Chaos in General Holographic Space-times

Nilanjan Sircar

WITS

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Outline

Outline of talk

Motivation -1

- Chaos refers to sensitive dependence on initial conditions, *i.e.* initially very similar states can evolve to be quite different.
- Chaos in context of thermalization.
- In Quantum Information Theory and Black holes this is also known as scrambling.
- Black Holes are fastest scramblers in nature: $t_* \sim \beta \log S$.
  Hayden & Preskill '07, Sekino & Susskind '08
- Largest Lyapunov exponent is bounded by Black hole result: $\lambda_L \leq \frac{2\pi k_B T}{\hbar}$. Theories where the bound is saturated should have a gravity dual. Maldacena, Shenker, & Stanford '15

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Bound on Chaos-1

- Diagnostic of Chaos: $C(t) = -\langle [W(t), V(0)]^2 \rangle_\beta \sim \mathcal{O}(1)$
- In semi classical limit, $V = p$ and $W = q(t)$: $C(t) = \hbar^2 \left(\frac{\partial q(t)}{\partial q(0)} \right)^2 \sim \hbar^2 e^{2\lambda_L t}$.
- $C(t) \sim \mathcal{O}(1)$ at $t_* \sim \frac{1}{\lambda_L} \log \frac{1}{\hbar}$
- Dissipation time t_d : $\langle V(0) V(t) \rangle \sim e^{-\frac{t}{t_d}}$.
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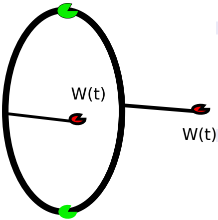

- $F(t) = \mathrm{Tr}\left[yVyW(t)yVyW(t)\right]$ where $y^4 = \frac{1}{Z}e^{-\beta H}$.
- Conjecture ($t \gg t_d$, $t \ll t_*$): $\frac{d}{dt}\left(F_d - F(t)\right) \leq \frac{2\pi}{\beta}\left(F_d - F(t)\right)$
- Chaotic system: $\left(F_d - F(t)\right) \sim \epsilon e^{\lambda_L t}$ $\rightarrow \lambda_L \leq \frac{2\pi}{\beta}$
- Holographically ($2+1$ dim bulk): $F(t) = f_0 - \frac{f_1}{N^2} e^{\frac{2\pi}{\beta}t}$ Shenker & Stanford '13

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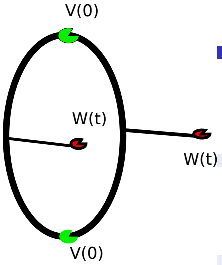

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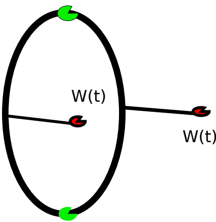

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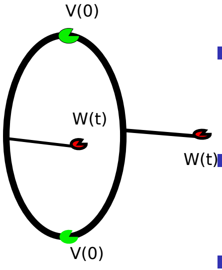

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Motivation -2

- For such chaotic/thermal systems, a small perturbation can change the pattern of correlation drastically : Disruption of entanglement/ Butterfly effect.
- The diagnostic used in this case is Thermo-Mutual Information Morrison & Roberts '12.
- It is recently studied in context of AdS/CFT by various people including Shenker, Stanford, Susskind, Roberts, Leichenauer, ...
- We have extended their work in context of:
  - Black Dp branes.
  - Lifshitz Black brane.
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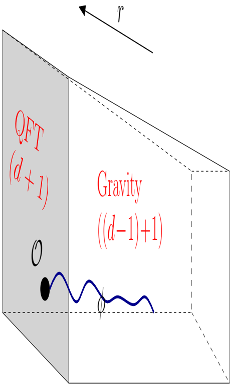
"AdS/CFT Correspondence" or Holography



