FINITE FEATURES IN GRAVITY

5th MANDELSTAM WORKSHOP

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Dionysios Anninos
King's College London

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discussions & collab. with:
Galante, Harris, Hofman, Mühlmann, Shaghoulian

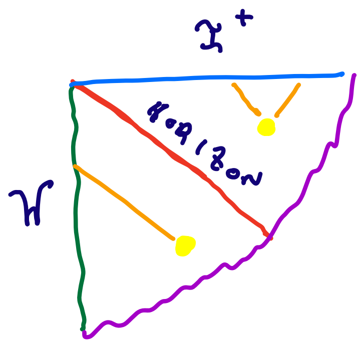
MOTIVATION

SPACETIME MAY EXHIBIT NO ASYMPTOTIA

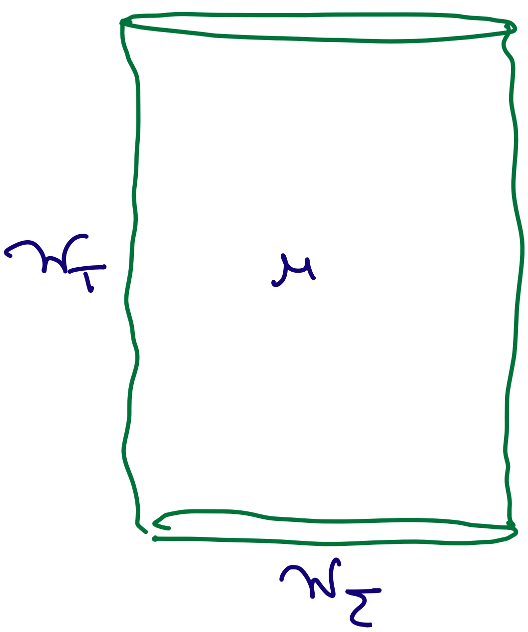
e.g.
* COSMOLOGICAL SPACES WITH COMPACT CAUCHY SURFACES;
* BIG BANG & BIG CRUNCH COSMOLOGIES
* COSMOLOGICAL EVENT HORIZONS

IN SUCH A SITUATION A MORE QUASILOCAL DESCRIPTION MIGHT BE NECESSARY.

INDEED, IN OUR OWN WORLD, PROVIDED $\Lambda > 0$ WE ARE CONFINED INSIDE THE DE SITTER HORIZON



FROM THE PERSPECTIVE OF GENERAL RELATIVITY WE CAN ASK WHETHER A WELL-POSED PROBLEM CAN BE FORMULATED ON A MANIFOLD $\mathcal{M}$ WITH A TIMELIKE BOUNDARY $\mathcal{W}$ OF FINITE SPATIAL SIZE.



$$\mathcal{W} = \mathcal{W}_T \cup \mathcal{W}_\Sigma$$

DOES THERE EXIST DATA ON $\mathcal{W}$ LEADING TO EXISTENCE AND/OR UNIQUENESS OF SOLUTION IN $\mathcal{M}$

DIRICHLET PROBLEM:

[Friedrich-Nagy; Anderson; An-Anderson; Witten; D.A., Galante, Mühlmann]

SPECIFY INDUCED METRIC h_{mn} ON $\mathcal{W}_T$

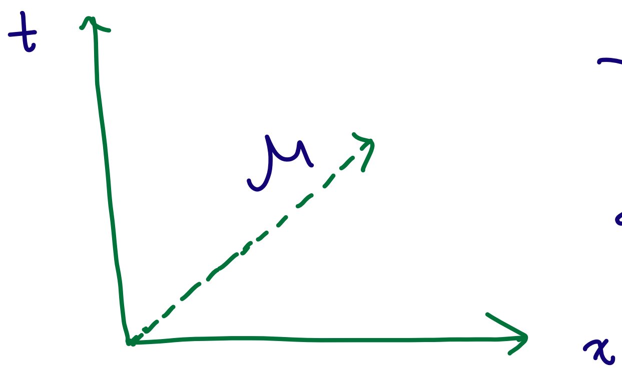
& $(\tilde{h}_{\mu\nu}, \tilde{K}_{\mu\nu})$ ON $\mathcal{W}_\Sigma$

THEOREM: (i) GENERIC DATA LEADS TO NON-EXISTENCE

[An-Anderson] (ii) DATA PERMITTING SOLUTION IN $\mathcal{M}$ PERMITS ∞ MANY SOLUTIONS.

