Density waves, bad metals and black holes

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Goal of the talk

- Discuss the (potential) relevance of (pseudo-)spontaneous symmetry breaking to transport across the phase diagram of cuprate high $T_c$ superconductors.

- Discuss the impact of (pseudo-)Goldstone dynamics on hydrodynamic transport and how this leads to bad metallic transport.

- Present an effective holographic toy-model of transport in cdw states and compute the dc resistivity in holographic critical phases.
References


- And ongoing work.
Central motivation: bad metallic transport

Two experimental challenges for theorists [Hussey, Takenaka & Takagi’04]:

- $T$-linear resistivity violating the Mott-Ioffe-Regel bound: **no quasiparticles**
- Optical conductivity: **far IR peak** ($\sim 10^2 \text{cm}^{-1}$) moving off axis as $T$ increases to room temperature.
Planckian dynamics

\[ \rho = \frac{m}{ne^2 \tau_{tr}} \sim T \quad \Rightarrow \quad \tau_{tr} = \tau_P \equiv \frac{\hbar}{k_B T} \]

**Universal scale** in all systems at finite temperature which follows from dimensional analysis

\[ [\hbar] = J.s, \quad [k_B] = J.K^{-1}, \quad [T] = K \quad \Rightarrow \quad \tau_P = \frac{\hbar}{k_B T} \]

In strongly-coupled, quantum systems, expected to be the **fastest equilibration time** allowed by Nature and Quantum Mechanics [Sachdev,Zaanen]. At room temperature

\[ \tau_P \sim 25fs \]
Planckian dynamics in the optical conductivity

\[ \hbar \omega_{\text{peak}} \sim k_B T , \quad \hbar \Delta \omega \sim k_B T , \]
These observations suggest that Planckian dynamics is a defining feature of both ac and dc transport in bad metals.

Planckian dynamics also emerge in the low energy effective description of strongly-coupled (holographic) quantum matter.

Universal low energy effective theory?
Universal low energy Planckian dynamics
I will offer a theory based on hydrodynamics and spontaneous translation symmetry breaking which

- leads to **small dc conductivities**, ie bad metal;
- accounts for the **far IR off-axis peak** in $\sigma(\omega)$;
- naturally **relates** the dc and ac transport timescales.

**Disclaimer**: effective low energy theory of transport, not a microscopic theory.
Spontaneous translation symmetry breaking

Temperature

Doping

Insulator

Superconductor

Charge order

Conventional metal

Strange metal

Doping

Charge order

Conventional metal

Strange metal

Insulator
Late time dynamics from hydrodynamics

Short-lived quasiparticles: **conserved quantities** are more fundamental for late-time transport

\[
\partial_t \epsilon + \vec{\nabla} \vec{j}_\epsilon = 0
\]

\[
\partial_t \pi^i + \nabla_k \tau^{ik} = 0
\]

\[
\partial_t \rho + \vec{\nabla} \vec{j} = 0
\]

Hydrodynamics: long wavelength description of the system

[credit: Beekman et al’16]
Electronic crystal

We also wish to include a CDW [Grüner'88, Chaikin & Lubensky]:

$$\rho(x) = \rho_0 \cos[Qx + \Psi(x, t)]$$

The phase $$\Psi(x, t)$$ is a new dof coming from the SSB of translations (Goldstone): ‘phonon’ of the electronic crystal.

$$[\pi, \Psi] = -i\delta$$

$$\Rightarrow \dot{\Psi} = i[H, \Psi] = i \left[ \int \pi \nu, \Psi \right] = \nu$$

Josephson relation for the phonon

[credit: Beekman et al’16]
Constitutive relation for the current

\[ j = \rho v - \sigma_o \nabla \mu + \ldots, \]

- \( \sigma_o \) is a diffusive transport coefficient: charge transport without momentum drag [Davison, Goutéraux & Hartnoll’15].