- Duality between $\mathcal{N} = 4$ $SU(N)$ Super-Yang Mills' theory in $(3+1)$-dim and type IIB Super-strings in $AdS_5 \times S^5$. Maldacena '97; Gubser, Klebanov, Polyakov '98 ; Witten '98
- Simplifies in the limit of large 't Hooft coupling ($\lambda = g_{YM}^2 N \gg 1$) and large $N \gg 1$ to a duality between classical type IIB super-gravity and full quantum Super-Yang Mills' theory at leading order in λ and N.
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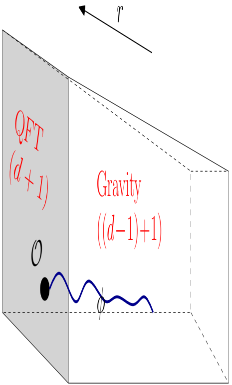

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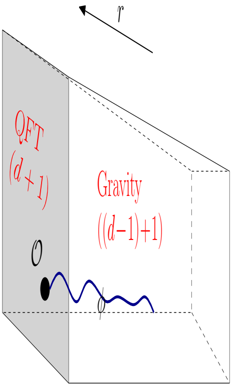

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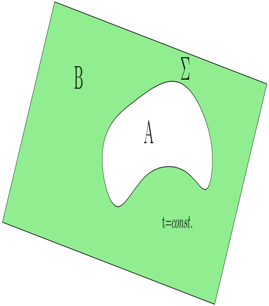

- Consider an entangling surface Σ which divides the space in to two separate sub-systems.
- Integrate out the the degrees of freedom living "outside" (region B).
- The reduced system is now described by a density matrix ρ_A.
- "Entanglement entropy" or von Neuman entropy: $S_{EE} = -\mathrm{Tr}\,(\rho_A \log \rho_A)$.

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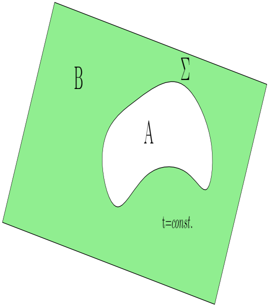

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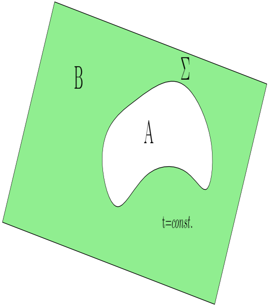

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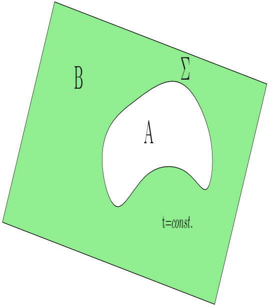

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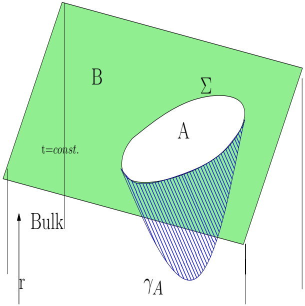

- $$S_{EE} = \min_{\partial\gamma_A = \Sigma}\left(\frac{\text{Area}(\gamma_A)}{4G_N}\right)$$ Ryu & Takayanagi '06
- Generalized to time dependent situations (Covariant prescription): min → extremum. Hubeny, Rangamani, & Takayanagi '07
- In presence of dilaton: Area → Area in Einstein frame Ryu & Takayanagi '06, Klebanov et al.. '07.
- For higher derivative gravity: Area → $\int_{\gamma_A} f[R_{\gamma_A}]$ de Boer et al.. '11, Hung et al.. '11, Dong '13, Camps '13

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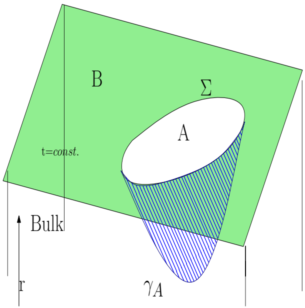

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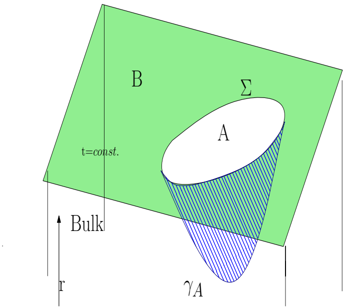