SIMPLE EXAMPLE:

LINEARIZED GRAVITY NEAR MINKOWSKI



$\mathcal{W}_T : \{x = 0,\ t > 0\}$

$\mathcal{W}_\Sigma : \{x > 0,\ t = 0\}$

IN DEDONDER GAUGE: $\nabla^\nu h_{\nu\mu} - \frac{1}{2}\partial_\mu h = 0$

& WITH DIRICHLET DATA:

$h_{mn} = 0$ ON $\mathcal{W}_T$ & $\tilde{h}_{\mu\nu} = \tilde{K}_{\mu\nu} = 0$ ON $\mathcal{W}_\Sigma$

ON FINDS ∞ MANY SOLUTIONS!

MANY OF THESE CAN BE WRITTEN AS: $\boxed{h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu}$

HOWEVER: ξ_μ IS NOT PURE GAUGE AS IT DOES NOT OBEY BOUNDARY CONDITIONS ON $\mathcal{W}$.

CONJECTURE: FIXING $\mathbf{h}$ & $\mathrm{tr}\,K$ ON $\mathcal{W}_T$ & $(\tilde{h}_{\mu\nu}, \tilde{K}_{\mu\nu})$ ON $\mathcal{W}_\Sigma$

[An-Anderson]

(WHERE $\mathbf{h}$ IS THE CONFORMAL CLASS OF h_{mn})

IS WELL-POSED.

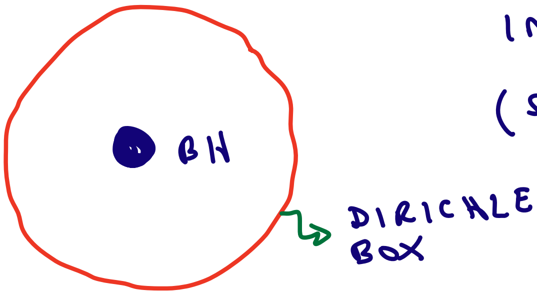
* EVIDENCE PROVIDED AT LINEARIZED LEVEL
* REMINISCENT OF AdS BOUNDARY DATA BUT CRUCIALLY HERE $\mathcal{W}$ IS A PRIORI OF FINITE SIZE

OPEN QUESTION: MATHEMATICAL STRUCTURE OF Bdy OBSERVABLES

(e.g. UNCLEAR IF AXIOMS OF LOCAL QFT APPLY)

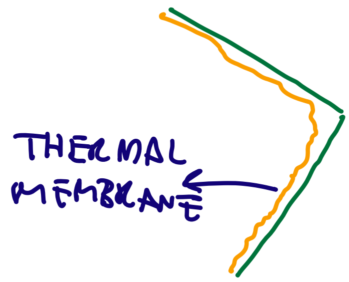
FINITE BOUNDARIES IN PHYSICS LITERATURE

YORK ~ 80s : BLACK HOLE THERMODYNAMICS
IN A BOX.
(S-WAVE SECTOR / DIRICHLET)



'T HOOFT; DAMOUR; ... ;
SUSSKIND-UGLUM-THORLACIUS;
PRICE-THORNE; ...
FLUID-GRAVITY LITERATURE

: STRETCHED HORIZON PICTURE.



JACOBSON: EINSTEIN EQNS FROM LOCAL HORIZON THERMODYNAMICS

MORE RECENT ATTEMPTS TO APPLY TO COSMOLOGICAL HORIZONS:

[Banks-Fischler; ... ; D.A., Anous, Bredberg, Ng; Banihashemi-Jacobson;
Coleman-Silverstein-Shyam-Torroba; Susskind; ...]