An analogy: particle-hole creation in a CFT.

\[ P = P_p + P_h = 0 \quad J = J_p + J_h = 2J_p \]
A CDW is ‘pinned’ by impurities: sliding only occurs beyond a threshold electric field.

More formally: the Goldstones acquire a small mass $\omega_o$.

Momentum is relaxed by impurities

$$\partial_t \pi^i + \nabla^j T^{ij} = -\Gamma \pi^i$$

[Thorne’96]
Consequences on charge transport

Weakly-disordered metal

\[ \sigma(\omega) = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{1}{\Gamma_{\pi} - i\omega} \]

\[ \sigma_{dc} \sim \Gamma_{\pi}^{-1} \]

The dc conductivity is dominated by momentum relaxation

Weakly-pinned CDW

\[ \sigma(\omega) = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{-i\omega}{-i\omega(\Gamma_{\pi} - i\omega) + \omega_o^2} \]

\[ \sigma_{dc} = \sigma_o + O(\Gamma_{\pi}) \]

The dc conductivity is set by the incoherent conductivity computed in the clean theory.
Phase disordering

- In 2d, crystals can **melt by proliferation of topological defects** in the crystalline structure [Nelson & Halperin’79].

- At $T = 0$: quantum melting [Kivelson et al’98, Beekman et al’16].

- The phase gets disordered ($\sim$ BKT) at a rate $\Omega$: **flow of mobile dislocations** [arXiv:1702.05104].

  $$\dot{\psi} + \Omega \psi = \nu$$

\[ \phi_n = 0 \quad \rightarrow \quad \phi_n = -\pi \]
\[ \phi_n = \pi \]
Now the conductivity reads

$$
\sigma = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{(\Omega - i\omega)}{(\Omega - i\omega)(\Gamma_\pi - i\omega) + \omega_o^2}, \quad \sigma_{dc} = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{1}{\Gamma_{CDW}}
$$

$$
\Gamma_{CDW} = \Gamma_\pi + \frac{\omega_o^2}{\Omega}
$$

New transport inverse timescale, non-zero even if $\Gamma_\pi \sim 0$.

- **Off-axis peak** for sufficiently small $\Omega$ or large pinning $\omega_o$

$$
\omega_o \geq \frac{\Omega^3}{\Gamma_\pi + 2\Omega}
$$
Strange metallic transport from fluctuating CDWs

- Neglect momentum relaxation $\Gamma_\pi \ll \omega_0, \Omega + \text{Galilean } \sigma_o = 0$:

  $$\sigma_{dc} = \frac{n e^2 \Omega}{m \omega_o^2}$$

- The width and position of the peak are controlled by $\Omega, \omega_o$. The data shows $\Omega \sim \omega_o \sim k_B T / \hbar$

  $$\Rightarrow \rho_{dc} = \frac{1}{\sigma_{dc}} \sim \frac{m}{n e^2} \frac{k_B T}{\hbar}$$

  $T$-linear resistivity!

- Hydrodynamics of fluctuating CDWs provide a natural mechanism whereby the ac and dc conductivities are controlled by the same Planckian timescale.
Some open questions

- Typical frequency scales of order $T$: at the edge of validity of hydrodynamics $\omega \ll T$.

- The role played by the Planckian timescale is indicative of quantum criticality: quantum critical computation.

- Work in progress: use Gauge/Gravity duality to compute non-hydrodynamic transport in phases with spontaneously broken translation symmetry.
Holographic model of spontaneous symmetry breaking

\[ S = \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} \partial \phi^2 - \frac{1}{4} Z(\phi) F^2 - V(\phi) - Y(\phi) \sum_{i=1}^{2} \partial \psi_i^2 \right] \]

- **Inspired by** [Donos & Gauntlett'13, Andrade & Withers'13].

- **Static Ansatz:** only radial dependence

\[ ds^2 = -D(r)dt^2 + B(r)dr^2 + C(r)d\vec{x}^2, \quad A = A(r)dt, \quad \phi = \phi(r) \]

except for \( \psi_I = k\delta_{ij}x^j \).