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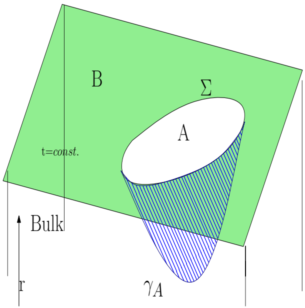

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Mutual Information

- $I(A;B) = S(A) + S(B) - S(A \cup B)$
- Finite quantity, UV divergence in EE cancels.
- Strong sub-additivity: $I(A;B) \geq 0$
- Measures total classical and quantum correlation between two regions.
- $I(A;B) \geq \frac{1}{2}\left(\frac{\langle O_A O_B\rangle - \langle O_A\rangle\langle O_B\rangle}{||O_A||\,||O_B||}\right)^2$ Wolf et al., '08 *i.e.*
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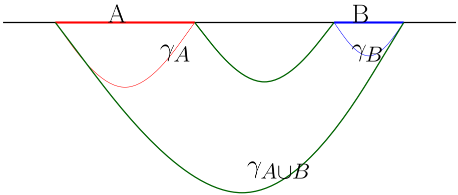

$$\begin{aligned} \min(\text{Area}(\gamma_{A\cup B})) &= \text{Area}(\gamma_{A\cup B}) \\ &\quad \text{if} \quad \text{Area}(\gamma_{A\cup B}) < \text{Area}(\gamma_A) + \text{Area}(\gamma_B) \\ &= \text{Area}(\gamma_A) + \text{Area}(\gamma_B) \quad \text{otherwise} \end{aligned}$$

For later case,

$$I(A;B) = S(A) + S(B) - S(A\cup B) = 0$$

Black Holes and Finite Temperature QFT

- QFT at Temperature $T \equiv$ Black Holes with Hawking temperature T.

- Black Hole metric:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + d\Sigma_\perp^2$$

$$f(r_H) = 0 \;;\quad T = \frac{1}{\beta} = \frac{|f'(r_H)|}{4\pi}$$

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Black Holes: Kruskal-Szekeres Coordinates

- Define Kruskal-Szekeres Coordinates (u, v):

$$uv = -e^{4\pi T r_*(r)}, \qquad u/v = e^{-4\pi T t}$$
$$ds^2 = -\frac{4f(r)}{16\pi^2 T^2} e^{-4\pi T r_*(r)} du dv + d\Sigma_\perp^2$$

$dr_* = \frac{dr}{f(r)}$.

- We can further re-define coordinates ("Penrose Diagram") region by: $U = \tan^{-1}(u)$, $V = \tan^{-1}(v)$.

- AdS black holes: AdS boundary at $uv = -1$ and Singularity at $uv = 1$.

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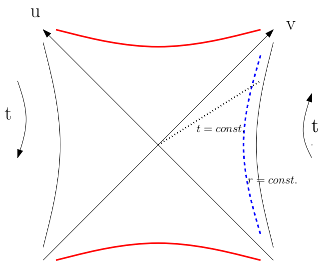
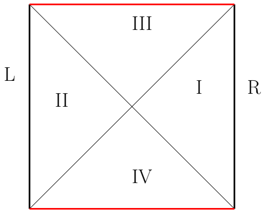
Eternal *AdS* Black Holes



Kruskal-Szekeres Diagram



Penrose Diagram

Thermo Field Double (TFD)

- Consider two QFTs with isomorphic Hilbert Spaces $\mathcal{H}_L$ and $\mathcal{H}_R$. Thermofield double is an particular entangled state in $\mathcal{H}_L \otimes \mathcal{H}_R$:

$$|TFD\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\frac{\beta}{2}E_n} |n\rangle_L |n\rangle_R$$

$Z(\beta) = \sum_n e^{-\beta E_n}$

- $\rho_L = \mathrm{Tr}_R |TFD\rangle \langle TFD| = \frac{1}{Z(\beta)} \sum_n e^{-\beta E_n} |n\rangle_L \langle n|_L$ $\leftarrow$ Thermal Density Matrix