→ INCOMPRESSIBLE NAVIER-STOKES EQN
↕
EINSTEIN-EQUATIONS

HOLOGRAPHIC PERSPECTIVE

AdS_{d+1}/CFT_d $\longleftrightarrow$ DUAL THEORY IS LOCAL FIELD THEORY.

$d \geq 2$.

$\Rightarrow$ BULK MUST ACCOMODATE SPATIAL CONTINUUM OF LOCAL OPERATORS

IT IS NATURAL TO POSTULATE THAT THESE OPERATORS ARE ACCOMODATED AT ASYMPTOTIC BOUNDARY OF BULK SPACETIME

i.e. BULK THEORIES DUAL TO LOCAL QFTS SPATIAL SLICES OF INFINITE PROPER VOLUME; AT LEAST CLASSICALLY.

QUANTUM MECHANICAL HOLOGRAPHY?

SOME HOLOGRAPHIC PAIRS INVOLVE QM DUAL

e.g. N D0 BRANE QM $\longleftrightarrow$ IIA D0 BACKGROUND

MATRIX QM $\longleftrightarrow$ NON-CRITICAL STRING

(near)-CFT_1 $\longleftrightarrow$ (near)-AdS_2.

IN THESE EXAMPLES BOUNDARY IS "WORLDLINE" & DUAL HAS FINITE D.O.F WHEN $N < \infty$.

BUT STILL ASYMPTOTIC...

... NONETHELESS, IF ONE CAN ISOLATE SOME IR SUBSECTOR, ONE MIGHT HAVE A COMPLETELY "FINITE" HOLOGRAPHY.

SIMPLE EXAMPLE: RG FLOWS OF (near)-CFT$_1$.

LET $$\hat{H}_q \equiv \sum_{i_1 \cdots i_q} J_{i_1 \cdots i_q} \hat{\psi}_{i_1} \cdots \hat{\psi}_{i_q}$$

WITH $$\langle J_{i_1 \cdots i_q} J_{j_1 \cdots j_q} \rangle = \frac{2^{q-1}}{q} \frac{J^2 (q-1)!}{N^{q-1}} \delta_{i_1 j_1} \cdots \delta_{i_q j_q}$$

(a la wigner)

AT ENERGY SCALES $\ll J$

THEORY IS NEAR FIXED POINT WITH

$\Delta_\psi \simeq 1/q$;

$$\boxed{\mathcal{O}^{(n)} \simeq \psi_i \partial_t^n \psi_i}$$ with $\Delta_n \gtrsim 1$

Sachdev-Ye; Kitaev; Maldacena-Stanford; ⋮

SUBSET OF OPERATORS APPEARING IN $\langle \psi_i \psi_i \psi_k \psi_k \rangle$

& SYMMETRY BREAKING SECTOR GOVERNED BY SCHWARZIAN ACTIONS:

$$S_{Sch} = \# \frac{N}{J} \int_{\mathbb{R}} dt \;\; \mathrm{Sch}\{f(t), t\}$$

CHARACTERISTIC SCALE OFFENDS CONFORMAL SYMMETRIES

LOW T THERMODYNAMICS FOLLOWS FROM $\mathbb{R} \longrightarrow S^1$ MAP

ONE FINDS : $\lim_{N \to \infty} \frac{S}{N} = S_0 + \underbrace{c \cdot T}_{\text{SPECIFIC HEAT.}} + \dots$