- **Internal shift and rotation symmetry of the \( \psi_I \) combines with spatial translations and rotations to preserve the translation and rotation symmetry of the Ansatz.**
$S = \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} \partial \phi^2 - \frac{1}{4} Z(\phi) F^2 - V(\phi) - Y(\phi) \sum_{i=1}^{2} \partial \psi_i^2 \right]$

- For simple choices of $Y = \phi^2$, $Y = (\sinh \phi)^2$, the real scalars can be rewritten as complex scalars $\Phi_I = \phi e^{i\psi_I}$ [Donos & Gauntlett’13], $\Phi_I = \tanh \phi e^{i\psi_I}$ [Donos & al’14].

- Can always be done asymptotically provided $Y_{UV} \sim \phi^2$

$$\mathcal{L}_{CFT} \rightarrow \mathcal{L}_{CFT} - \frac{1}{2} \left( \lambda^I \mathcal{O}_I^* + \lambda^*_I \mathcal{O}^I \right)$$

Same as in mean-field treatments of CDWs [Grüner’88].

- If $\lambda_I = 0$, spontaneous breaking.
Match to crystal stress tensor

Dual stress-tensor: equilibrium stress-tensor for an isotropic crystal

\[
\langle T^{ij}_{eq} \rangle = [p - (G + K) \partial \cdot \Psi] \delta^{ij} - 2G \left[ \partial^{(i} \Psi^{j)} - \delta^{ij} \partial \cdot \Psi \right], \quad \Psi^i = x^i
\]

with the **bulk modulus** (elastic resistance to compression)

\[
K = -\frac{k^2}{2} \int_{r_h}^0 dr \sqrt{BDY}
\]

Isotropy ⇒ The **shear modulus** $G$ does not appear at equilibrium.
Recall that the conductivity of a static, pinned CDW is

\[ \sigma = \sigma_o + \frac{\rho^2}{\chi_{PP}} \frac{-i\omega}{(-i\omega)(\Gamma_\pi - i\omega) + \omega_o^2} \]

\[ \sigma_{dc} = \sigma_o + O(\Gamma_\pi) \]

We computed the incoherent conductivity analytically.

At low temperatures:

\[ \sigma_o(T \to 0) = \frac{4K^2}{(\mu\rho - 2K)^2} \left( Z_h + \frac{4\pi\rho^2}{s\kappa^2 Y_h} \right) \]
Long story short: RG flows between a UV CFT ($\phi = 0$) and a **hyperscaling violating** IR ($\phi \to \infty$) \[\text{[GOUTÉRAUX'14]}\]

\[V_{IR} = V_0 e^{-\delta \phi}, \quad Z_{IR} = Z_0 e^{\gamma \phi}, \quad Y_{IR} = Y_0 e^{\lambda \phi}\]

\[ds^2 = r^\theta \left[ -\frac{dt^2}{r^{2z}} + \frac{L^2 d^2 r}{r^2} + \frac{d\vec{x}^2}{r^2} \right], \quad A = A_0 r^{\xi-z} dt\]

\[\psi_i = k x^i, \quad \phi = \kappa \log r\]

- The solution is **scale covariant**
  \[t \to \lambda^z t, \quad (r, x) \to \lambda (r, x)\]
- Typical observables **scale**, eg \(s \sim T^{\frac{d-\theta}{z}}\)
Two interesting cases

[Dumm et al., PRL 88 14 (2002)]

$z \to \infty$: $\text{AdS}_2 \times \mathbb{R}^2$, underdoped cuprates?

$$\sigma_o(T \to 0) \to T^0$$

$z, \theta \to \infty, \theta = -z$: conformal to $\text{AdS}_2 \times \mathbb{R}^2$

$$\sigma_o(T \to 0) \to T^{-1}$$

Optimally doped cuprates? ([Davison, Schalm & Zaanen’13])