Generalized hydrodynamics, local prethermalization, and hydrodynamization in ultracold 1D gases

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Outline

1. Introduction
   - Eigenstate thermalization
   - Generalized Gibbs ensemble
   - Onset of quantum chaos
   - Integrability in experiments
2. Near-integrable quantum dynamics
   - Generalized hydrodynamics
   - Local prethermalization and hydrodynamization
3. Summary

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Eigenstate thermalization hypothesis

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M. Srednicki, J. Phys. A **32**, 1163 (1999), L. D'Alessio *et al.*, Adv. Phys. **65**, 239 (2016).

$$O_{\alpha\beta} = O(E)\delta_{\alpha\beta} + e^{-S(E)/2} f_O(E,\omega) R_{\alpha\beta}$$

where $E \equiv (E_\alpha + E_\beta)/2$, $\omega \equiv E_\alpha - E_\beta$, $S(E)$ is the thermodynamic entropy at energy E, and $R_{\alpha\beta}$ is a random number with zero mean and unit variance.

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Spin-1/2 XXZ chain with nearest- and next-nearest-neighbor interactions

$$\hat{H} = \sum_{i=1}^{L} \left[\frac{1}{2} \left(\hat{S}_i^+ \hat{S}_{i+1}^- + \text{H.c.} \right) + \Delta \hat{S}_i^z \hat{S}_{i+1}^z \right] + \lambda \sum_{i=1}^{L} \left[\frac{1}{2} \left(\hat{S}_i^+ \hat{S}_{i+2}^- + \text{H.c.} \right) + \frac{1}{2} \hat{S}_i^z \hat{S}_{i+2}^z \right].$$

The model has $U(1)$ and translational symmetry
(Z_2 at $M^z = 0$ and parity at $k = 0, \pi$).

The model is "special" (integrable) for $\lambda = 0$.
(It can be solved exactly in that limit using the Bethe ansatz.)

T. LeBlond, K. Mallayya, L. Vidmar, and MR, PRE **100**, 062134 (2019).

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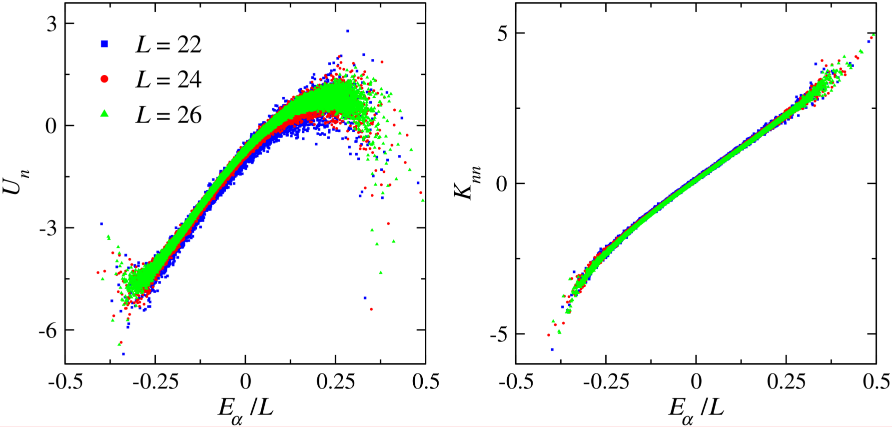
Diagonal matrix elements ($\Delta = 0.55$ and $\lambda = 1.0$)



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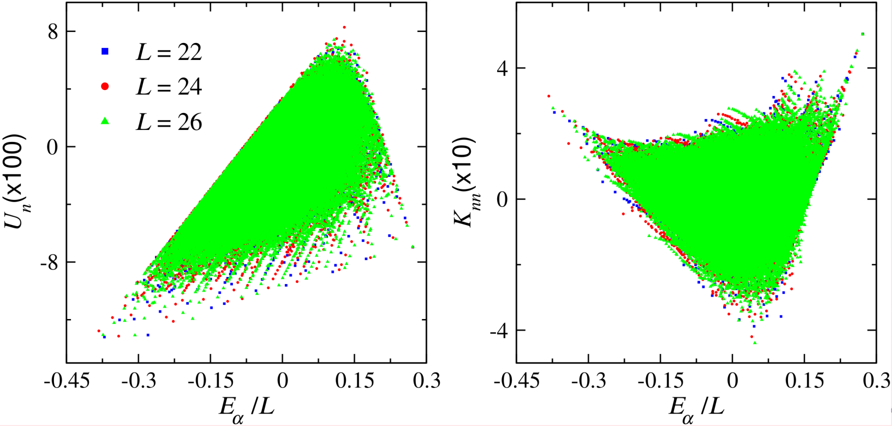
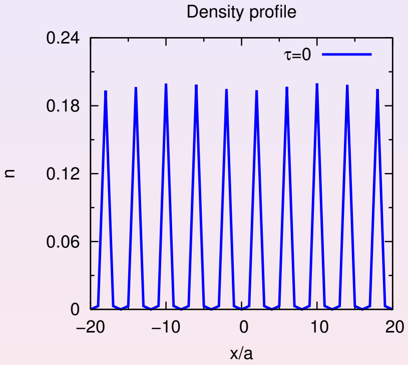
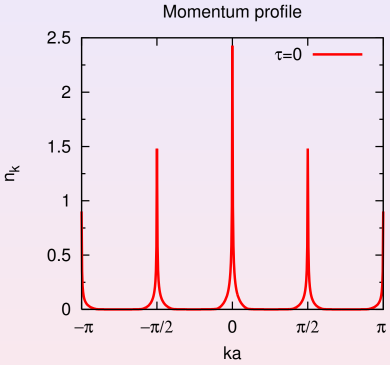