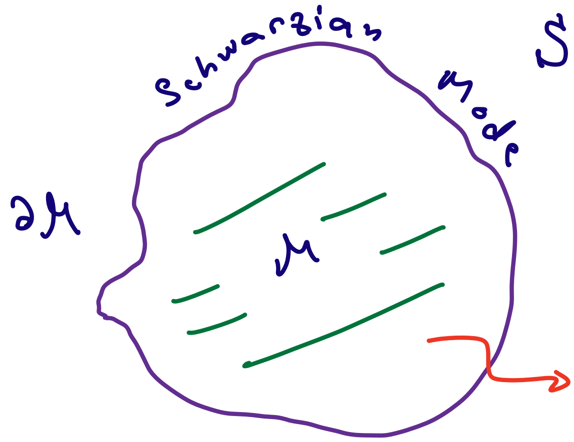
THERMAL ENTROPY

T=0 DEGENERACY

(SEMI) - HOLOGRAPHIC PICTURE

$$S = -\frac{1}{2\kappa}\int_{\partial M} d^2x \sqrt{g}\,\phi(R+2) - \frac{1}{\kappa}\int_{\partial M} d^2x \sqrt{h}\,\phi K$$

+ matter fields.



POINCARÉ DISK.

THE BULK GEOMETRY IS CLASSICALLY FEATURELESS.

STRUCTURE COMES FROM:

RELEVANT DEFORMATIONS

$\mathcal{O}^{(n)}$ ARE ALL IRRELEVANT!

HOWEVER $\Delta_{\psi_i} = 1/q$

$\Rightarrow$ AT LARGE q ROOM FOR MANY RELEVANT OPERATORS WITH $\Delta < 1$.

CONCRETELY WE STUDY: [D.A., Galante, Sheory] Jiang, Yang; ...

$$\boxed{\hat{H} = \hat{H}_q + s\,\hat{H}_{\tilde{q}}} \qquad \tilde{q}/q < 1.$$

(NAÏVE) INTUITION IS THAT $\hat{H}_{\tilde{q}} = J_{i_1 \ldots i_{\tilde{q}}}\, \psi_{i_1} \ldots \psi_{i_{\tilde{q}}}$ MIGHT FLOW TO RELEVANT DEFORMATIONS OF ORIGINAL (near)-CFT WITH $\Delta = \tilde{q}/q < 1$.

RESULTS:

$\tilde{q}/q = 1/2$ PERMITS ANALYTIC TREATMENT FOR $N \gg q \gg 1$.

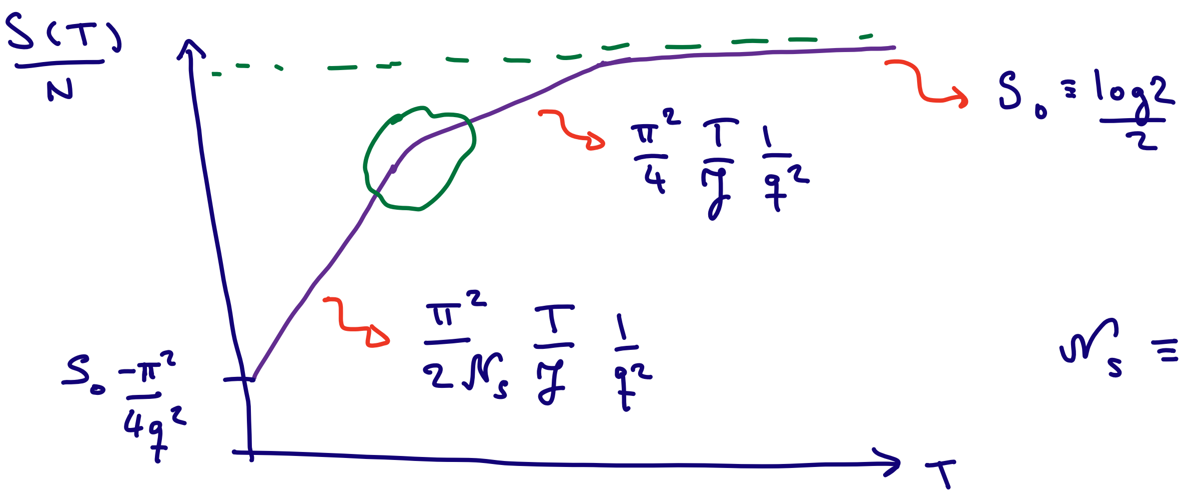
e.g. $$\frac{1}{N}\langle \psi_i(t)\psi_i(0)\rangle = \frac{\mathrm{sgn}\, t}{2}\left(\frac{1}{1+\mathcal{J}|t|\sqrt{1+4s^2}+s^2\mathcal{J}^2|t|^2}\right)^{1/q}$$

