EXACT, FINITE, MODEL RENORMALIZATION

of NON-PERTURBATIVE, GAUGE-INVARIANT, REALISTIC QCD,

by

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DESCRIPTIONS OF PREVIOUS WORK

a) Starting Point: Schwinger Generating Functional (GF) for QCD, with gluon operators in an Arbitrary (Relativistic) Gauge.

b) Re-arrange this GF in terms of a "Reciprocity Relation", and a "Gaussian Linkage Operation"; and the GF now depends upon two functionals of A,
$G_c(x,y|A) = [m + \gamma\cdot(\partial - igA\cdot\tau)]^{-1}$ and $L[A] = \mathrm{Tr}\,\ln[1 - ig\gamma\cdot A\cdot\tau\, G_c[A]]$.

c) Insert Halpern's half-century-old representation

$$e^{-\frac{i}{4}\int F^2} = N\int d[\chi]\, e^{\frac{i}{4}\int \chi^2 + \frac{i}{2}\int F\cdot\chi}, \quad \chi^a_{\mu\nu} = -\chi^a_{\nu\mu}.$$

[[[The next two steps were overlooked for decades.]]]

d) A second and trivial re-arrangement can now be made to <u>formally</u> insure gauge-invariance, even though the GF still apparently contains gauge-dependent gluon propagators.

e) Insert and employ Fradkin's functional representations for $G_c[A]$ and $L[A]$, which are Gaussian in A.

f) All the Gaussian Linkage operations can then be carried through exactly, corresponding to the summation of all gluons exchanged between any pair of quark (and/or antiquark) lines, and including cubic and quartic gluon interactions.

THE RESULT:

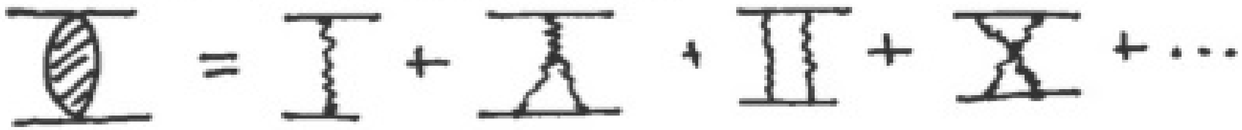
Explicit cancellation of all gauge-dependent gluon propagators, with the resulting GF exhibiting Manifest Gauge Independence...

AND

one finds a new, exact property of Non-Perturbative, Gauge-Invariant QCD,

EFFECTIVE LOCALITY (EL):

Define a "Gluon Bundle" (GB) as the Sum over all gluon exchanges between any pair of quark lines,



The space-time coordinates of both ends of a GB are equal, modulo small uncertainties in their transverse coordinates.

What this means is that, at high energies, the Halpern FI can be reduced to sets of ordinary integrals, yielding a vast simplification in the calculation of all QCD correlation functions. (Pencil-and-paper + a desktop computer can now replace huge, multiple-processing, lattice estimations.)

CONCERNING TRANSVERSE QUARK FLUCTUATIONS...

Starting from conventional, quark-field-operator equations of motion,

one can define "IN" and "OUT" operator fields, $\psi_{\substack{IN\\OUT}}(x)$, as in any Abelian Theory; QCD just has a more complicated interaction, non?

But this is absurd! For decades we have known that all asymptotic quark states are hadronic bound states of quarks; and for such a bound state we can specify longitudinal and time coordinates, but not transverse coordinates, since they are always fluctuating.

NB: The conventional "static quark" approximation used in lattice and other model binding-potential calculations is fundamentally wrong,

and in 2 ways. Without taking such "transverse imprecision" into account, all non-perturbative amplitudes are plagued with absurdities.

How to introduce transverse quark fluctuations from First Principles?

We believe we know how to do this, but work still underway.

What we have done is to introduce phenomenological transverse fluctuation amplitudes for every quark-gluon vertex, replacing the usual gluon-quark current interaction at the same space-time point,

$$\int d^4x\, \bar{\psi}(x)\gamma_\mu A^a_\mu(x)\tau_a\psi(x) \quad \text{by} \quad \int d^2x'_\perp \int d^4x\, a(x_\perp - x'_\perp)\,\bar{\psi}(x)\gamma_\mu \tau_a A^a_\mu(x)\,\psi(x'),$$

with $a(x_\perp - x'_\perp)$ real and symmetric, and $x'_\mu = (x'_\perp\ , x_L\ , x_0\ )$.

The probability of finding two quarks separated by a transverse (or impact parameter) distance is then: $\varphi(b) = \int \frac{d^2q}{(2\pi)^2}\, e^{iq\cdot b}\, |\tilde{a}(q)|^2$.

Is this a Violation of Lorentz Covariance, or of SU(3) Invariance?