Outline

Relaxation dynamics in an integrable system





MR, V. Dunjko, V. Yurovsky, and M. Olshanii, PRL **98**, 050405 (2007).

Bose-Fermi mapping in a 1D lattice

Hard-core boson Hamiltonian in an external potential

$$\hat{H} = -J \sum_i \left(\hat{b}_i^\dagger \hat{b}_{i+1} + \text{H.c.} \right) + \sum_i v_i \; \hat{n}_i$$

Constraints on the bosonic operators

$$\hat{b}_i^{\dagger 2} = \hat{b}_i^2 = 0$$

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$$\Downarrow$$

Map to spins and then to fermions (Jordan-Wigner transformation)

$$\hat{\sigma}_i^+ = \hat{f}_i^\dagger \prod_{\beta=1}^{i-1} e^{-i\pi \hat{f}_\beta^\dagger \hat{f}_\beta}, \quad \hat{\sigma}_i^- = \prod_{\beta=1}^{i-1} e^{i\pi \hat{f}_\beta^\dagger \hat{f}_\beta} \hat{f}_i$$

$$\Downarrow$$

Non-interacting fermion Hamiltonian

$$\hat{H}_F = -J \sum_i \left(\hat{f}_i^\dagger \hat{f}_{i+1} + \text{H.c.} \right) + \sum_i v_i \ \hat{n}_i^f$$

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Set of conserved quantitites

(Occupations of the single-particle energy eigenstates of the noninteracting fermions)

$$\hat{H}_F \hat{\gamma}_m^\dagger |0\rangle = E_m \hat{\gamma}_m^\dagger |0\rangle$$
$$\left\{\hat{I}_m\right\} = \{\hat{\gamma}_m^\dagger \hat{\gamma}_m\}$$

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$$\hat{b}_i^{\dagger 2} = \hat{b}_i^2 = 0$$

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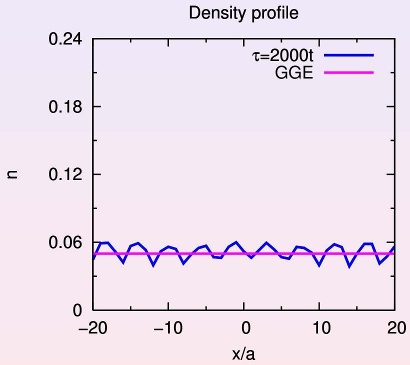
Generalized Gibbs ensemble

$$\hat{\rho}_{\text{GGE}} = Z_c^{-1} \exp\left[-\sum_m \lambda_m \hat{I}_m \right], \quad \text{where} \quad Z_c = \text{Tr}\left\{ \exp\left[-\sum_m \lambda_m \hat{I}_m \right] \right\}$$

The Lagrange multipliers are determined from

$$\text{Tr}\left\{ \hat{I}_m \hat{\rho}_{\text{GGE}} \right\} = \langle \hat{I}_m \rangle_{\tau=0} \quad \Longrightarrow \quad \lambda_m = \ln\left[\frac{1 - \langle \hat{I}_m \rangle_{\tau=0}}{\langle \hat{I}_m \rangle_{\tau=0}} \right]$$

Generalized Gibbs ensemble (GGE)



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Onset of quantum chaos in finite systems

Spin-1/2 XXZ chain with nearest- and next-nearest-neighbor interactions

T. LeBlond, D. Sels, A. Polkovnikov, and MR, PRB **104**, L201117 (2021).

$$\hat{H} = \sum_{i=1}^{L} \left[\frac{J}{2} \left(\hat{S}_i^+ \hat{S}_{i+1}^- + \text{H.c.} \right) + \Delta \hat{S}_i^z \hat{S}_{i+1}^z + \Delta' \hat{S}_i^z \hat{S}_{i+2}^z \right]$$

where $J = \sqrt{2}$, $\Delta = (\sqrt{5}+1)/4$, and we change Δ'.

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Typical fidelity susceptibility [D. Sels and A. Polkovnikov, PRE **104**, 054105 (2021)]:

$$\chi_{\text{typ}}(O) = \exp(\overline{\ln[\chi_m(O)]}),$$

where, for each eigenstate α,

$$\chi_\alpha(O) = L \sum_{\beta \neq \alpha} \frac{|\langle \alpha | \hat{O} | \beta \rangle|^2}{(E_\alpha - E_\beta)^2}.$$

The average $\overline{\ln[\chi_\alpha(O)]}$ is computed over the central 50% of eigenstates in each total quasi-momentum sector, and then $\chi_{\text{typ}}(O)$ is averaged over quasi-momentum sectors.