FOR SUFFICIENTLY SMALL s

$$\left.\begin{array}{ll} \text{A.} & \dfrac{1}{\mathcal{J}\sqrt{1+4s^2}} \ll |t| \ll \dfrac{\sqrt{1+4s^2}}{s^2\mathcal{J}} \\ & \\ \text{B.} & |t| \gg \dfrac{\sqrt{1+4s^2}}{s^2\mathcal{J}} \end{array}\right\}$$

EXHIBIT NEAR-CFT BEHAVIOUR.

FINITE TEMPERATURE RG FLOW



$$\nu_s \equiv \frac{s^2}{\sqrt{1+4s^2}}$$

TWO-REGIMES WITH LINEAR IN T ENTROPY

NUMERICAL EVIDENCE SUPPORTS

PICTURE FOR FINITE $\tilde{q}$ & VARIOUS $\tilde{q}/q < 1/2$

WHEN $q/\tilde{q} < 2$ $\rightsquigarrow$ S_0 depends on s

MOREOVER, USING:

CONFORMAL PERTURBATION THEORY

WE CAN STUDY CORRECTION TO THERMODYNAMICS NEAR FIXED POINT $\underline{\underline{A}}$.

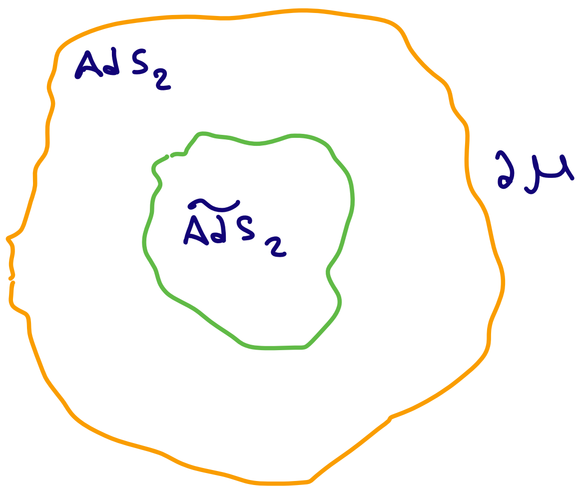
CORRECTION (NON-ANALYTIC BEHAVIOUR IN T)

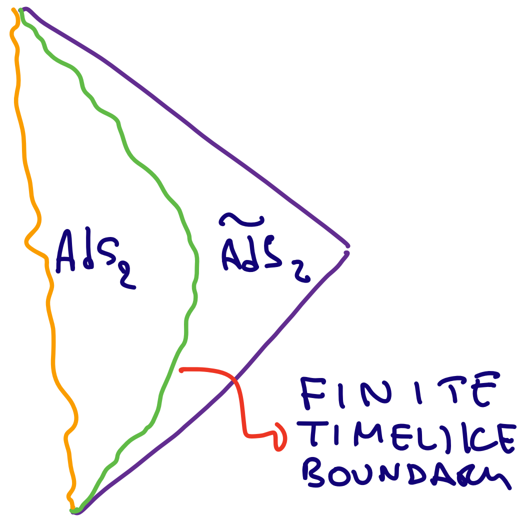
COMPATIBLE WITH: $\boxed{\Delta_{rel} = 1/n}$ $\rightsquigarrow$ PARTONIC PICTURE