Q: What is the First Rule of Quantum Mechanics?

A: One cannot believe any statement unless it can be measured.

Q: What is the Corollary to that Rule?

A: If one cannot, in Principle, measure something, then any useful idea, not violating other explanations, cannot be rejected.

- In fact, Lorentz covariance of hadron motion is preserved, and all hadrons are color singlets.

How to Choose φ (b)? It is directly related to quark binding; how does φ (b) produce V(r), the q-$\bar{q}$ binding potential whose lowest bound state represents the pion?

First try: A Gaussian, $\varphi(b) \simeq e^{-(\mu b)^2}$, where μ^{-1} sets the scale of transverse fluctuations. Then, all absurdities of correlation functions disappear. But this distribution is "too symmetric", and gives a zero V(r).

Second try: A "deformed" Gaussian, $\varphi(b) = \varphi(0)\, e^{-(\mu b)^{2+\xi}}$ with ξ a "deformation parameter", real and small.

A straight-forward calculation then yields, for small ξ : $V(r) \simeq \xi \mu (\mu r)^{1+\xi}$.

Substituting this potential into a Schrodinger binding equation, using the "quantic" approximation, then yields $\mu \sim m_\pi$, $\xi \simeq .1$. This is sensible, since the max. b fluctuations should be $\lesssim$ than m_π^{-1}.

Our result encompasses two different lattice calculations, $V \sim r$ and $V \sim r \ln(r)$. But all lattice and other model calculations of q-$\bar{q}$ binding correspond to an amplitude containing only one of the two Casimir SU(3) invariants; our amplitude contains both.

What method do we use to pass from φ (b) to V(r)?

Imagine that a q and a $\bar{q}$ are scattering at high energy. One can write an Eikonal approximation, valid in the limit of s >> ltl, for the conventional scattering amplitude. (Details for QCD eikonals were worked out by HMF, YG, JA, and BMcK in two papers circa 1983.)

It has been well-known for a half-century that, assuming a specific V(**r**), in ordinary QM, or in Abelian QFTs, the corresponding eikonal function E(**b**) is given by

$$E(b) = \gamma(s) \int_{-\infty}^{+\infty} dz_L\, V(\vec{b} + \hat{p}_L z_L),$$

where γ (s) is a constant depending on CM energy and the type of interaction.

We can write the non-perturbative amplitude corresponding to a GB exchanged between a q and a $\bar{q}$; and we see that the Eikonal limit of this amplitude has E(**b**) defined in terms of φ (b), and proportional to: ln{ φ (b)}. Here, E(**b**) = E(b), and V(**r**)= V(r).

Our method: Calculate the 2-D Fourier transform $\tilde{E}(k_\perp^2)$ of E(b). Extend $k_\perp^2 \to k_\perp^2 + k_L^2 = \vec{k}^2$, so that we now have $\tilde{E}(\vec{k}^2)$; and then calculate the 3-D transform of this $\tilde{E}(\vec{k}^2)$, which will yield V(r).

No static approximation required! Our analysis gives E(b) explicitly, in terms of φ (b), so that we can calculate V(r) for any choice of φ (b). The minimal bound state energy representing the pion shows that most of the pion mass comes from the gluons forming the GB, and relatively little from the quark masses,

NB: The 3-body problem of qqq binding remains to be calculated. What is μ here? Intuitively, one expects $\mu \sim m_n$. And since each q-q GB interaction turns out to have the same form, $V_{ij} \sim \xi\mu(\mu r_{ij})^{1+\xi}$ where $r_{ij} = | \vec{r}_i - \vec{r}_j |$ is the distance between any two quarks, one has a rough estimation of each V_{ij}.

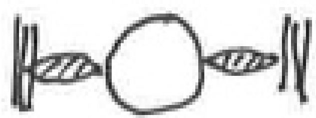
Then, if each $r_{ij} \sim m_\pi^{-1}$, $V_{ij}(r_{ij}) \sim m_n \sim \xi\mu\left(\frac{\mu}{m_\pi}\right)^{1+\xi}$ or $m_n \sim \frac{m_n}{10}\left(\frac{m_n}{m_\pi}\right)^{1+\xi} \sim m_n$

which is reasonable for an intuitive, qualitative estimate.

If you look up Nuclear Forces on Wikipedia, you'll find the statement that there exists no derivation on the basis of QCD.

Here is the first (to our knowledge) example of nucleon binding (for a Model deuteron) from basic QCD. The Model neglects electrical charge, and nucleon spins (which can always be added in); this is a Qualitative model, describing the essence of Nuclear Physics.

Question of Scale: Quark binding takes place for $r_{ij} \sim m_{\pi}^{-1}$, but nucleon binding takes place at 2, or 3, or 4 times that distance. How to achieve this?

