Horizon Fluff

A Proposal for Black Hole Microstates

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Outline

- Brief introduction to black hole thermodynamics and the need for black hole microstates.

- Residual Symmetries, charges, and the need for refinement of equivalence principle.

- Near horizon and asymptotic symmetries.

- Horizon Fluff proposal.

- Concluding remarks.

- In General Relativity any observer is associated with a coordinate system, the way we parameterize spacetime.

- Equivalence Principle is the pillar of GR and states that physics should be independent of the observer.

- Equivalence principle in the formulation of GR is made manifest in General Covariance:

  - Dynamical equations should be covariant under general coordinate transformations,

  - all physical observables should be of the form of local diffeomorphism invariant quantities.

- GR solutions may in general have causally disconnected regions separated by a horizon.

- Horizons divide the spacetime into two regions, in and out (or may be back and forth).

- Horizons are not necessarily an essential property of spacetime, they depend both on spacetime and the observer.

- The simplest and most important case of observer dependent horizon is Rindler horizon, associated with a uniformly accelerated observer moving in flat Minkowski spacetime.

- There are cases where horizon is an essential property of spacetime, visible to a large class of observers, especially all of those sitting at infinity, or any observer who is sitting outside the horizon.

- Such horizons are called event horizon.

- To us, black holes are spacetimes with event horizon.

- Equivalence principle implies that there are infalling observers who pass the event horizon without even noting it.

- These observers at the horizon are locally Rindler observers, for whom spacetime is locally flat.

- Acceleration of these observers is called surface gravity.

- Black holes as solutions of GR can dynamically form from usual matter fields within GR dynamics through gravitational collapse.

- Black holes are usually unique solutions to GR equations if we impose some mild symmetries like stationarity and/or axisymmetry and specify asymptotic behavior.

- All details of the information of the matter disappears due to the collapse; gravitational collapse behaves like an information eraser.

Black holes and thermodynamics

- Since early 1970's and works of J. Bekenstein & Bardeen-Carter-Hawking, it is established that black holes have an entropy and they obey laws of black hole thermodynamics.

- The Bekenstein-Hawking entropy is proportional to horizon area and the temperature to surface gravity.

- Considering quantum field theory on a classical black hole background leads to radiation [Hawking (1975)].

- Emerging picture:
  - *Semiclassical black holes behave like a usual-standard thermodynamical system.*
  - Black hole formation is a thermalization process.
  - It generates an entropy which is proportional to the horizon area, unlike usual thermodynamic systems.
  - Within classical GR, black holes can form and evaporate.
- Hawking radiation generically lasts until black hole (completely) evaporates.
- Process of formation and evaporation of black hole is *not* described by a unitary S-matrix.

- Noting that
  - Decoupling of scales seems to be a cornerstone of all physical formluations, and
  - energy scale in which quantum gravity effects become important seems to be Planckian,

- quantum gravity seems to be irrelevant to the Hawking process and to the information paradox.

- The resolution is then to view black hole as thermodynamic limit of an underlying microscopic system with unitary dynamics.

Black Hole Microstates

- Classical Einstein GR cannot accommodate black hole microstates (recall uniqueness theorems).

- One needs to go beyond classical GR......

- But this is generically against the lores mentioned above.

- Black hole problem may be viewed as a window to quantum gravity.

- E.g., in string theory, AdS/CFT or loop quantum gravity this has been used as a crucial check.

- It is apparent that strict classical GR does not bear a solution to identification of black hole microstates.

- It is more desirable to find a semiclassical resolution, rather than a full quantum one, something like "Bohr atom."

- If possible we may also get a better handle on the quantum gravity.

- The idea we try to realize here is

black hole microstates can be identified within semiclassical GR, with relaxing a bit, the strict statement of general covariance.

Einstein Equivalence Principle needs a refinement

- There are various arguments and analyses indicating that the strict statement of the general covariance should be relaxed and revised.

- Certain geometries which are diffeomorphic to each other are physically distinct.

- These geometries are labelled by conserved charges associated with the very particular subset of residual diffeomorphisms.

- These charges are given by surface integrals and are in general non-local physical observables.

- Therefore, these are not in violation of equivalence principle, which is meant for local physical quantities.

Einstein Equivalence Principle needs a refinement

- That is, there are (infinitely many) distinct geometries associated with a black hole solution with a given mass, angular momentum and electric charge.