i.e. $(J/T)^{2-2/n}$

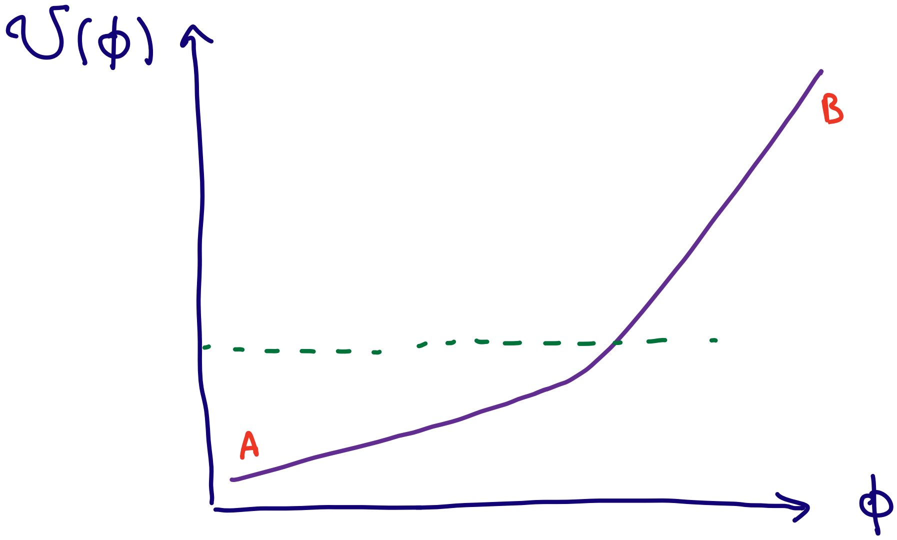
ALSO FIXED POINT $\underline{\underline{B}}$ HAS EMERGENT SCHWARZIAN MODE EMBEDDED ENTIRELY INSIDE STRONG COUPLING SECTOR.

GEOMETRIZATION OF RG FLOW





$$S_{deformed} = -\frac{1}{2\kappa}\int d^2x\sqrt{g}\left[\phi R + U(\phi)\right] - \frac{1}{\kappa}\int dx\sqrt{h}\,\phi K$$



$$\boxed{R = -\partial_\phi U(\phi)} \longrightarrow 2\ AdS_2\ \text{REGIONS}$$

BOUNDARY OF INTERIOR AdS_2 GOVERNED BY EMERGENT SCHWARZIAN MODE RESIDING ENTIRELY WITHIN GEOMETRIC DESCRIPTION.

IN LORENTZIAN SIGNATURE WE HAVE A FINITE TIMELIKE BOUNDARY

APPENDED WITH SIMPLE OBSERVABLES

LANDSCAPE OF RG FLOWS

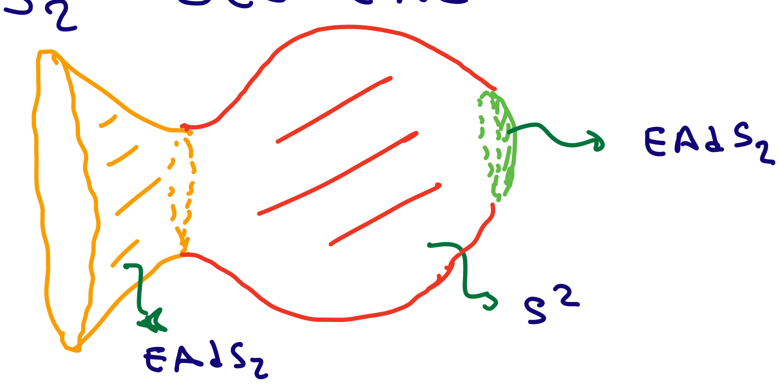
IN A SIMILAR SPIRIT ONE CAN IMAGINE RICHER CLASSES OF RG-FLOWS

e.g. $$\boxed{\hat{H} = \sum_{i=1}^{n} s_i \hat{H}_{q_i}}$$ with $q_1 > q_2 > \dots > q_n$.

LEADING TO INCREASINGLY RICH STRUCTURE SUSPENDED IN INTERIOR OF AdS_2

OF PARTICULAR INTEREST IS TO OBTAIN A PORTION OF S^2 SUSPENDED BETWEEN TWO $EAdS_2$ GEOMETRIES

e.g.