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The average $\overline{\ln[\chi_\alpha(O)]}$ is computed over the central 50% of eigenstates in each total quasi-momentum sector, and then $\chi_{\text{typ}}(O)$ is averaged over quasi-momentum sectors.

Using the ETH, we can predict the scaling of $\chi_{\text{typ}}(O)$ with the mean level spacing ω_H and the Hilbert space dimension D if the system is quantum chaotic.

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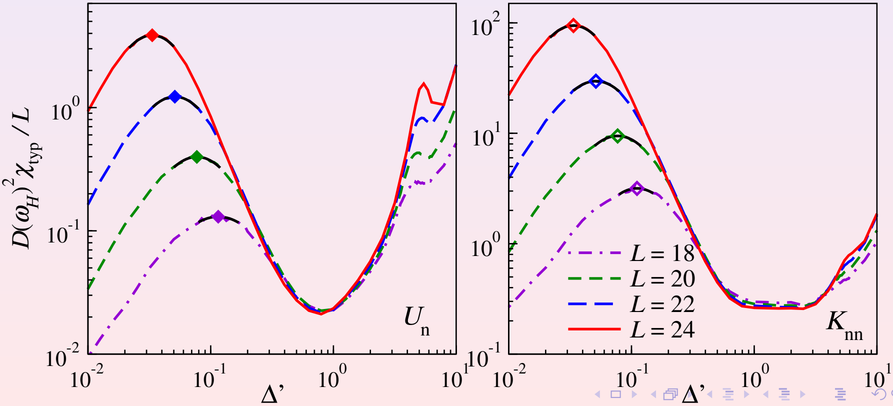

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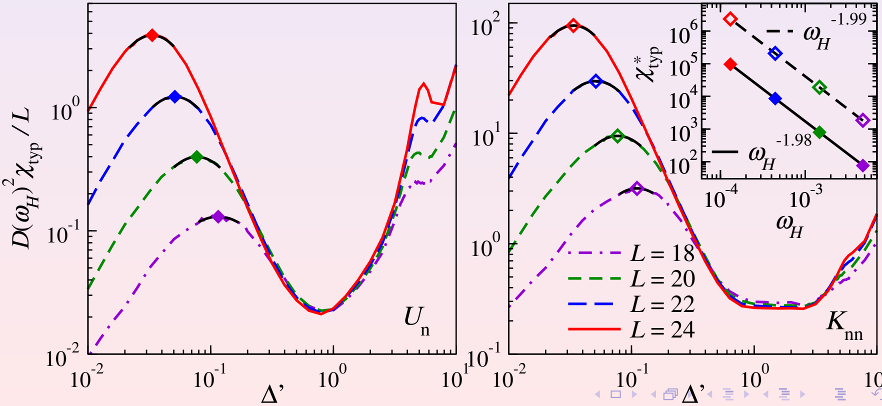

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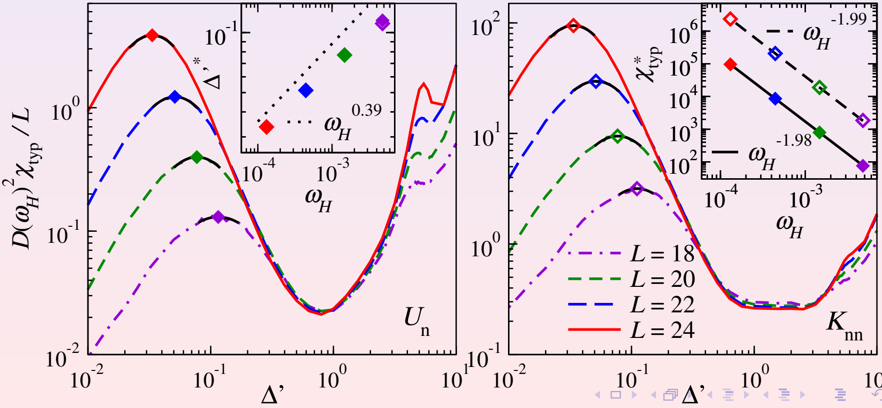

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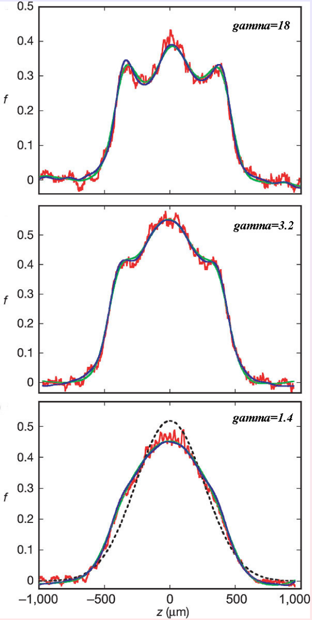
Quantum Newton's cradle: after equilibration



