Charmonia melting in a magnetic field: a preliminary view from a phenomenological soft wall model

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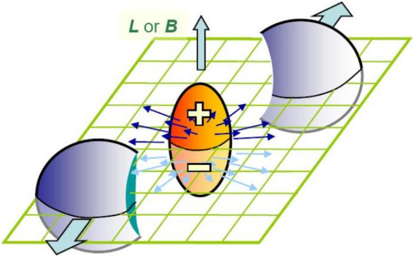
Overview

Overview

Why study strong magnetic fields?

- experimental relevance: appearance in QGP after a (noncentral) heavy ion collision (order $eB \sim 1 - 15 m_\pi^2$) (Skokov, Tuchin, Kharzeev, McLerran,



Deng, Huang . ,

- lifetime$_{\text{constant}B} \sim 10$ fm McLerran, Skokov, NPA929 (2014); Tuchin, PRC88(2013) . lifetime$_{QGP} \sim 1 - 10$ fm $\rightarrow$ Incentive to take $\vec{B}$ constant (ignoring "spatial decay" as well!) Typical LHC value: $\sim 0.2 - 0.3 \text{GeV}^2$
- From a (holographic) model viewpoint: interesting for comparison with recent lattice efforts

Why study strong magnetic fields?

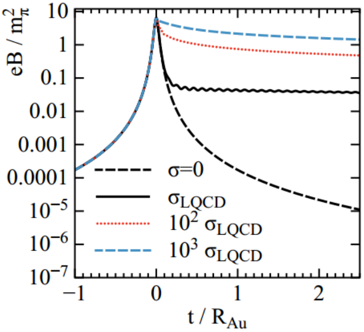


Figure : McLerran, Skokov (NPA929 (2014))

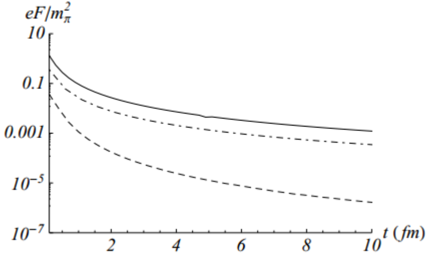
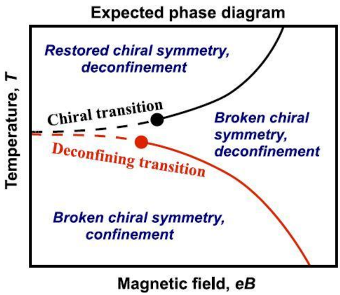


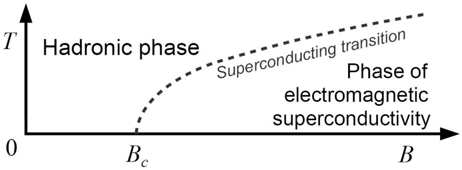
Figure : Tuchin PRC88(2013)

Studied effects

- split between $T_c(eB)$ and $T_\chi(eB)$? T_c depending on B?



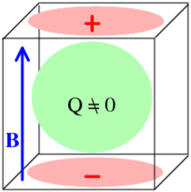
- ρ meson condensation + vacuum superconductor? (Chernodub et al)

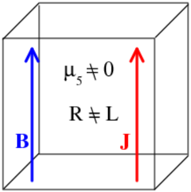


Studied effects

- (C)P odd effects: chiral magnetic effect, charge separation, etc. (work of Kharzeev, Fukushima, Warringa etc).

Assume overabundance in right quarks. Intuitively: the *B*-field will align spins,the differently charged right quarks will move in opposite directions, so imbalance in left-right can cause net current, $J \sim B$.





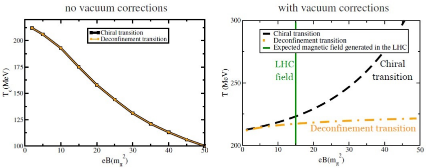
This work

Focus on deconfinement transition in a magnetic background

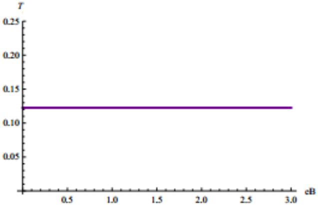
Using a modified soft wall model (see later).