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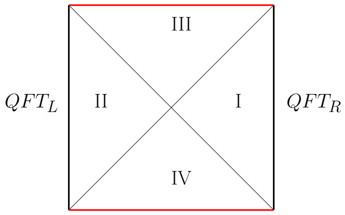
TFD ≡ Maximally extended Black Holes



- TFD is dual to the maximal extension of the eternal black hole. The pair of QFTs living on the two boundaries correspond to the two QFTs in the definition of TFD. Israel '76, Maldacena '01
- Entanglement between the two QFTs is given by the thermal entropy of the black hole.

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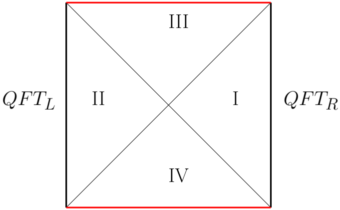

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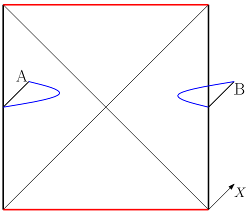
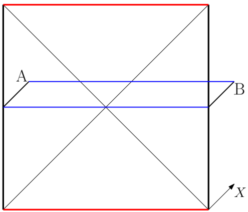
Black Hole and (Thermo-) Mutual Information





Consider the mutual information for strips of size L:

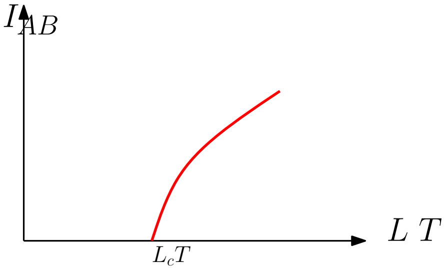
$$I_{AB} = S_A + S_B - S_{A\cup B}$$

For BTZ Black Hole ($2+1$-dimension) :

$$I_{AB} = \max\left(\frac{L_{AdS}}{G_N} \log \sinh(\pi L T), 0\right)$$

Shenker & Stanford '13

Black Hole and (Thermo-) Mutual Information: Plot

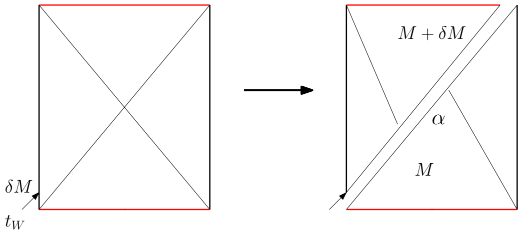


There exists a critical strip size $L = L_c$ beyond which the Mutual Information is non zero.

Shenker & Stanford '13, Leichenauer '14, Sonnenschein-NS-Tangarife '16

Outline

Perturbing TFD: Shock Wave geometry

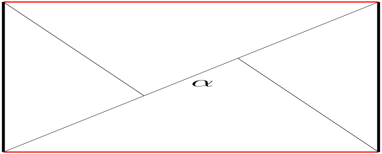


Consider the Kruskal coordinates $(\tilde{u}, \tilde{v})$, (u, v) to the left and right of the perturbation respectively. In the limit of small perturbation $\frac{\delta M}{M} \ll 1$ and large $t_w \gg 1$: $\tilde{v} = v + \alpha, \tilde{u} = u$ with,

$$\alpha = \frac{c_2}{c_1} \frac{\delta M \beta}{S_{BH}} e^{-\frac{2\pi}{\beta}(r_*(\infty) - t_w)}$$

Using Bekenstein-Hawking Formula and First Law of Thermodynamics. Shenker & Stanford '13

Heuristic Calculation of Scrambling time



t_W should correspond to scrambling time t_* when the effect of the perturbation is order one.

$$\boxed{\alpha \sim 1}$$

which gives for perturbation $\delta M \sim T$,

$$\boxed{t_* = \frac{\beta}{2\pi} \log S_{BH} + \frac{\beta}{2\pi} \log \left(\frac{c_1}{c_2} e^{r_*(\infty)} \right)}$$

$S_{BH} \sim N^2 \gg 1$. Shenker & Stanford '13

Diagnostics for Chaos -I

- We are interested in looking for signature of disruption of entanglement/ correlation due to a small perturbation.
- So in the Shockwave geometry corresponding to a small perturbation of the Thermofield double we can calculate two sided mutual information (thermo-mutual information (TMI)).
- As seen before the TMI is non zero only for beyond some critical strip size $L > L_c$.
- Now with $L > L_c$ we can calculate the TMI in the Shockwave geometry.