THESE GEOMETRIES IN LORENTIAN SIGNATURE INTERPOLATE BETWEEN AdS_2 & dS_2

[D.A., Hofman; Galante; Harris]

SOME MICROPHYSICAL EVIDENCE FOR THESE COMES FROM SIMPLIFIED DESCRIPTION OF JT GRAVITY IN TERMS OF MATRIX INTEGRALS WITH GENERALIZED POTENTIAL.

[Witten; Maxfield-Turiaci]

NOT YET EMBEDDED IN SYK...

GENERAL COMMENTS ON SPHERE

MOST "FINITE" GEOMETRY SUPPORTED BY GRAVITY. SADDLE POINT SOLUTION TO

$$\boxed{\mathcal{Z} = \int [Dg]\, e^{-\frac{1}{16\pi G}\int d^4x\sqrt{g}(-R+2\Lambda)}}$$

$\Lambda > 0$ CRUCIAL TO SUPRESS LARGE VOLUME. Diff, Gauge, Field Redef. invariant.

PHYSICAL MEANING OBSCURED BY ABSENCE OF BOUNDARY.

GIBBONS-HAWKING POSTULATE

$$\boxed{Z = e^{S}}$$

ENTROPY OF DE SITTER HORIZON.

CLASSICALLY $\boxed{S_0 = \frac{A_{\text{HOR}}}{4G} = \frac{3\pi}{\Lambda G}}$

* ABSENCE OF BOUNDARY $\Rightarrow$ NO TEMPERATURE PARAMETER.
* ALSO ADM ENERGY VANISHES.

SOME SIMILARITIES TO BPS STORY OF BLACK HOLES

$$\boxed{S \overset{?}{=} \dim \mathcal{H}_{dS}}$$

[Fischler - Banks - Fiol, ...] Verlinde, Susskind, ...

GENERAL QUANTUM STRUCTURE:

$$\log Z = S_0 - \underbrace{\frac{\dim SO(5)}{2}}_{\text{VOLUME OF RESIDUAL DIFFEOS.}} \log S_0 - \left(\frac{331}{90} + \frac{8}{3}\right) \log \frac{3}{\Lambda \ell_{ref}^2} + c + \mathcal{O}(S_0^{-1})$$

ONE-LOOP PIECE: [D.A., Denef, Law, Sun]

$$\boxed{S^{(1)} = \int_0^\infty d\omega \left(-\log\left[e^{b\omega/2} - e^{-b\omega/2}\right]\rho(\omega)\right)}$$

WHERE $\rho(\omega)$ FOLLOWS FROM HARISH-CHANDRA GROUP CHARACTER OF $SO(D,1)$; $b = 2\pi\ell$

$\Longrightarrow$ APPEARANCE OF THERMAL STRUCTURE

MAKING SENSE OF $\mathcal{Z}$ (THROUGH A VARIETY OF TOOLS) IS AN ADDITIONAL PATH TOWARD FINITE HOLOGRAPHY

e.g.

IN 3D: $T\bar{T}$ DEFORMATIONS OF AdS_3/CFT_2 INTO dS_3 PROPOSED BY
[Coleman-Mazenc-Shyam-Silverstein-Soni-Torroba-Yang]
REPRODUCE LOGARITHMIC CORRECTIONS.

IN 2D: CONNECTION TO OLD MATRIX INTEGRALS...
[Polyakov-Knizhnik-Zamolodchikov-Kazakov- ... - D.A., Mühlmann]
Ambjörn, Distler-Kawai, David

OUTLOOK

* NEW AVENUES (GR; HOLOGRAPHY; EUCLIDEAN GRAVITY, ...)

  TO EXPLORE FINITE FEATURES IN Q.G.

* MUCH TO EXPLORE, CONNECTING DIFFERENT PICTURES.

* ULTIMATELY, IT MAY BE ABOUT A CLEARER UNDERSTANDING OF THE ROLE OF AN OBSERVER IN Q.G. ?

THANK YOU!