Some results in different models

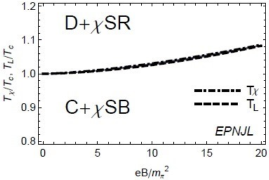
PLSM$_q$ model Mizher, Chernodub, Fraga, PRD82 (2010) 105016

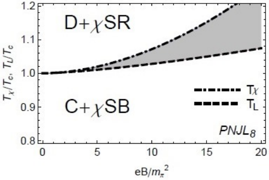


Sakai-Sugimoto model Dudal, Callebaut, PRD87(2013)



Different PNJL models Gatto, Ruggieri, PRD82 (2010) 054027; PRD83 (2011) 034016





Competing predictions!

State of the art lattice data disagree with most previous results: $T(eB) \searrow$

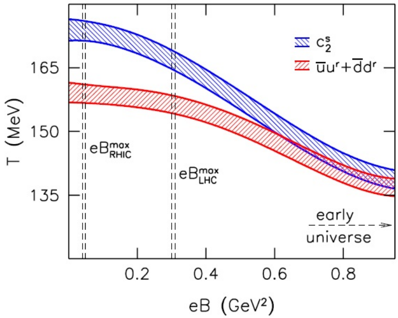


Figure : Lattice (Bali et al JHEP 1202 (2012); PRD86 (2012)

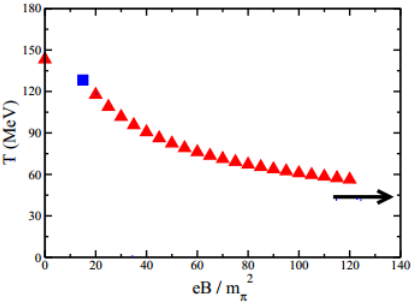


Figure : MIT bag (Fraga, Palhares PRD86 (2012)

$\rightarrow$ quenched vs. true QCD

Holographic QCD

- **What is holographic QCD?**

"QCD" $\overset{dual}{\longleftrightarrow}$ (super)gravity in a higher-dimensional background: 4D QCD "lives" on the boundary of a five-dimensional space wherein the (super)gravity theory is defined

- **Origin of the QCD/gravity duality correspondence?**

Anti de Sitter/Conformal Field Theory (AdS/CFT)-correspondence (Maldacena 1997): conformal $\mathcal{N}$=4 SYM theory $\overset{dual}{\longleftrightarrow}$ supergravity in $AdS_5 \times S_5$ space

- **Many holographic QCDish theories have seen the light**

string motivated models (Sakai-Sugimoto,...), wall models, light front holography, ...

How to make magnetic predictions in holographic QCD models?

- Confinement is modeled in/described by background metric (e.g. AdS with a cut-off)

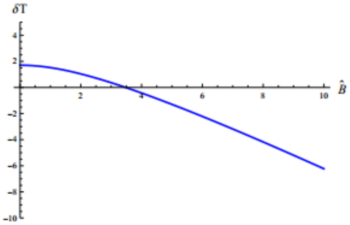
  Cut-off scale is needed to feed string tension
- Quark physics is mostly modeled in via probe branes/effective (DBI) actions ($\rightarrow$ “quenched QCD”)
- problematic to capture all magnetic field effects: B can only couple to neutral glue/geometric background if charged quark dynamics is taken into account.

$\Rightarrow$ reason why it is hard to study the deconfinement transition in a B-field (amongst other things)

- In general: deconfinement transition $\leftrightarrow$ interpreted as Hawking-Page transition in dual picture [free energy/pressure comparison].

Back reaction of quarks on (de)confining background?

- Genuine back reaction in string-QCD model is not easy to consider, usually expansions in N_f/N_c using smeared probe brane configurations (never done with B-field).
- In wall models: possibility (to try) to construct B-dependent wall, e.g. by using Einstein-Maxwell setup? (not yet done either)
- In-between approach by Ballon-Bayona (JHEP 1311 (2013)): pressure correction of probe flavours taken into account: $T_c(eB) \searrow$. Back reaction effects on geometry argued to be subleading.