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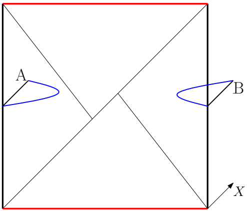
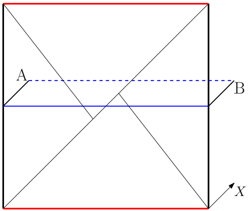
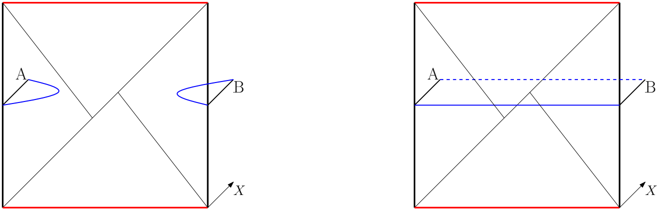
Diagnostics for Chaos-II





- In this geometry TMI is function of $I_{AB}(L,\alpha(t_W)) = S_A(L) + S_B(L) - S_{A\cup B}(L,\alpha(t_w))$.
- We define scrambling time (t_*) as when $I_{AB}(L,\alpha(t_W = t_*)) = 0$ for a given $L > L_c$.
- $t_* = \frac{\beta}{2\pi}\log S_{BH} + \mathcal{O}(N^0)$, same conclusion as $\alpha \sim 1$ analysis.

Diagnostics for Chaos-II



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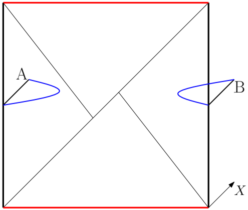

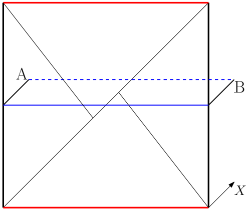

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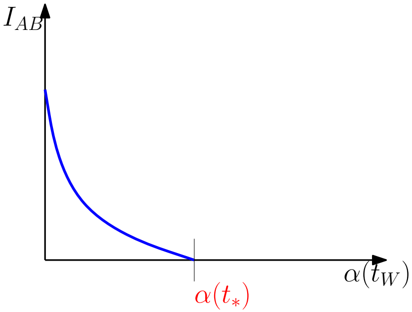
Diagnostics for Chaos-Plot



General behavior of TMI in Shockwave geometries except for some parameter region of higher derivative gravity.

Shenker & Stanford '13, Leichenauer '14, Sonnenschein-NS-Tangarife '16

Analytic Results in BTZ black hole

- In $2+1$-dimensional BTZ black hole calculation can be performed analytically. Shenker & Stanford '13
- For $L > L_c$: $I(A,B) = \frac{L_{AdS}}{G_N}\left(\log \sinh \frac{\pi L}{\beta} - \log\left(1+\frac{\alpha}{2}\right)\right)$
- $\alpha = \frac{E\,\beta}{2S_{BH}} e^{2\pi\, t_w/\beta}$
- For high temperature, $LT \gg 1$: $t_* = \frac{L}{2} + \frac{\beta}{2\pi}\log\frac{2S_{BH}}{\beta E}$.

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$F(t)$ in BTZ Black Hole

In geodesic approximation Cornalba et al '06, Shenker & Stanford '13,

$$F(t) \sim \left(\frac{1}{1+\frac{\alpha}{2}}\right)^{2ml}$$

where $\alpha = \frac{E\,\beta}{2S_{BH}} e^{2\pi\, t_w/\beta}$.
$F(t)$ is initially order 1 but starts decaying exponentialy at time $t \sim t_*$.
Note in BTZ case, the calculation of $F(t)$ and Mutual Information are essentially same and given by geodesics.