- Not all such geometries may be called black hole microstates: if a distant observer can see and distinguish them they may not qualify as microstates of the same black hole.

- However, there are conserved surface charges associated with the horizon, defined only in the near horizon region.

- The idea may then be refined, considering the near horizon vs elsewhere (roughly asymptotic) charges.

Horizon Fluff, Sketch of the proposal

Black hole microstates are labelled by the set of charges defined only on the horizon which are not distinguishable by generic observers outside the horizon.

- We need to identify the near horizon (NH) and black hole asymptotic (Asymp.) charges and their algebra.

- Construct all states charged under these algebras.

- Check which of them qualify as microstate defined above.

We explicitly carry this out in a simple toy model of 3d black holes.

Horizon Fluff at work: Example of 3d black holes

- To explicitly show how this idea works we focus on the simplified example of AdS_3 black holes.

- AdS_3 gravity:

$$S = \frac{1}{16\pi G}\int d^3x\ \sqrt{-g}(R - \frac{2}{\ell^2}), \qquad R_{\mu\nu} = -\frac{2}{\ell^2}g_{\mu\nu},$$

  is the right place to ask these questions. While it does not have propagating gravitons, it has non-trivial black hole solutions.

- It has two parameters of dimension length: G, ℓ.

Horizon Fluffs at work: Introduction to BTZ black holes

- Metric in BTZ coordinates

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\phi - \frac{r_+ r_-}{\ell r^2}dt)^2$$

$$f(r) = \frac{(r^2 - r_-^2)(r^2 - r_+^2)}{\ell^2 r^2}$$

- BTZ black holes are described by two parameters $r_\pm$ or mass M and angular momentum J:

$$\Delta_\pm = \frac{1}{2}(\ell M \pm J) = \frac{1}{16G\ell}(r_+ \pm r_-)^2$$

- BTZ black holes are locally AdS_3.

- Their (bifurcation surface of) horizon is a circle of radius $r_\pm$, and

$$S_{\text{Bekenstein-Hawking}} = \frac{2\pi r_+}{4G}$$

- Their (horizon) temperature and angular velocity are

$$T = \frac{r_+^2 - r_-^2}{2\pi\ell^2 r_+}, \quad \Omega = \frac{r_-}{\ell r_+}; \qquad \beta_\pm = \frac{r_+ \pm r_-}{2\ell} = \frac{1}{\pi\ell T}(1 \mp \ell\Omega)$$

- They satisfy the first law and Smarr relation

$$TdS = dM - \Omega dJ, \qquad TS = 2(M - \Omega J)$$

Phase Space of AdS$_3$ black holes

- Metric for the most general locally BTZ geometry is

$$ds^2 = \frac{\ell^2 dw^2}{w^2} - \left(w\mathcal{A}_+ - \frac{\ell^2 \mathcal{A}_-}{w}\right)\left(w\mathcal{A}_- - \frac{\ell^2 \mathcal{A}_+}{w}\right).$$

- One-forms $\mathcal{A}_\pm$ are defined as

$$\mathcal{A}_\pm = \zeta^\pm dt \pm J^\pm d\varphi\,, \qquad \zeta^\pm = \zeta^\pm(t,\varphi), J^\pm = J^\pm(t,\varphi)$$

- The two arbitrary functions $\zeta^\pm, J^\pm$ are subject to Einstein field equations

$$d\mathcal{A}_\pm = 0.$$

The most general solution is hence specified by two functions.

Phase Space of AdS$_3$ black holes, Canonical descritpion

- Solutions with

$$\zeta^{\pm} = const., \qquad J^{\pm} = J^{\pm}(\varphi)$$

define canonical phase space of black holes.

- These are black holes with

$$T^{\pm} = \zeta^{\pm}$$

- Smoothness (absence of conical deficit outside the horizon):

$$\ell\zeta_{\pm} = J_0^{\pm} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\varphi \; J_{\pm}(\varphi)\,.$$

- These are family of black holes all with a given temperature

$$M = \frac{1}{4G}\left(J_0^{+2} + J_0^{-2}\right) \qquad J = \frac{\ell}{4G}\left(J_0^{+2} - J_0^{-2}\right)$$

and

$$\Delta^{\pm} = \frac{\ell}{4G}(J_0^{\pm})^2, \quad S = \frac{\pi\ell}{G}(J_0^{+} + J_0^{-})$$

- If $J_0^{\pm}$ are imaginary valued $J_0 = i\nu/2$, these solutions correspond to

  – conic deficits if $\nu \in (0,1)$

  – global AdS_3 if $\nu = 1$

- For both real or imaginary $J_0^{\pm}$, we have a family of solutions, each of specified by two functions $J_0^{\pm}(\varphi)$. While all locally AdS_3, they are distinct due to existence of conserve charges.

