alpha \frac{\partial \tilde{\mathcal{L}}}{\partial P_u^\beta} - \partial_\beta \frac{\partial \tilde{\mathcal{L}}}{\partial P_u^\alpha} = 0$

follows from EOM of eta (indep. ws field)

Non-relativistic world-sheet limit

consider backgrounds of the form (in d+2 dimensions)

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = 2\tau(du - m) + h_{ab}dx^a dx^b$$

$$\tau = dx^+ + \tau_a dx^a \ , \quad m = m_a dx^a$$

c to infinity limit with transverse part fixed x^a, m_a, τ_a, h_{ab}

rescale $x^+ = c^2\tilde{t}$ then: $\lim_{c\to\infty} \frac{1}{c^2}\tau = d\tilde{t}$

with t-tilde fixed in limit

world-sheet theory becomes a sigma-model on NC-like target space

$$\tilde{\mathcal{L}} = -P_u^\alpha m_\alpha - \frac{1}{2}(\partial_\gamma \tilde{t}) P_u^\gamma e^\alpha e^\beta h_{\alpha\beta}$$

$$\tilde{S} = T\int d^2\sigma e\left(v^\alpha m_\alpha - \frac{1}{2}e^\alpha e^\beta h_{\alpha\beta}\right)$$

not invariant under the standard local TNC symmetries
(target space geometry like: gauging Galilean x U(1))

Relation to tractable limits of strings on AdS5xS5

the new covariant (doubly) non-rel. world-sheet action appears in the SMT (string matrix theory) limit of the AdS/CFT correspondence !

SMT limit $\lambda \to 0\ ,\quad J_i,\, N \text{ fixed}\ ,\quad H \equiv \dfrac{E-J}{\lambda} \text{ fixed}$

[Harmark,Kristjansson,Orselli(0806)]
[Harmark,Orselli(1409)[

for one J: "SU(2) sector"

N=4 SYM side: semiclassically (large J): this corresponds to Kruczenski (0311) limit which shows that from Heisenberg XXX spin chain one gets Landau-Lifshitz model

$$\mathcal{S} = \int d^2x \left[C_t(\vec{n}) - \frac{1}{4}(\partial_x \vec{n})^2 \right]$$

AdS5XS5 side: can implement limit on geometry (gives same sigma-model)

Thus: LL model is in fact a non-relativistic string on a NC-like target space !

non-relativistic nature already noticed before:
general magnon dispersion relation [Beisert(0511)]

$$E = \sqrt{1 + \lambda \sin^2(p)}$$

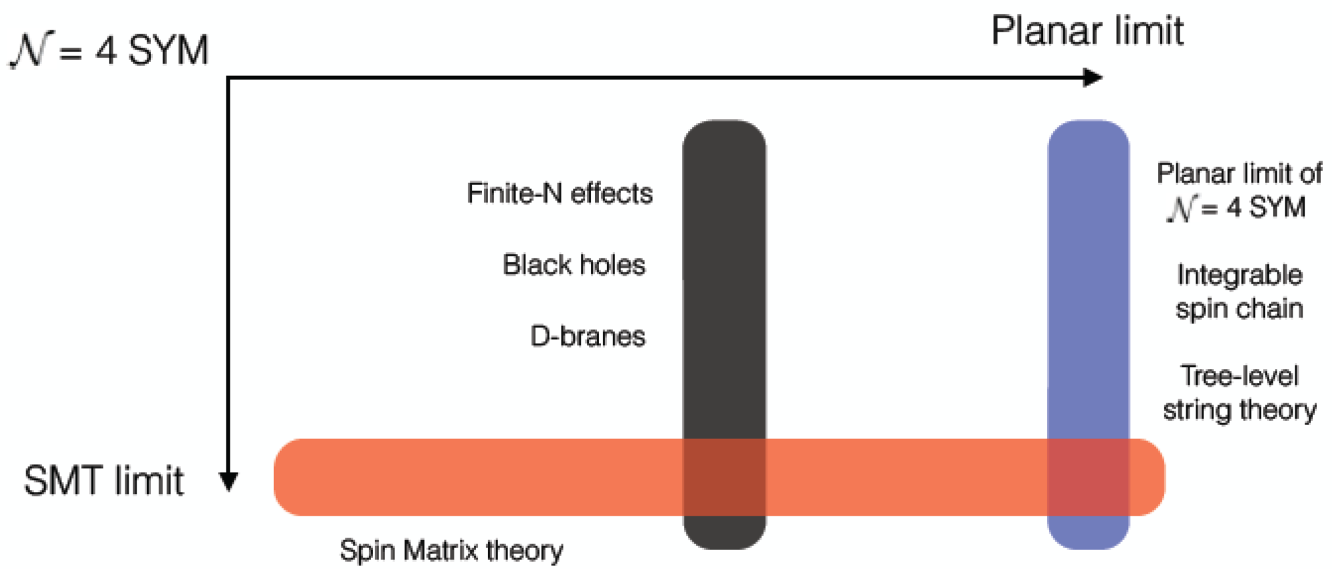
limit $\lambda \to 0$ with $H = (E-1)/\lambda$ fixed gives $H = \frac{1}{2}\sin^2(p)$

Other SMT limits

beyond SU(2) sector, all other sectors
(w. spins on sphere and angular mometa on AdS) work out as well:
- sigma-models in the literature are of the form the
non-relativistic string on NC-like geometry

checked: SU(1,1), SU(1,1|2), SU(1,2|3)

(dimension of target spaces vary)



Outlook

further examine non-Lorentzian CS theories:

- asymptotic symmetry groups, central charges, thermodynamics, "BH solutions"
- holographic dictionary, applications, connection to WCFT, higher spin,
- relation between Carroll and flat space holography
- coupling to matter

non-rel string theory:

- "beta functions" (target space dimension?), dynamics of the target space geometry
- higher-derivative corrections to the sigma model
- comparison to other non-rel limits
- inclusion of WS fermions
- SUSY on WS/target space
- non-rel limits of DBI