Outline

Black Dp Branes

- Black Dp Branes correspond to $p+1$-dimensional Super Yang Mills' theories with 16 supercharges. $p=3$ corresponds to usual AdS/CFT. Itzhaki, Maldacena, Sonnenschein, & Yankielowicz
- $p \neq 3$ corresponds to non-conformal field theories. The Yang-Mills' coupling is dimension full for $p \neq 3$.
- $p \neq 3$ has a non-trivial dilaton which couples to metric. So we need modified Ryu-Takayanagi principle to calculate entanglement entropy. Ryu & Takayanagi '06, Klebanov et al.. '07.
- Validity of super-gravity solution requires $1 \ll g_{eff}^2 \ll N^{\frac{4}{7-p}}$ where, $g_{eff}^2 = g_{YM}^2 N \frac{r^{p-3}}{l_s^{2(p-3)}}$.

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Black Dp Branes contd.

- So we need to put an UV-cutoff Λ and the validity at the other end can be achieved by choosing a appropriate temperature.
- Temperature and Entropy is given as,

$$s_{BH} = c(p)(g_{YM}^2 N)^{\frac{p-3}{5-p}} N^2 T^{\frac{9-p}{5-p}}$$

- Holography is not well defined for $p \geq 5$.

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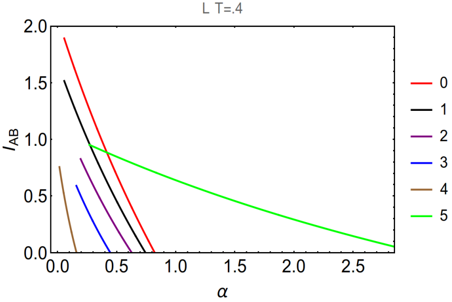
Black Dp Branes: Results



$$t_* = r_*(\Lambda) + \frac{\beta}{2\pi}\left(\log s_{BH}(\beta, p, \lambda, N) + \log\frac{c_1}{c_2}(p) + \log\alpha_*(\frac{L}{\beta}, p)\right)$$

$$r_*(\Lambda) = \frac{2\ell}{n(p-5)}\left(\frac{\Lambda}{\ell}\right)^{\frac{n}{2}(p-5)}, \quad p \neq 5$$

$$= \ell\log\left(\frac{\Lambda}{r_h}\right), \quad p = 5$$

$n = \frac{8}{8+(7-p)(3-p)} > 0$

Lifshitz Black Branes

- We are here interested in non-relativistic scale invariant theories with:

$$t \to \lambda^z t \quad ; \quad x \to \lambda t \ \text{ for } \ z \neq 1.$$

- The holographic dual to such field theories at finite temperature is generically called Lifshitz Black Holes. Kachru, Liu, & Mulligan '08

- $s_{BH} = \frac{1}{4G_N^{(4)}} \left(\frac{2\pi\ell}{z}\right)^{\frac{2}{z}} T^{\frac{2}{z}}.$

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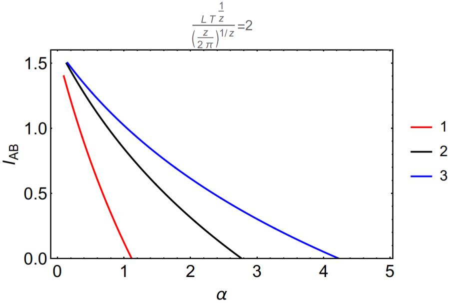

$$t_* = \frac{\beta}{2\pi}\log s_{BH} - \frac{\ell^2}{z^2\Lambda} + \frac{\beta}{2\pi}\log\left(\frac{1}{2}e^{-\psi\left(\frac{z}{2}\right)-\gamma}\right) + \frac{\beta}{2\pi}\log\alpha_*\left(LT^{\frac{1}{z}}\right)$$