AdS$_3$ black holes, Microcanonical descritpion

- Solutions with

$$\zeta^{\pm} = J^{\pm}, \qquad J^{\pm} = J^{\pm}(t/\ell \pm \varphi)$$

or

$$\mathcal{A}_{\pm} = J_0^{\pm}\, dh_{\pm}(x^{\pm}) \qquad \text{with} \qquad h_{\pm}(x^{\pm} + 2\pi) = h_{\pm}(x^{\pm}) + 2\pi\,.$$

define microcanonical phase space of black holes.

- These are black holes with given mass and angular momentum:

$$\Delta^{\pm} = \frac{\ell}{4G}(J_0^{\pm})^2, \quad S = \frac{\pi\ell}{G}(J_0^{+} + J_0^{-})$$

- Conserved charge of this family are

$$L^{\pm}(x^{\pm}) = {J_0^{\pm}}^2 {h'_{\pm}}^2 - \frac{1}{2}\left[\frac{h'''_{\pm}}{h'_{\pm}} - \frac{3}{2}\left(\frac{h''_{\pm}}{h'_{\pm}}\right)^2\right].$$

- These are obtained from solutions $\mathcal{A}_{\pm} = J_0^{\pm}\, dx^{\pm}$ upon a coordinate transformation

$$x^{\pm} \to h_{\pm}(x^{\pm}), \qquad h' > 0$$

- One can show that the canonical family of black holes are mapped to the Fefferman-Graham asymptotic form with

$$L_{\pm} = J'_{\pm} + J^2_{\pm}.$$

Canonical to Microcanonical map

- Canonical and microcanonical *energy-momentum tensors* are

$$L_{\mathrm{Can}}(\phi) = J'(\phi) + J^2(\phi)$$

$$L_{\mathrm{mCan}}(\phi) = J_0^2 h'(\phi)^2 - \frac{h'''}{2h'} + \frac{3h''^2}{4h'^2}.$$

yielding

$$\boxed{J(\phi) = \pm J_0 h'(\phi) - \frac{1}{2}\frac{h''}{h'}}$$

- It will become clear that, it is convenient to define a new field Φ:

$$\Phi \equiv \int^{\phi} J = \pm J_0 h - \frac{1}{2} \ln h'.$$

- Since $h' > 0$, log-term is always real-valued and well-defined and

$$\Phi(\phi + 2\pi) = \Phi(\phi) \pm 2\pi J_0.$$

Symplectic Symmetries of canonical phase space

Asymptotic algebra for BTZ black holes

- $J(\Phi)$ functions constitute conserved charges associated with the geometries, with commutation relation [Afshar et al, March 2016]

$$\{J(\varphi_1), J(\varphi_2)\} = \frac{c}{12} \cdot 2\pi \partial_\varphi \delta(\varphi_1 - \varphi_2)$$

- Upon quantization, the Fourier modes

$$\boldsymbol{J}(\varphi) = \sum_{n \in \mathbb{Z}} \boldsymbol{J}_n e^{in\varphi}$$

satisfy creation-annihilation algebras

$$[\boldsymbol{J}^+_m, \boldsymbol{J}^-_m] = 0, \qquad [\boldsymbol{J}^\pm_m, \boldsymbol{J}^\pm_n] = \frac{c}{12} n \delta_{m,-n}, \qquad c = \frac{3\ell}{2G}.$$

where c is the Brown-Henneaux central charge.

- Canonical energy momentum tensor for the chiral 2d CFT at Brown-Henneaux central charge with field Φ and its conjugate momentum J is

$$\boldsymbol{L}_n = in\boldsymbol{J}_n + \frac{6}{c}\sum_{p\in\mathbb{Z}} \boldsymbol{J}_p \boldsymbol{J}_{n-p}$$

- One may then verify that

$$[\boldsymbol{L}_n, \boldsymbol{L}_m] = (n-m)\boldsymbol{L}_{m+n} + \frac{c}{12} n^3 \delta_{m,-n},$$

$$[\boldsymbol{L}_n, \boldsymbol{J}_m] = -m\boldsymbol{J}_{m+n} + i\frac{c}{12} m^2 \delta_{m,-n}$$

- There are two chiral copies of these algebras.