T. Kinoshita, T. Wenger, and D. S. Weiss, Nature **440**, 900 (2006).

$$\gamma = \frac{mg_{1D}}{\hbar^2 n_{1D}}$$

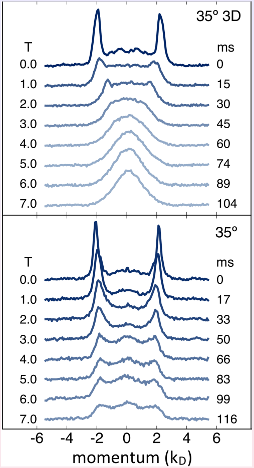
g_{1D}: Contact interaction strength
n_{1D}: One-dimensional density

If $\gamma \gg 1$ the system is in the strongly correlated Tonks-Girardeau regime

If $\gamma \ll 1$ the system is in the weakly interacting regime

Review of related work in atom chips:
T. Langen, T. Gasenzer, and J. Schmiedmayer, J. Stat. Mech. 064009 (2016).

Prethermalization & thermalization (QNC Dysprosium)



Y. Tang, W. Kao, K.-Y. Li, S. Seo, K. Mallayya, MR, S. Gopalakrishnan, and B. L. Lev, Phys. Rev. X **8**, 021030 (2018).

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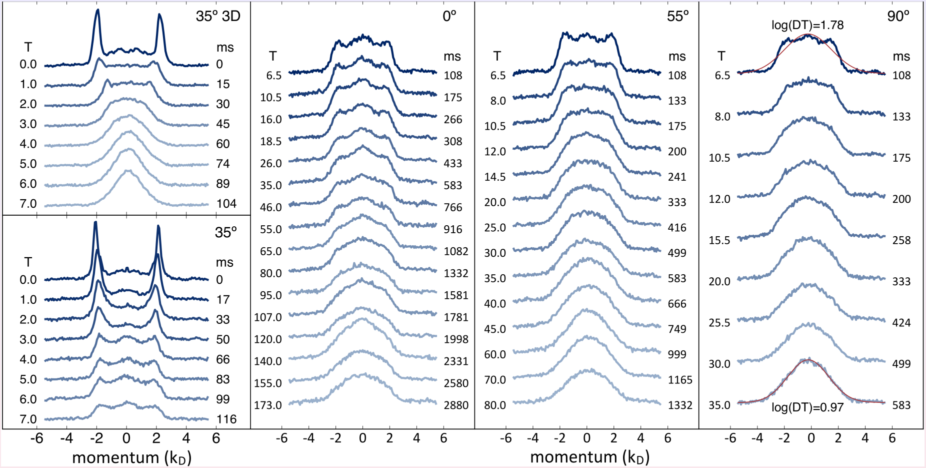

$$\text{DT} = \sqrt{\sum_k \left[n(k) - n_G(k)\right]^2}$$

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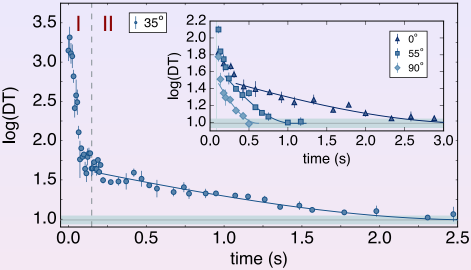
Approach to thermal predictions:



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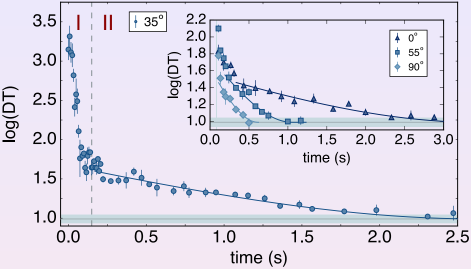
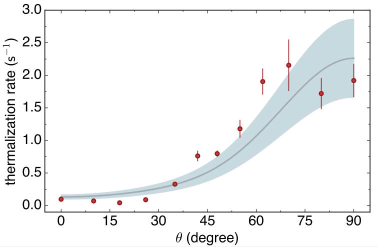


Consistent with FGR $\propto U^2_{\text{total}}(\theta)$



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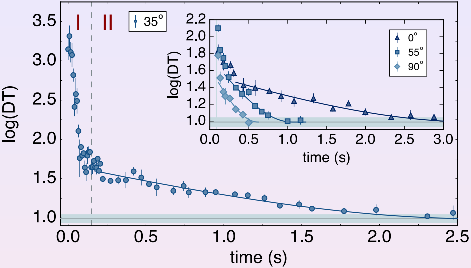