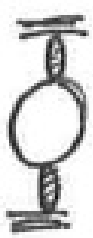
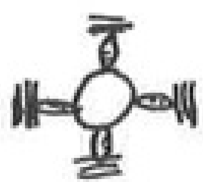
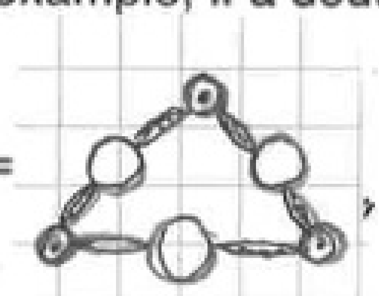
Consider: 

This requires extraction and regularization of the logarithmic UV divergence of the loop, which contributes two essential features:

a) It "stretches", so that distances larger than m_π^{-1} can easily enter.

b) It provides a crucial change of sign for the effective n-n binding potential.

This sign change can be the basis of nucleon-binding to form nuclei. Expect (and hope) that nuclear physicists will employ such effective potentials to discuss heavier nuclei. For example, if a deuteron is

 , is α =  , but tritium =  , ?

Can one make a nuclear shell model out of something like this?

Virtual Closed-Quark Loops, and their Interactions with GBs.

All the basic, "radiative correction" structure of non-perturbative QCD comes from interacting closed-quark-loops with GBs. How can this be efficiently described? I will try to do this in words, describing the functional operations that need to be performed.

A single 'dressed' quark has an amplitude proportional to

$$\mathcal{N}\int d[\chi]\, e^{\frac{i}{4}\int\chi^2}\left(\det(g f\cdot\chi)^{-\frac{1}{2}}\right) e^{\hat{\mathfrak{D}}_A} \cdot G_c[A]\, e^{L[A]}\Big|_{A\to 0}, \quad \hat{\mathfrak{D}}_A = \frac{i}{2}\int \frac{\delta}{\delta A}(g f\cdot\chi)^{-1}\frac{\delta}{\delta A},$$

while two scattering quarks are described by

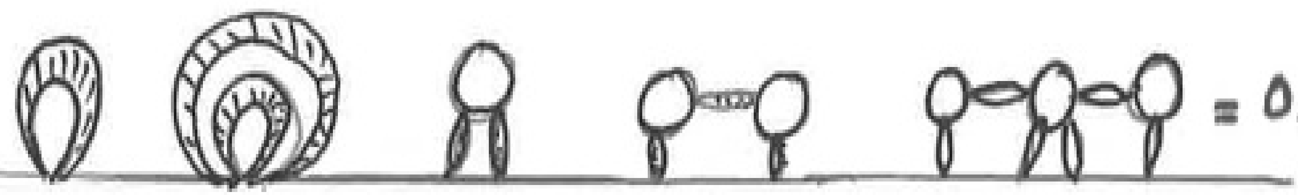
$$\mathcal{N}\int d[\chi]\, e^{\frac{i}{4}\int\chi^2}\left(\det(g f\cdot\chi)^{-\frac{1}{2}}\right) \cdot e^{\hat{\mathfrak{D}}_A}\, G_c^{(1)}[A]\, G_c^{(2)}[A]\, e^{L[A]}\Big|_{A\to 0};$$

and $\chi(x)$ is the Halpern functional variable originally used to represent exp[(i/4)$\int d^4x\, F^2(x)$].

Every GB exchanged is represented by the linkage operator

connecting the two $G_c[A]$s to each other, and the $G_c[A]$s to L[A].

And here the relative simplicity of non-perturbative QCD shows itself clearly, for all of its "self-energy" graphs vanish, either by the asymmetry of the (f·Xi) color and coordinate indices, or by explicit loop integration.



What is the Z_q of a (non-perturbative) quark? $Z_q = 1$.
Non-Perturbative QCD is far simpler than QED !!

The 'radiative corrections' of QCD enter when there is momentum transfer between one quark and another quark; and the procedure may occur when the momentum transfer passes through intermediate GBs and/or closed quark loops.

For simplicity, let us suppress possible quark binding into hadrons, and just consider two quarks exchanging momentum transfer in their CM.