A complementary view

- Theoretical view on deconfinement: VEV of Polyakov loop (breaking or not of $\mathbb{Z}_N$ center "symmetry").
- Phenomenological view on deconfinement: bound QCD-states are "melted away".
- Prototypical example: charm bound state, J/ψ.

  Its suppression known to be a good signal for the plasma phase

  (Matsui/Satz PLB178 (1986))

  Known to only melt "beyond deconfinement"; lattice spectral function analysis (Asakawa/Hatsuda PRL92 (2004))

Let us investigate how B influences the J/Ψ melting!

Overview

(original) Soft wall metric

Erlich, Katz, Son, Stephanov PRL95 (2005)/ Karch, Katz, Son, Stephanov PRD74 (2006)

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + d\mathbf{x}^2 + dz^2\right), \qquad e^{-\Phi} = e^{-cz^2},$$

for low T confined phase, $z = 0$ (the boundary) $\ldots\infty$.

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right), \qquad e^{-\Phi} = e^{-cz^2}$$

for high T deconfined phase where $f(z) = 1 - z^4/z_h^4$. $z = 0\ldots z_h$;
(AdS black hole horizon). $T = \frac{1}{\pi z_h}$.
Soft wall action:

$$S \propto \int d^5x\sqrt{-g}e^{-\Phi}\mathrm{tr}\left[F^{L,\mu\nu}F_{L,\mu\nu} + F^{R,\mu\nu}F_{R,\mu\nu}\right],$$

Inclusion of charmonia

- We follow original setup of Fukushima et al PRD80 (2009), PRD81 (2010): extra $U(1)$ action for heavy charm

$$S_c = -\int d^5x\sqrt{-g}\,\mathrm{tr}\; e^{-c_\rho z^2}\mathcal{L}_{light} + e^{-c_{J/\psi}z^2}\mathcal{L}_{charm}.$$

  $c_\rho = 0.151$ GeV^2 and $c_{J/\psi} = 2.43$ GeV^2 to reproduce masses.

- $c_{J/\psi} \gg c_\rho \rightarrow$ almost irrelevant for prediction of $T_c = 0.492\sqrt{c_\rho} = 0.191$ GeV. (Herzog, PRL98 (2007))
- Effective model, in principle $c =$ universal and relation to σ_{QCD}. Back reaction effects of heavy flavour m_c could make $c(m_c)$ “run”.
- Analyze spectral function (peaks) for J/ψ in terms of T and compare to T_c.

How to couple B to J/ψ?

- "Global" EM coupling is trivial since J/ψ is neutral.
- We need to couple B to its charged c-constituents.
- Treat J/ψ as a composite object, with "smeared charge" $\rightarrow$ nonlinear electrodynamics (Dirac-Born-Infeld)

  Use DBI-action

$$S = -\frac{1}{4\pi^2\alpha' g_5^2}\int d^D x e^{-\Phi}\sqrt{-\det\left(g_{\mu\nu} + 2\pi\alpha' i F_{\mu\nu}\right)}$$

as generalization of charm $U(1)$ model, working up to quadratic order in the fluctuations with

$$F_{\mu\nu} \rightarrow F_{\mu\nu} + \overline{F}_{\mu\nu}$$

with $\overline{F}_{12} = -2/3ieB$ (corresponding to $\vec{B} = B\vec{e}_z$)

Internal consistency: constant B solves the DBI EOM

$$S \sim \int d^D x e^{-\Phi} \sqrt{-\det(g + 2\pi\alpha' iF)}$$

for a fixed background dilaton and (string) metric. Varying this action with respect to A_μ yields

$$\delta S \sim -\int d^D x \left(\frac{1}{g + 2\pi\alpha' iF}\right)^{\nu\lambda} \delta F_{\lambda\nu} e^{-\Phi} \sqrt{-\det(g + 2\pi\alpha' iF)},$$

which gives

$$\partial_\lambda \left[\left(\frac{1}{g + 2\pi\alpha' iF}\right)^{\nu\lambda} e^{-\Phi} \sqrt{\det(g + 2\pi\alpha' iF)} - \left(\frac{1}{g + 2\pi\alpha' iF}\right)^{\lambda\nu} e^{-\Phi} \sqrt{\det(g + 2\pi\alpha' iF)} \right] = 0$$

The specific form of g, Φ and the candidate $\overline{F}$ fulfills this EOM.