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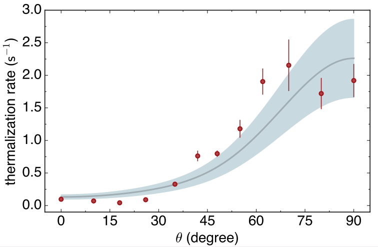
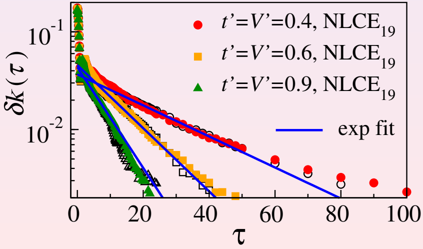


Breaking integrability in the XXZ model (NLCEs)



K. Mallayya and MR, PRL **120**, 070603 (2018).

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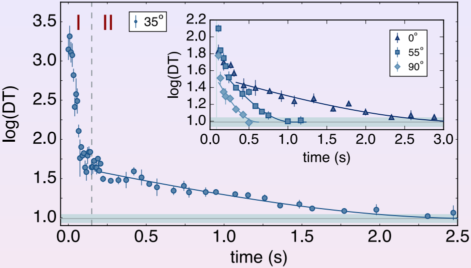

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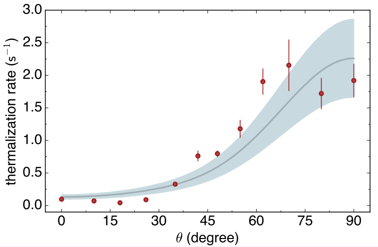

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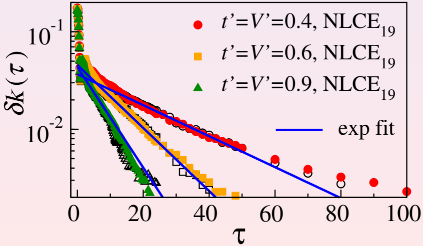

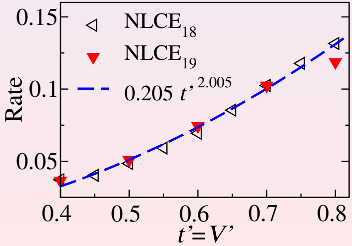

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Particle-number conservation (nonintegrable model)

Fermi's golden rule:

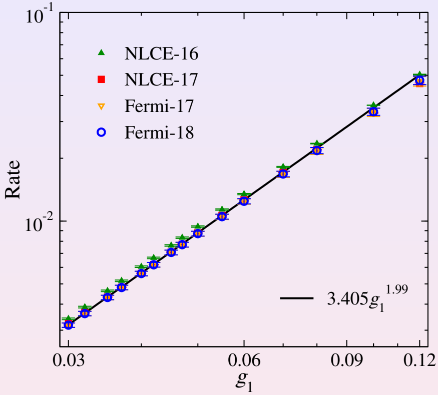
$$\dot{n}(\tau) = \frac{2\pi g_1^2}{L} \sum_{i,j} \delta(E_j^0 - E_i^0)\,(N_j - N_i)\,P_i^0(\tau) \times \left|\langle E_j^0|\hat{U}_1|E_i^0\rangle\right|^2 ,$$

where

$$N_i = \langle E_i^0|\hat{N}|E_i^0\rangle$$
$$P_i^0(\tau) = \langle E_i^0|\hat{\rho}(\tau)|E_i^0\rangle$$

Thermalization rate

$$\Gamma^{\text{Fermi}}(g_1) = -\frac{\dot{n}(\tau)}{n(\tau) - 0.5}$$



K. Mallayya, MR, and W. De Roeck, PRX **9**, 021027 (2019).

K. Mallayya and MR, PRB **104**, 184302 (2021).

Outline

Generalized hydrodynamics

Large-distances long-times dynamics:

- View the system as a continuum of fluid cells, each of which is spatially homogeneous, integrable, and contains many particles.
- Slow evolution of local quantities $\Rightarrow$ each fluid cell is locally equilibrated to a GGE parameterized by the distribution of rapidities.

O. A. Castro-Alvaredo, B. Doyon, T. Yoshimura, PRX **6**, 041065 (2016).

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GHD equation in an external potential $V(x)$:

$$\partial_t \rho_{\text{qp}}(\theta, x, t) + \partial_x [v_{\text{eff}}(\theta, x, t) \rho_{\text{qp}}(\theta, x, t)] = \left(\frac{\partial_x V(x)}{m} \right) \partial_\theta \rho_{\text{qp}}(\theta, x, t).$$

where θ are the rapidities, $\rho_{\text{qp}}(\theta, x, t)$ is the density of quasi-particles with rapidity θ, at position x, and time t, and $v_{\text{eff}}(\theta, x, t)$ is the effective velocity of the quasi-particles.

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$v_{\text{eff}}(\theta, x, t)$ depends on $\rho_{\text{qp}}(\theta, x, t)$:

$$v_{\text{eff}}(\theta, x, t) = \frac{\theta}{m} + \int d\theta' \varphi(\theta - \theta') \rho_{\text{qp}}(\theta', x, t) [v_{\text{eff}}(\theta', x, t) - v_{\text{eff}}(\theta, x, t)].$$

It encodes all the interaction effects [two-body scattering $\rightarrow \varphi(\theta - \theta')$].