The BTZ Hilbert space $\mathcal{H}_{\mathsf{BTZ}}$

States labelled by the asymptotic algebra are constructed as follows:

- Vacuum States

$$\boldsymbol{J}_n^{\pm}|J_0^{\pm}\rangle = 0 \quad (n > 0), \qquad \boldsymbol{J}_0^{\pm}|J_0^{\pm}\rangle = J_0^{\pm}.$$

- All the other states are then a linear combination of

$$|\{n_i^{\pm}\}; J_0^{\pm}\rangle = \Big(\prod_{\{n_i^{\pm}\}} \boldsymbol{J}^{+}_{-n_i^{+}} \cdot \boldsymbol{J}^{-}_{-n_i^{-}}\Big)|J_0^{\pm}\rangle, \;\; n_i^{\pm} > 0, \quad \forall |\{n_i^{\pm}\}; J_0^{\pm}\rangle \in \mathcal{H}_{\mathsf{BTZ}}$$

- BTZ black hole state $|BTZ\rangle \in \mathcal{H}_{\mathsf{BTZ}}$, is defined as:

$$\langle BTZ|\boldsymbol{J}_n^{\pm}|BTZ\rangle = J_0^{\pm}\delta_{n,0}, \qquad \Delta^{\pm} = \frac{c}{6}(J_0^{\pm})^2,$$

from which we learn $\qquad |BTZ\rangle = |J_0^{\pm}\rangle.$

- J_0 can take imaginary values too. But the corresponding geometries are not black holes.

- For $J_0 = \pm i\nu/2$ with $\nu \in (0,1)$,

  - we have conic spaces (particles on AdS$_3$) and,

  - for $\nu = 1$ we have global AdS$_3$.

  - states in the family of conic or global AdS$_3$:

$$|\{n_i^\pm\};\nu^\pm\rangle = \Big(\prod_{\{n_i^\pm\}} \boldsymbol{J}^+_{-n_i^+}\cdot \boldsymbol{J}^-_{-n_i^-}\Big)|\nu^\pm\rangle, \quad n_i^\pm > 0,$$

    constitute $\mathcal{H}_{\text{Conic}}, \mathcal{H}_{\text{gAdS}}$.

- We define

$$\mathcal{H}_{\text{CG}} = \mathcal{H}_{\text{Conic}} \cup \mathcal{H}_{\text{gAdS}}$$

More on Conic+golbal AdS$_3$ states, Near Horizon algebra

- Desirable to understand better imaginary J_0 (anti-hermitian $\boldsymbol{J}_0$).

- Also, $J(\phi)$ is not a primary field (w.r.t. $L(\phi)$).

- The $\mathcal{W}(\phi)$ field

$$\mathcal{W}(\phi) \equiv \mathcal{P}\left(e^{\int^{\phi} J(\phi)}\right)$$

  is a priamry field of weight one.

- Twisted boundary condition:

$$\mathcal{W}_{\pm}(\phi + 2\pi) = e^{\pm 2i\nu}\mathcal{W}_{\pm}(\phi), \qquad (\mathcal{W}_{\pm})^{\dagger} = \mathcal{W}_{\mp}$$

- The most natural theory is then to view $\mathcal{W}_{\pm}$ as conjugate momenta to a canonical field for a chiral 2d CFT:

$$\{\mathcal{W}_{\pm\nu}(\phi), \mathcal{W}_{\mp\nu}(\phi')\} = \frac{6\pi}{c}\partial_{\phi}\delta(\phi - \phi),$$

with Fourier modes

$$\mathcal{W}_{\pm\nu}(\phi) = \sum_{n} \mathcal{W}_{n}^{\pm\nu} e^{i(n\pm\nu)\phi}.$$

- **Quantized $\mathcal{W}$ fields**:

$$[\boldsymbol{\mathcal{W}}_{n}^{\pm\nu}, \boldsymbol{\mathcal{W}}_{m}^{\pm\nu'}] = 0, \qquad [\boldsymbol{\mathcal{W}}_{n}^{\pm\nu}, \boldsymbol{\mathcal{W}}_{m}^{\mp\nu'}] = \frac{c}{12}(n \pm \nu)\ \delta(\nu - \nu')\delta_{n+m,0}.$$

- ν is a real number in $(0, 1]$. We assume quantization of ν in units of $1/c$, like a Bohr-atom type quantization.

$$\nu = \frac{r}{c}, \quad r = 1, 2, \cdots, c.$$

we expect this quantization to come out in a full quantum gravity theory (e.g. in the example of D1D5 CFT).