How to fix the extra parameter α'

- Soft wall models cannot predict area law for Wilson loop Karch et al, JHEP 1104 (2011).
- Let us look at Polyakov loop: no problem! And lattice data for heavy test quark, compatible with our choice of charm!

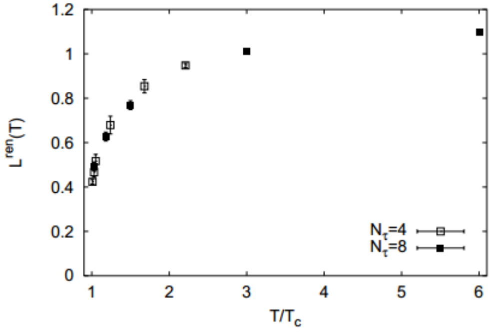
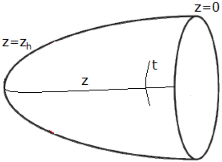


Figure : Kaczmarek et al, PLB543 (2002)

How to fix the extra parameter α'?



P-loop $\sim$ string around thermal circle; $\langle P \rangle$= (on-shell) Nambu-Goto action in AdS$_5$ BH metric.

$$S = \frac{L^2}{2\pi\alpha' T}\int_0^{z_h} dz \frac{1}{z^2}\sqrt{1+f\mathbf{x}'^2} = \frac{L^2}{2\pi\alpha' T}\int_0^{z_h} dz \frac{1}{z^2}$$

hence (with cut-off at $z=\varepsilon$)

$$\langle L(T)\rangle = e^{-S} = e^{\frac{L^2}{2\alpha'} - \frac{L^2}{2\pi\alpha' T\varepsilon}}.$$

After P-loop renormalization (piece $\propto$ circumference dropped)

$$\langle L(T)\rangle_{\text{ren}} = e^{\frac{L^2}{2\alpha'}}.$$

Matching on high T lattice loop (units $L=1$): $\alpha' = (2.29)^2$.

EOM for the vector modes

Gauge $A_z = 0$ and $2V = A_L + A_R$, with $V \propto e^{-i\omega t}$ (zero momentum).

The equation of motion for $V_{1,2}$ is given by

$\boxed{\perp \vec{B}}$

$$\partial_z^2 V_{1,2} + \partial_z \left(\ln \left(\sqrt{-\mathcal{G}} e^{-cz^2} G^{zz} G^{11} \right) \right) \partial_z V_{1,2} - \frac{G^{tt}}{G^{zz}} \omega^2 V_{1,2} = 0$$

$\boxed{\parallel \vec{B}}$

$$\partial_z^2 V_3 + \partial_z \left(\ln \left(\sqrt{-\mathcal{G}} e^{-cz^2} G^{zz} G^{33} \right) \right) \partial_z V_3 - \frac{G^{tt}}{G^{zz}} \omega^2 V_3 = 0$$

EOM for the vector modes

$$\mathcal{G} = g_{00} g_{33} g_{zz} \left(g_{11} g_{22} - (2\pi\alpha')^2 \bar{F}_{12}^2 \right)$$

$$\mathcal{G}^{\mu\nu} = \begin{bmatrix} \frac{1}{g_{00}} & 0 & 0 & 0 & 0 \\ 0 & \frac{g_{22}}{X} & -\frac{2\pi\alpha' i \bar{F}_{12}}{X} & 0 & 0 \\ 0 & \frac{2\pi\alpha' i \bar{F}_{12}}{X} & \frac{g_{11}}{X} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{g_{33}} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{g_{zz}} \end{bmatrix}$$

where $X = g_{11} g_{22} - (2\pi\alpha')^2 \bar{F}_{12}^2$.
$G =$ the symmetric part of the metric tensor $\mathcal{G}$.