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where θ are the rapidities, $\rho_{\text{qp}}(\theta, x, t)$ is the density of quasi-particles with rapidity θ, at position x, and time t, and $v_{\text{eff}}(\theta, x, t)$ is the effective velocity of the quasi-particles.

Densities of conserved quantities:

$$q(x,t) = \int d\theta \, h_q(\theta) \rho_r(\theta, x, t)$$

where $h_q(\theta)$ depends on the quantity q, for the particle density $h_n(\theta) = 1$.

O. A. Castro-Alvaredo, B. Doyon, T. Yoshimura, PRX **6**, 041065 (2016).

B. Bertini, M. Collura, J. De Nardis, M. Fagotti, PRL **117**, 207201 (2016).

Generalized hydrodynamics: trap-quench dynamics

Dynamics after increasing the strength of the confining potential

$$H_{\mathsf{LL}} = \sum_{j=1}^{N} \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_j^2} + U(x_j) \right] + g \sum_{1 \le j < l \le N} \delta(x_j - x_l) \, ,$$

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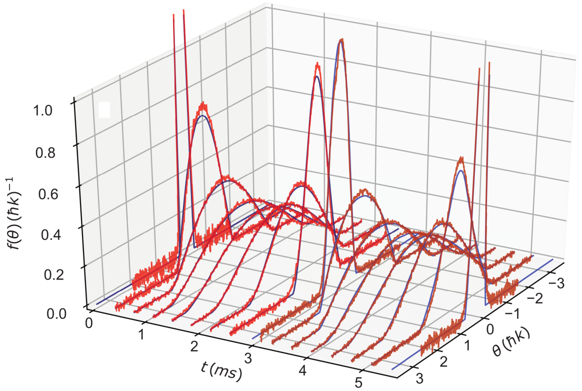
Evolution of the rapidity distributions, $U(x) \to 100U(x)$:



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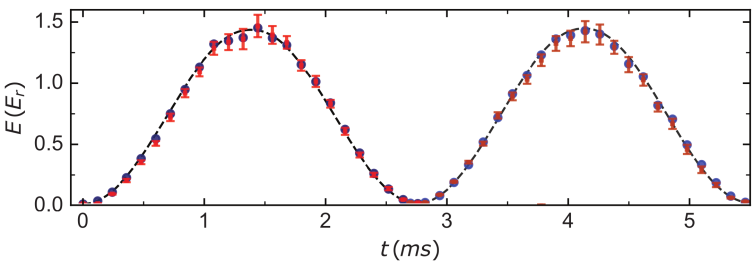


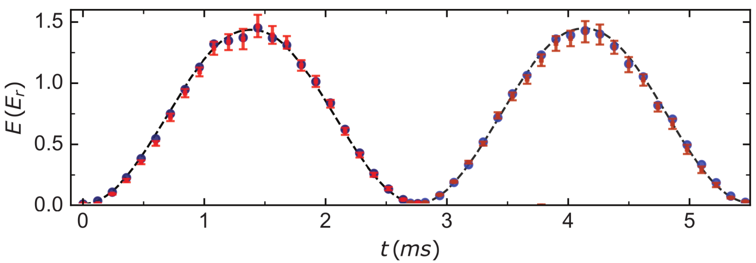
Oscillations

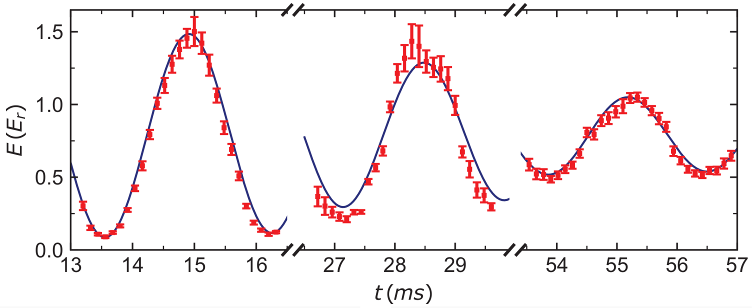
1st

and

2nd

Generalized hydrodynamics: trap-quench dynamics





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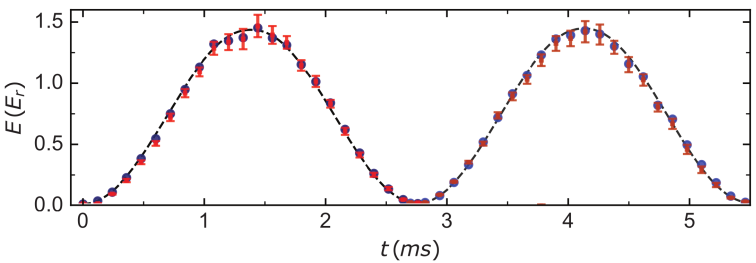
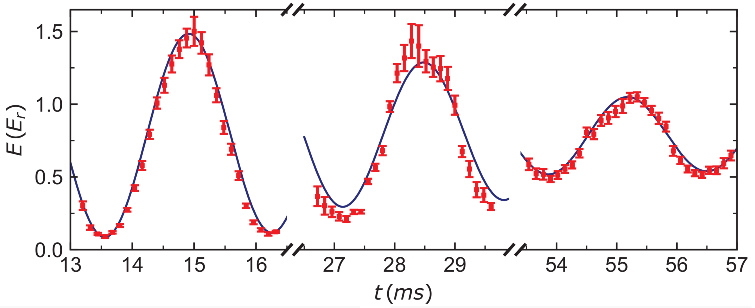