- One may then define

$$\mathfrak{J}_{c(n\pm\nu)} \equiv \sqrt{6}\mathcal{W}_n^{\pm\nu}, \qquad \nu = \frac{1}{c}, \frac{2}{c}, \cdots, 1.$$

- One readily sees that

$$[\mathfrak{J}_m, \mathfrak{J}_m] = 0, \quad [\mathfrak{J}_m, \mathfrak{J}_n] = \frac{n}{2}\delta_{m,-n},$$

Note that we have two copies, left and right modes, of $\mathfrak{J}$-algebra.

- One may then construct

$$\mathcal{L}_n = \sum_{p\in\mathbb{Z}} :\mathfrak{J}_p \mathfrak{J}_{n-p}:$$

and verify that

$$[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{m+n} + \frac{1}{12}(n^3 - n)\delta_{m,-n},$$

$$[\mathcal{L}_n, \mathfrak{J}_m] = -m\mathfrak{J}_{m+n}$$

Black holes as solitonic condensates

- $\mathcal{J}_n$ algebra is exactly the same as $\boldsymbol{J}_n$ algebra, up to some normalization. Both describe a free field representation of a chiral 2d CFT.

- Nonetheless, $\mathcal{J}_n$ have been constructed from $\mathcal{W}$'s which are **non-perturbative, non-local** fields in terms of $\boldsymbol{J}$'s:

  – $\mathcal{W}$ fields and hence the $\mathcal{J}_n$ are "good" for imaginary J_0, the conic+global AdS$_3$ cases,

  – while $\boldsymbol{J}_n$ are "good" for real J_0 cases, the black holes.

- $\mathcal{J}_n$ algebra does not have any trace of the AdS$_3$ background and may also be found in the Rindler wedge [Afshar et al, Nov. 2016]. We will hence call it the NH algebra.

Hilbert space of NH Algebra, *NH soft hairs* $\mathcal{H}_{\text{CG}}$

NH soft hair states are constructed as follows:

- Vacuum State

$$\mathcal{J}_n|0\rangle = 0 \quad (n \geq 0),$$

- One may check that $\mathcal{L}_0|0\rangle = 0$.

- All the other states are then a linear combination of

$$|\Psi\rangle = \Big(\prod_{\{n_i^\pm\}} \mathcal{J}^+_{-n_i^+} \cdot \mathcal{J}^-_{-n_i^-}\Big)|0\rangle, \quad n_i^\pm > 0, \quad \forall|\Psi\rangle \in \mathcal{H}_{\text{CG}}$$

- One can show that NH soft hair states in $\mathcal{H}_{\text{CG}}$ are denoting the states associated with Conical defect or Global AdS_3 and their Virasoro descendants.

The Horizon Fluff proposal

Main idea:

The BTZ black hole microstates are the set of all states in $\mathcal{H}_{CG}$ which have the same $\Delta^{\pm}$

Main input/remaining part:

Specify how $\boldsymbol{J}_n$ operators defined on $\mathcal{H}_{BTZ}$, are acting/defined on $\mathcal{H}_{CG}$

For the above we need a black hole NH/asymptotic *duality*
This is in the same spirit as Sine-Gordon–Thirring duality.