Overview

Near horizon limit: $z = z_h$

We follow the Minkowski AdS/CFT dictionary of Policastro, Son, Starinets, JHEP0209 (2002) see also Teaney PRD74 (2006)

- We work in rescaled variables: $\xi = \sqrt{c}z, \quad \tilde{\omega} = \frac{\omega}{\sqrt{c}}, \quad \tilde{D} = \frac{D}{c}$
- Frobenius analysis leads to $V_{1,2,3} \sim \left(1 - \frac{\xi}{\xi_h}\right)^{\pm i\tilde{\omega}\frac{\xi_h}{4}}$
- (No extra singularities if B is present)

Near boundary limit: $z = 0$

- Frobenius analysis yields

$$\Phi_2(\xi) = \xi^2 \sum_{k=0}^{+\infty} a_k \xi^k,$$

$$\Phi_1(\xi) = C \ln(\xi) \Phi_2(\xi) + \sum_{k=0}^{+\infty} b_k \xi^k.$$

  $a_0 = b_0 = 1$. b_2 can be chosen at will (= adding Φ_2 to Φ_1).
- $a_2 = -\frac{\tilde{\omega}^2}{8} + \frac{1}{2}$ and $C = -\frac{\tilde{\omega}^2}{2}$. Odd-indexed parameters $= 0$.
- Physics will be encoded in

$$\Phi_2'(\varepsilon) = 2\varepsilon,$$
$$\Phi_1'(\varepsilon) = \partial_\xi \left(C \ln(\xi) \xi^2 + b_2 \xi^2 \right) \Big|_{\xi=\varepsilon},$$

Real time AdS/CFT $\rightarrow$ ingoing solution

We may choose boundary conditions

$$\Phi_1(\varepsilon) = 1, \quad \Phi_1'(\varepsilon) = -\tilde{\omega}^2 \ln(\varepsilon)\varepsilon,$$
$$\Phi_2(\varepsilon) = \varepsilon^2, \quad \Phi_2'(\varepsilon) = 2\varepsilon.$$

Out-/ingoing solutions also complete set ($\phi_\pm \sim \left(1 - \frac{\xi}{\xi_h}\right)^{\pm i\tilde{\omega}\frac{\xi_h}{4}}$ near the horizon):

$$\Phi_1 = \alpha\phi_- + \alpha^*\phi_+$$
$$\Phi_2 = \beta\phi_- + \beta^*\phi_+.$$

Normalize the desired solution v as

$$v = \Phi_1 + B\Phi_2 = (\alpha + \mathcal{B}\beta)\phi_- + \underbrace{(\alpha^* + \mathcal{B}\beta^*)}_{0}\phi_+,$$

we obtain $\mathcal{B} = -\frac{\alpha^*}{\beta^*}$.

Boundary on-shell action $\rightarrow$ retarded propagator

The boundary contribution [obtained by integrating by parts and retaining *only* the contribution at $z = 0$ (PSS prescription)]

$$S_{\text{on-shell, bdy}} \sim \lim_{z\to 0} \int d^4x \sqrt{-\mathcal{G}}\, G^{zz} G^{\nu\sigma} A_\nu \partial_z A_\sigma .$$

leading to

$$S_{\text{on-shell, bdy}} \sim \lim_{\xi\to 0} \int d^4x \frac{L}{\xi} v \partial_\xi v ,$$

or, in Fourier space,

$$\lim_{\xi\to 0} \int d^4x \frac{L}{\xi} v \partial_\xi v = \lim_{\xi\to 0} V_4 \frac{L}{\xi} \tilde{v} \partial_\xi \tilde{v} ,$$

$$D_R(\omega, \mathbf{q} = 0) \sim \lim_{\xi\to 0} \frac{\tilde{v}\partial_\xi \tilde{v}}{\xi} \quad \text{(PSS prescription)}$$

$$\rho(\omega, \mathbf{q} = 0) = -\frac{\Im D_R(\omega, \mathbf{q} = 0)}{\pi} \sim \Im \left(\frac{(1 + \mathcal{B}\varepsilon^2)(\Phi_1'(\varepsilon) + 2\mathcal{B}\varepsilon)}{\varepsilon} \right) = 2\Im \mathcal{B}$$

Numerical results for spectral function $\parallel \vec{B}$

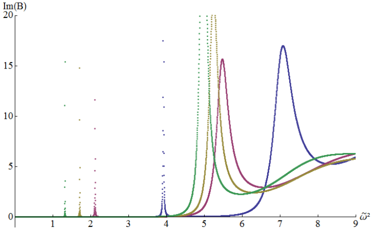


Figure : The temperature is fixed at $t = T/\sqrt{c} = 0.07$. Blue: $qB = 0$, purple: $qB = 0.2$ GeV^2, yellow: $qB = 0.4$ GeV^2, green: $qB = 1.0$ GeV^2.