Oscillations

1st

and

2nd

6th

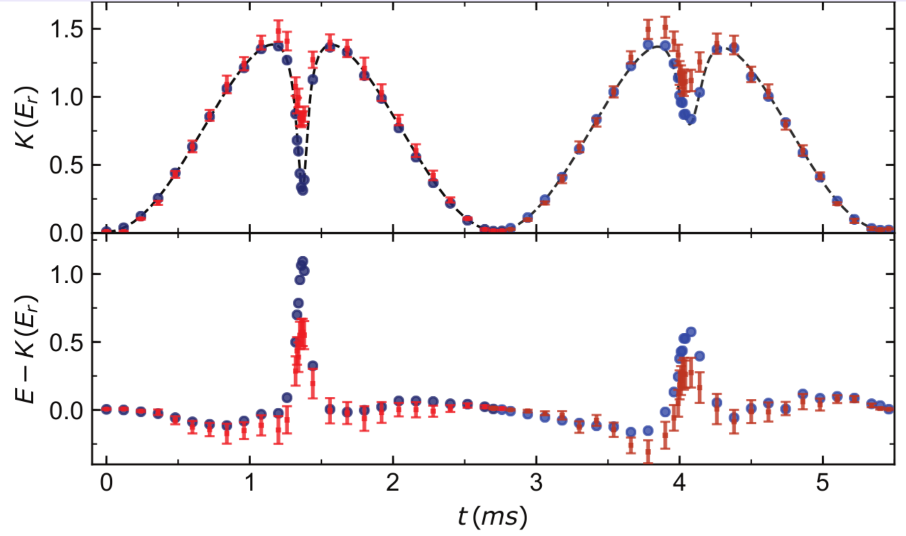
11th

and

21st

Generalized hydrodynamics: trap-quench dynamics

Evolution of the kinetic and interaction energies:



Outline

Time evolution of the momentum distribution

Dynamics after a Bragg pulse, $U(x) \to U(x) + U_{pulse} \cos^2(kx)$

$$H_{\mathrm{LL}} = \sum_{j=1}^{N} \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_j^2} + U(x_j) \right] + g \sum_{1 \le j < l \le N} \delta(x_j - x_l)\,,$$

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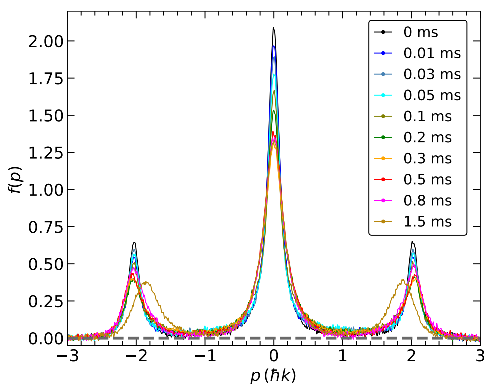
Evolution of the momentum distribution after the pulse, $t_{\mathrm{pulse}} = 6\,\mu\mathrm{s}$:



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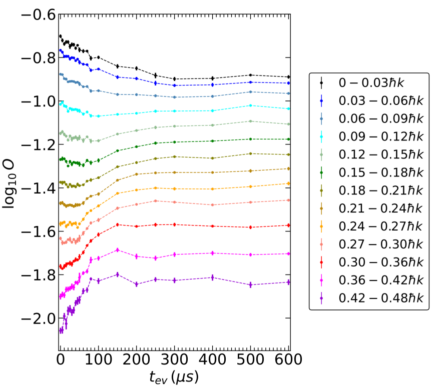

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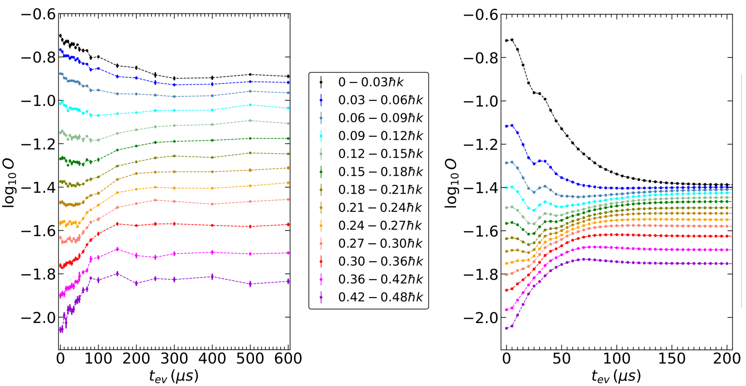