Higher derivative Black Branes

- We consider AdS black-brane solutions in case of Einstein gravity corrected with higher curvature terms.
- It corresponds to finite T Conformal Theories with central charges $a \neq c$.
- Most general higher curvature gravity theory with second order equation of motion is known as Lovelock theory.
- We will consider $4+1$-dimensional Lovelock theory, which corresponds to just addition of Gauss-Bonnet term along with usual Einstein-Hilbert term in the action.
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Gauss Bonnet Action

$$S_{\rm grav} = \frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2} + \frac{\lambda_{\rm GB}L^2}{2}\mathcal{X}_4\right)$$

$$\mathcal{X}_4 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

The entropy density of Gauss-Bonnet black brane: Cai '02

$$s_{BH} = \frac{L^3}{4G_N}s(\lambda_{GB})T^3$$

The value of λ_{GB} is bounded by causality constraints: Brigante et al. '08, Buchel et al. '09

$$-7/36 \leq \lambda_{\rm GB} \leq 9/100$$

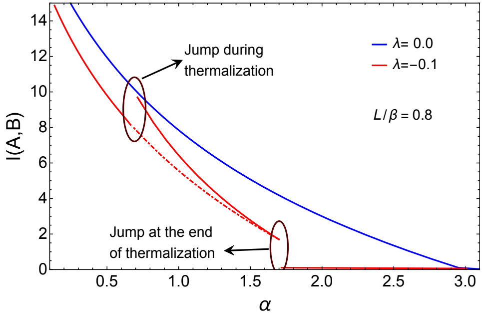
It is recently pointed out that Gauss-Bonnet as an exact theory violates causality for any λ_{GB}. Camanho et al. '14

Holographic Entanglement Entropy in Higher Derivative theories

$$S_{\rm EE} = \frac{1}{4G_N}\int_\gamma d^3\sigma \sqrt{\tilde{\gamma}}\left(1 + \lambda_{\rm GB} L^2 R_\gamma\right) + \lambda_{\rm GB} L^2 \frac{1}{2G_N}\int_{\partial\gamma} d^2\sigma \sqrt{h} K$$

de Boer et al. '11, Hung et al. '11

Higher derivative Black Branes: Results



For positive λ_{GB} the result is qualitatively similar to $\lambda_{GB} = 0$ case.

$$t_* = \frac{\beta}{2\pi} \log s_{BH} + \frac{\beta}{2\pi} \left(\frac{\pi}{2} - 4\lambda_{\rm GB} \right)$$

Similar jumps was noticed in time evolution of Entanglement entropy with higher derivative correction Caceres et. al. '15.

Outline

Summary

- We discussed the concept of Chaos/ Scrambling/ Butterfly Effect in context of Holography.
- We have shown that the dual thermal field theory scrambles information at time scales given by $\frac{\beta}{2\pi} \log S_{BH}$ for various geometries.
- We have used vanishing of thermo-Mutual Information as a signature for scrambling.
- The results are qualitatively similar in conformal $(a = c)$, non-conformal and non-relativistic cases.
- For conformal theories with $(a \neq c)$, dual to Gauss-Bonnet Black hole the behavior can be very different depending on sign of coupling.

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Future directions-1

- Thermo-Mutual Information, used as a probe for scrambling involves regions in two different copies of the field theory. It would be useful to express this as a probe in a single field theory.

- Exponential fall of out of time 4 point correlation function is also used as a probe of Scrambling. Connection to Chaos is more transparent in this definition, also definition of Lyapunov exponent is natural. In case of BTZ black hole, the two definitions can be shown to be equivalent, at least for some heavy operators. Precise connection in general is missing.

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Future directions-2

Chaos in General Holographic Space-times

Nilanjan Sircar

- Whether the higher derivative correction beyond GB term can smooth out the jumps? If not, significance of such jumps?
- Extension to non-commutative theories, General Higher Derivative gravity confining theories... Reynolds et al 1604.04099, Huang et al 1609.08841, Alishahiha et al 1610.02890

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Thank You