Black hole NH/asymptotic *duality*

- Any Virasoro algebra at central charge c,

$$[\boldsymbol{L}_n, \boldsymbol{L}_m] = (n-m)\boldsymbol{L}_{n+m} + \frac{c}{12} n^3 \delta_{n+m},$$

has a subalgebra at central charge cN, for any integer N:

$$\tilde{\boldsymbol{L}}_n \equiv \frac{1}{N} \boldsymbol{L}_{Nn}$$

then one may readily show

$$[\tilde{\boldsymbol{L}}_n, \tilde{\boldsymbol{L}}_m] = (n-m)\tilde{\boldsymbol{L}}_{n+m} + \frac{cN}{12} n^3 \delta_{n+m},$$

- The NH and BH Virasoro algebras respectively with $c=1$ and $c = 3\ell/(2G)$ are also related by a similar relation, if c is an integer:

$$\boldsymbol{L}_n = \frac{1}{c} \mathcal{L}_{nc} \quad n \neq 0, \qquad \boldsymbol{L}_0 = \frac{1}{c}(\mathcal{L}_0 - 1/24)$$

- The above implies

$$\frac{1}{c}\sum_{p\in\mathbb{Z}} \mathfrak{J}_{nc-p}\mathfrak{J}_{p} = i\boldsymbol{J}_{n} + \sum_{p\in\mathbb{Z}} \boldsymbol{J}_{n-p}\boldsymbol{J}_{p},$$

and relates/maps $\mathcal{H}_{\mathsf{BTZ}}$ onto $\mathcal{H}_{\mathsf{CG}}$.

- The "asymptotic vacuum state" $|J_0^{\pm}\rangle$ (which is nothing but a BTZ state $|BTZ\rangle$) is a highly excited state in the NH Hilbert space $\mathcal{H}_{\mathsf{CG}}$.

- One may exploite this to identify BTZ black hole microstates.

BTZ black hole microstates

- We define BTZ black hole microstates $|\mathcal{B}\rangle$ as states in $\mathcal{H}_{\text{CG}}$ solving

$$\langle\mathcal{B}'|\boldsymbol{J}_n^{\pm}|\mathcal{B}\rangle = J_0^{\pm}\ \delta_{n,0}\ \delta_{\mathcal{B}',\mathcal{B}}, \qquad \Delta^{\pm} = \frac{c}{6}(J_0^{\pm})^2.$$

- Recalling the Asympto. to NH embedding map, we get:

$$\boxed{|\mathcal{B}\rangle = \Big(\prod\ \mathcal{J}^{+}_{-n_i^{+}} \cdot \mathcal{J}^{-}_{-n_i^{-}}\Big)|0\rangle, \qquad \textstyle\sum_i n_i^{\pm} = c\Delta^{\pm}}$$

The black hole is a ***condensate*** *of the NH fluffs.*

For the case of BTZ black hole the horizon fluff are nothing but conic spaces (particles on AdS_3) or their Virasoro descendants.

BTZ black hole microstate counting

- Having identified all of microstates of a BTZ black hole of given mass and angular momentum, $\Delta^{\pm}$, we can count the number of linearly independent microstates.

- This problem is the Ramanujan-Hardey problem: Number of partition of a given integer N into non-negative integers. For $N \gg 1$, this is $p(N)$:

$$p(N) \simeq \frac{4}{N\sqrt{3}}\; e^{2\pi\sqrt{\frac{N}{6}}}$$

- Therefore, the microcanonic entropy is $\ln p(N),\; N = c\Delta$:

$$S_{BTZ} = 2\pi\left(\sqrt{\frac{c\Delta^+}{6}} + \sqrt{\frac{c\Delta^-}{6}}\right) - \ln(c\Delta^+) - \ln(c\Delta^-) + \cdots$$

where $\cdots$ stands for $1/N$ corrections.

Horizon Fluff and the Log-corrections

- An important and non-trivial check for our proposal is that it give the correct logarithmic correction to the BH entropy.

- The correct log-corrections for BTZ black hole is

$$S(E) = S_0(E) - \frac{3}{2}\ln(S_0(E)), \qquad S_0(E) = 2\pi\sqrt{cE}$$

  where $S_0(E)$ is the Cardy part of the entropy.

- The above correction, the factor of 3/2, comes from modular invariance of the presumed dual 2d CFT.

- Modular invariance is crucial property of a 2d CFT dual to AdS_3 QGr; it is a part of diff. invariance of the gravity theory.

- The Hardy-Ramanujan (HR) counting gives -2 instead of -3/2:

$$S(E) = S_0(E) - 2\ln(S_0(E)),$$

- The discrepancy comes from the fact that HR counting is done in canonical description while the -3/2 is obtained in microcanonical description.