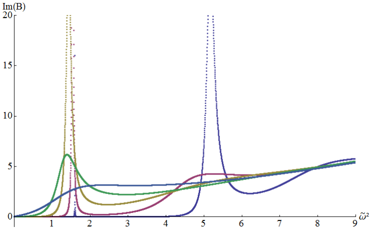
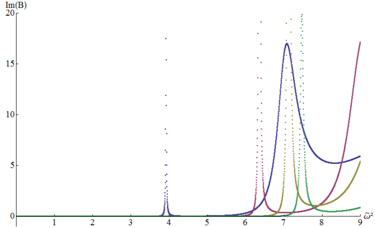


Figure : The magnetic field is fixed at $qB = 0.5$ GeV^2. Blue: $t = 0.07$, purple: $t = 0.09$, yellow: $t = 0.11$, green: $t = 0.14$, lightblue: $t = 0.20$.

Spectral peaks melt at higher temperatures than if $B = 0$ (melting temperature $t = 0.14$, $t_c = 0.122$). Peaks also shift towards lower ω^2.

Numerical results for spectral function $\perp \vec{B}$



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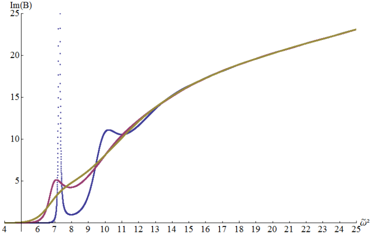

Figure : The magnetic field is fixed at $qB = 0.5$ GeV^2. Blue: $t = 0.07$, purple: $t = 0.09$, yellow: $t = 0.11$.

$\perp \leftrightarrow \|$: transverse polarizations become heavier, but melt faster.

Picture that transverse get heavier compared to longitudinal polarizations confirmed by sum rules analysis Cho et al, arXiv:1411.7675

Linking to other magnetic QCD observations

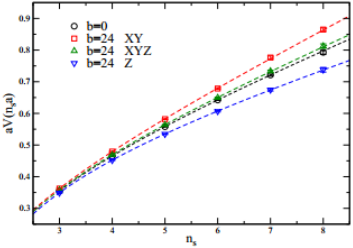

Figure : D'Elia et al, PRD89 (2014) (lattice) + Chernodub arXiv:1001.0570 (pheno): static quark potential stronger in $\perp$ direction than in $\parallel$ one

To our understanding, no direct link between polarizations of J/Ψ vs. quark pair direction.

Thought experiment: random distribution of (anti)quark pairs $\rightarrow$ $2/3 \perp \vec{B}$; $1/3 \parallel B \rightarrow$ expect 2/3 to be heavier using soft wall result $m \propto \sigma n$.

Intermediate conclusion

Magnetic field vs. (de)confinement

At least in our soft wall setting, the different polarizations are sensitive to B. Also the melting is considerably influenced. The longitudinal polarization survives longer, and beyond $T_{Polyakov}$.

Quite complicated picture how B influences (de)confinement. More involved than P-loop studies suggest?

We expect similar results in more evolved holoQCD models with genuine DBI action.

Overview

Transport properties in the QGP

Our interest here: how well do the quarks "follow" the medium?

$$D = \frac{1}{6\chi}\lim_{\omega\to 0}\sum_{i=1}^{3}\frac{\rho_{ii}^{V}}{\omega},$$

$$\vec{B} \to \text{anisotropy} \to \begin{cases} D_{\perp} \\ D_{\parallel} \end{cases}$$

χ = heavy quark number susceptibility $\sim \rho_{00}$. We did not compute this quantity yet (more difficult). Although χ dependent on B, only a single quantity. So we can get rid of χ by working with the relative diffusion.

