Construction of the Emergent Yang-Mills Theory

Shaun de Carvalho

University of the Witwatersrand

Supervisor: Prof. R. de Mello Koch

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**Main aim:** to construct the **emergent Yang-Mills theory** that is dual to the *low energy description* of **open string excitations** and **giant graviton branes**.

- We make use of the **spin chain description** of the CFT operators.
- Why? It is a very effective approach to the **planar limit** of the SYM theory.
- This description maps:
  1. Each CFT operator $\rightarrow$ state of a spin chain, and
  2. Dilatation operator $\rightarrow$ Hamiltonian of the spin chain.
- The *dynamics* of the spin chain are naturally described in terms of excitations known as **magnons**.
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We have evaluated the action of the one loop dilatation operator $D_2$ in the $SU(3)$ sector:

$$D_2 = -g_{YM}^2 \text{Tr} \left( [Y, Z] \left[ \frac{d}{dY}, \frac{d}{dZ} \right] + [X, Z] \left[ \frac{d}{dX}, \frac{d}{dZ} \right] + [Y, X] \left[ \frac{d}{dY}, \frac{d}{dX} \right] \right)$$

1. Initially in the restricted Schur polynomial basis

$$\chi_{R,(r,s,t)\bar{\mu}\bar{\nu}} = \frac{1}{n!m!p!} \sum_{\sigma \in S_{n+m+p}} \text{Tr}_{(r,s,t)\bar{\mu}\bar{\nu}} \left( \Gamma^R(\sigma) \right) X_{i\sigma(1)}^{i_1} \cdots X_{i\sigma(p)}^{i_p} \times$$

$$\times Y_{i\sigma(p+1)}^{i_{p+1}} \cdots Y_{i\sigma(p+m)}^{i_{p+m}} Z_{i\sigma(p+m+1)}^{i_{p+m+1}} \cdots Z_{i\sigma(p+m+n)}^{i_{p+m+n}}$$

2. Improved the result $\rightarrow$ Gauss graph basis (result is diagonalised over some irreducible representation labels).

$$O_{R,r}(\sigma) = N \chi_{R,(r,s,t)\bar{\mu}\bar{\nu}}, \quad \sigma \in H\backslash S_m/H.$$
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A number of things to note:

- We mainly focused on the subleading piece, i.e. the mixing of the $X$ and $Y$ fields - small deformations of $\frac{1}{2}$-BPS operators; cannot be ignored.
- Many concepts of Group theory and Representation theory were used.
- Once we had a general result, we then considered the large $N$ limit.
- Computations in the Gauss graph basis are made more precise with the help of Gauss graphs.


- Quick, step-by-step review.
- Show novel work. Careful cases considered.
- Diagonalizing result $\rightarrow$ system of bosons hopping on a lattice.
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Future work

- We intend to study the $SU(2|2)$ group, its algebra and its connection to magnons.
- Why? To gain some insight to the low level description of the super Yang-Mills theory.
- This group involves fermions which we did not consider previously.
- Study supergroups and superalgebra to determine the **anomalous dimension of the $SU(2|2)$ group** and the **magnon scattering matrix** $S_{12}$.
- Later on, we also want to look at the gravity theory - consider the $9 + 1$ dimensional metric and action. Why?
  - Helps build an understanding of the open strings attached to the $S^3$ worldvolume, and
  - it helps us think about what **magnon bound states** we are studying.
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Thank you for your time, attention and for the opportunity.