- The missing factor of -1/2 can be understood recalling the canonical to microcanonical map

$$\boxed{J(\phi) = \pm J_0 h'(\phi) - \frac{1}{2}\frac{h''}{h'}}$$

- HR counting:

$$S^{can.} = S_0(E_{can.}) - 2\ln S_0(E_{can.})$$

$$S_0(E_{can.}) = \frac{1}{2\pi}\langle\int_0^{2\pi} d\phi\ \boldsymbol{J}(\phi)\rangle_{BTZ}$$

- Modular invariance:

$$S^{mc.} = S_0(E_{mc.}) - \frac{3}{2}\ln S_0(E_{mc.})$$

- The canonical to microcanonical map implies

$$\langle\int_0^{2\pi}\ d\phi\boldsymbol{J}(\phi)\rangle_{BTZ} = J_0\langle\boldsymbol{h}(\phi)\Big|_0^{2\pi}\rangle_{BTZ} - \frac{1}{2}\langle\ln(\boldsymbol{h}'(\phi)\Big|_0^{2\pi}\rangle_{BTZ}.$$

- To compute the log-term, we can safely work in the "large-J_0" limit, where $h(\phi)$ provides a convenient free-field representation with the Lagrangian $L(\phi) \simeq J_0^2 h'(\phi)^2$.

- Using the fact that

$$\langle \ln(\boldsymbol{h}'(\phi))\Big|_0^{2\pi}\rangle_{BTZ} = \frac{d}{dn}\langle(\boldsymbol{h}'(\phi)^n)\Big|_0^{2\pi}\rangle_{BTZ}\Bigg|_{n=0}.$$

we get

$$\langle \ln(\boldsymbol{h}'(\phi))\Big|_0^{2\pi}\rangle_{BTZ} = \frac{d}{dn}\left[\left(\int Dh\ h'^n e^{i\int J_0^2 h'^2}\right)\Bigg|_0^{2\pi}\right]\Big|_{n=0}$$

which is a straightforward free field theory computation of an n-point function in the "pinching" limit.

- Regularizing with standard point-splitting method, yields

$$\langle \ln(\boldsymbol{h}'(\phi))\Big|_0^{2\pi}\rangle_{BTZ} = -\ln J_0 + \ J_0 \text{ independent terms.}$$

and hence the desired result. $\mathfrak{Q.E.D.}$ ■

Summary and Outlook

- *Horizon fluff are subset of states/configurations labeled by the near horizon residual symmetries which cannot be distinguished by their residual symmetry charges associated with diffeomorphisms away from horizon of the black hole.*

- Horizon Fluff Proposal states that

Horizon fluff are microstates of a black hole specified by its asymptotic residual symmetry algebra charges.
Black hole microstates are a subset of the near horizon residual diffeomorphisms which may **not** *be extended to beyond horizon.*

- *Our intuition is that a black hole is a* **condensate** *of the NH fluffs.*

The six steps to the horizon fluff

1. Identify the asymptotic symmetry algebra.

2. Identify the near hrozion symmetry algebra.

3. Construct the asymptotic black hole Hilbert space $\mathcal{H}_{\mathrm{BH}}$.

4. Construct the near horizon Hilbert space $\mathcal{H}_{\mathrm{NH}}$.

5. Find the map embedding the asymptotic algebra into the NH one.

6. Finally, solve the equation defining the black hole on the $\mathcal{H}_{\mathrm{NH}}$ to get the microstates.

Summary and Outlook

- We explicitly showed how this proposal works for the case of AdS_3 BTZ black holes.

- It has been shown how this proposal works for more general family of AdS_3 black holes [M.M. Sh-J & H. Yavartanoo].

- *For the case of BTZ black hole the horizon fluff are nothing but conic spaces (particles on AdS_3) or their Virasoro descendants.*

- For this example the step 5 was introduced through a duality.

- One can argue for this duality more for the case of AdS_3.

- It is desirable to understand and establish it better.

Horizon Fluff and the Log-corrections

- As an important and non-trivial check for our proposal we showed that it produces the exact and correct logarithmic-correction to the BTZ BH entropy, if we note the canonical vs. microcanonical descriptions.

- Horizon Fluff for other black holes?!
  - It works for the 4d Extremal Kerr black hole,
  - We have proposals for generic 4d Kerr black hole

You are warmly invited to search for

Horizon Fluff

for generic black holes.

Thank You
For Your Attention

Supplementary Material

Causal diagram of BTZ black hole.