Numerical results for heavy quark diffusion

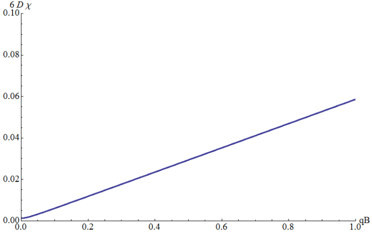

Figure : $D_{\parallel}$ for $t = 0.14$.

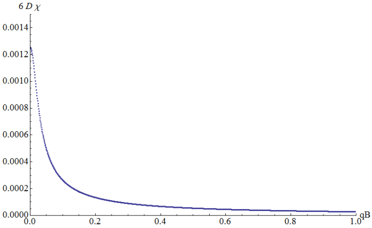

Figure : $D_{\perp}$ for $t = 0.14$.

Relative diffusion

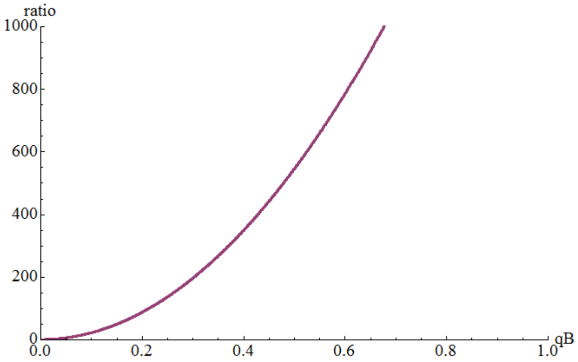

Figure : $\frac{D_{\parallel}}{D_{\perp}}$.

Diffusion along $\vec{B}$ much stronger!

Heavy quark diffusion: analytical treatment

Differential equation for the longitudinal polarization:

$$V_3'' + \frac{g'}{g} V_3' - \frac{G^{tt}}{G^{33}} \omega^2 V_3 = 0,$$

$$g = \sqrt{-\mathcal{G}} e^{-cz^2} G^{zz} G^{33} = \sqrt{\frac{L^{10}}{z^{10}} + \frac{L^6}{z^6} D^2} \frac{z^4}{L^4} e^{-cz^2} \left(1 - \frac{z^4}{z_h^4}\right)$$

We only need $\omega \to 0$ information for diffusion, so we work in a hydrodynamic (small ω) expansion

$$V_3(z) = \left(1 - \frac{z}{z_h}\right)^{-\frac{i\omega z_h}{4}} \left(F_0(z) + \omega F_\omega(z) + \ldots\right)$$

Heavy quark diffusion: analytical treatment

Avoiding singularities at the horizon $z = z_h$ in $F_{0,\omega}$ + direct integration (order per order) yields, at the end, for $\omega \to 0$

$$6\chi D_{\parallel} \sim \frac{e^{-cz_h^2}}{z_h\pi}\sqrt{1+\frac{4(2\pi\alpha')^2 z_h^4}{9}(qB)^2},$$

$$6\chi D_{\perp} \sim \frac{e^{-cz_h^2}}{z_h\pi}\frac{1}{\sqrt{1+\frac{4(2\pi\alpha')^2 z_h^4}{9}(qB)^2}}.$$

based on e.g.

$$\chi D_{\parallel} \sim \Im \lim_{\omega\to 0}\frac{1}{\omega}\lim_{z\to 0}\frac{V_3\partial_z V_3}{z}$$

Also

$$\frac{D_{\parallel}}{D_{\perp}} = \frac{L^4 + D^2 z_h^4}{L^4} = 1 + \frac{4(2\pi\alpha')^2 z_h^4}{9}(qB)^2.$$

Overview

Things to do

In the proposed magnetized soft wall model

- Study $\vec{q} \neq 0$ spectral functions, to see if rotational symmetry is restored. (somewhat unexpected prediction at $B = 0$ by Fukushima et al in PRD81 (2010)).
- Add (anomalous) mixing with η_c (only relevant for $\parallel$ sector)
- Study of heavy quark susceptibility.
- Phenomenological relevance, e.g. to elliptic flow? ($\rightarrow$ experimental relevance of B itself!)
- Usage of improved holographic charmonium model! (cf. problem with "universal" scale c)

In general for magnetized QCD

- Construct a dual background that contains quark/B-effects!

The End.

Thanks!