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Local prethermalization

Pre-quench ground state (TG limit):

$\prod_{|\theta|<\theta_F} \hat{c}^\dagger_\theta |0\rangle$

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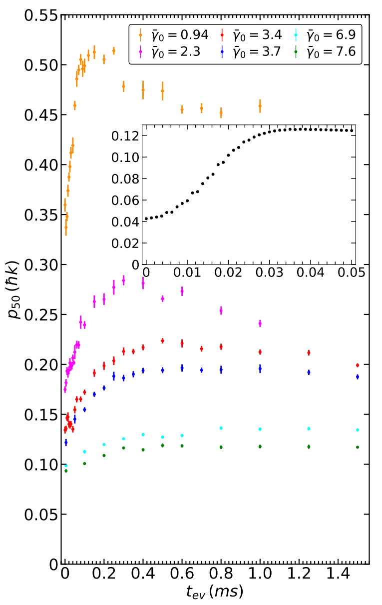

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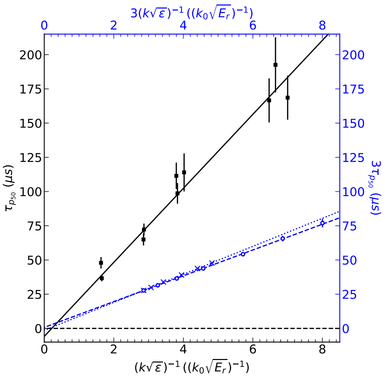

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Energy difference between n=0 and ± 1:

$$\Delta E = (2\hbar k)^2/2m$$

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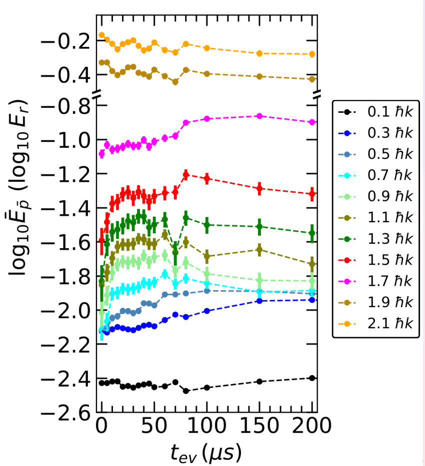

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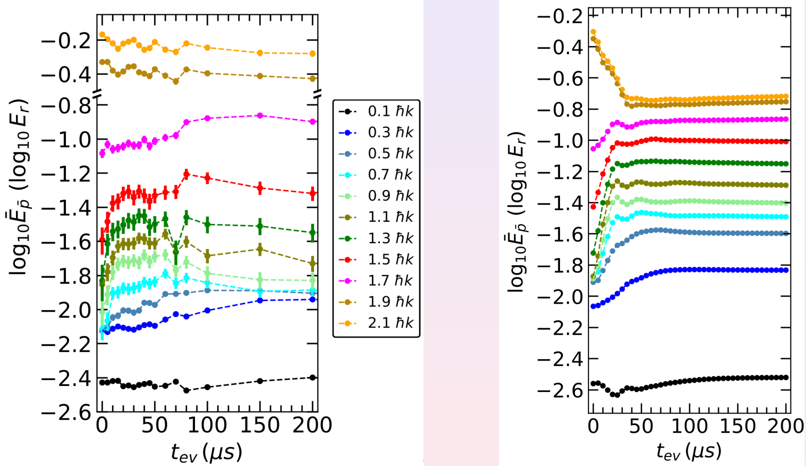

Summary

- Studied quantum dynamics in near-integrable 1D quantum systems.
  ★Experiment and theory went hand in hand.
- Two time scales in the short-time dynamics after Bragg scattering quenches.
- Hydrodynamization: Energy is redistributed among distant momentum modes and short-distance density variations smooth out ($t_{hd} \propto \frac{1}{k^2}$).
  ★Long-lived quasiparticles and the conservation of rapidities not necessary!
- Prethermalization: Energy is redistributed among nearby momentum modes and equilibration occurs to the local generalized Gibbs ensemble ($t_p \propto \frac{1}{pk}$).
- The large-distance long-time dynamics is accurately described by generalized hydrodynamics, even if the number of particles is ~ 10 and even if there are rapid variations in the nature of the quasi-particles.

Thank you!

Collaborators

- Tyler LeBlond ($\rightarrow$ ORNL)
- Krishna Mallayya ($\rightarrow$ Cornell)
- Lev Vidmar ($\rightarrow$ Jožef Stefan Institute)
- *Yicheng Zhang* ($\rightarrow$ U Oklahoma)
- David Weiss & group (Penn State)
- Wojciech De Roeck (KULeuven)
- Jerome Dubail (Lorraine, CNRS)
- Sarang Gopalakrishnan (Princeton)
- Benjamin Lev & group (Stanford)
- Anatoli Polkovnikov (Boston U)
- Dries Sels (NYU)

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