A useful technique is the exact Functional Cluster Expansion,

$$e^{\hat{\mathfrak{D}}_A} \cdot e^{L[A]} = \exp\left[\sum_{n=1}^{\infty} Q_n/n!\right], \quad Q_n = e^{\hat{\mathfrak{D}}_A} (L[A])^n \Big|_{conn.},$$

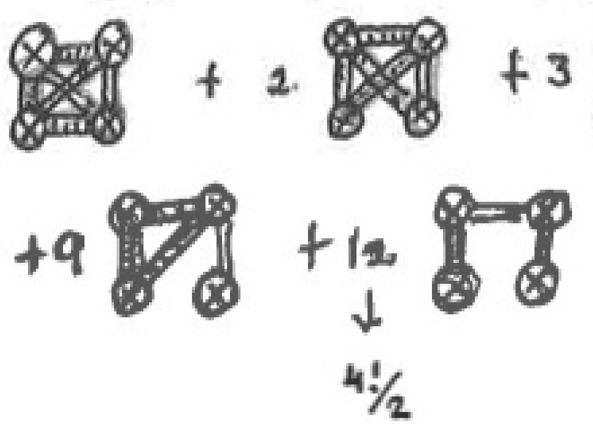
with linkage operator

$$\hat{\mathfrak{D}}_A = \frac{i}{2} \int \frac{\delta}{\delta A} \hat{K} \frac{\delta}{\delta A}, \quad (\hat{K})^{ab}_{\mu\nu} = (g f_{abc} \chi^c_{\mu\nu})^{-1} \Leftarrow GB.$$

For example, $Q_1 = e^{\hat{D}_A} L[A] = \bigcirc + \ominus + \oplus + \cdots = \bar{L}[A]$

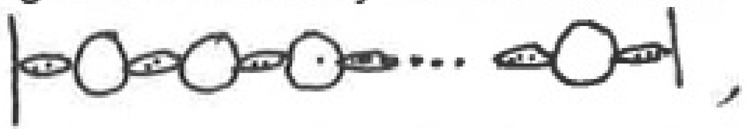
and $Q_2 = \bar{L}[A]\left(e^{\overleftrightarrow{D}_A} - 1\right)\bar{L}[A]$.

Things get complicated very quickly; e.g. Q_4 is given by



, $\bar{L}[A] = \otimes$, $[e^{\overleftrightarrow{D}} - 1] =$  .

Even functionally, it is a horrid mess. But, there exists one way of reducing this to an easily-calculated set of 'chain-graph-loops',



which form a geometric series that can be summed.

But this depends crucially on the definition of renormalization…

At this point, let's 'take stock' of where we stand. We began with a Theory of quarks and gluons; but the gluons have disappeared, and only their GB sums remain.

What to do? Renormalize the GBs! But how?
What is done in QED?

The 'dressed' photon propagator has the form

$D'_C = Z_3 D_C + \int$(other things with no mass shell pole), where in

every perturbation order, $Z_3^{-1} \to \infty$, or $Z_3 \to 0$.

To renormalize, one <u>divides</u> by Z_3 and defines

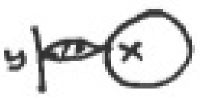
$D_{aR} = D_C + (1/Z_3) \int$(other things), where the 2nd RHS term ->0

upon 'mass-shell-amputation', and does not contribute to S-matrix elements.

Slides 3 and 4 mentioned the new, exact property of Effective Locality.

To see how this enters, consider one side of a typical process, the connection of a quark to one side of a closed loop, represented by the factors

$$N \int d[\chi] \exp\Big[\frac{i}{4}\int \chi^2 + ig\int_0^s ds'\, u'_\mu(s')\,\Omega_a(s')\,\left(f\cdot\chi(w)\right)^{-1}\Big|^{ab}_{\mu\alpha} \int_0^t dt'\, v'_\alpha(t')\,\hat{\Omega}_b(t') \cdot \int d^2x'_\perp \int d^2y'_\perp\, a(x_\perp - x'_\perp)\cdot a(y_\perp - y'_\perp)\cdot \delta(y' - u(s') - x' + v(t'))\Big]$$



where $z'_\mu = (z_0, z_L, z'_\perp)$, $w = y' - u(s') = x' - v(t')$, $(f.Xi)^{ab}_{\mu\alpha} = f_{abc}\cdot\chi^c_{\mu\alpha}$

with $\chi^c_{\mu\alpha}$ antisymmetric in μ, α , and $(f.Xi)^{-1}$ representing the GB.

The FI $\int d[\chi] \Rightarrow \prod_i^{\infty} N_i \int d^n\chi(w)$ corresponding to a summation over all color coordinates, and a sum of all possible Xi values in the tiny four-volume element labeled by the subscript i, of 4-volume δ^4.
It is understood that at the end of the calculation, N -> ∞ and δ -> 0.

Because of EL, represented by the $\delta^{(4)}(y'-u(s')-x'+v(t'))$, the interaction is confined to only one space-time point, w_i, and the normalized FIs over all <u>other</u> such points reduce to products of 1: $\prod_{j\neq i}^{N} N_j \int d^4\chi_j \exp[\frac{i}{4}\delta^4\chi_i^2] = (1)^N = 1$.

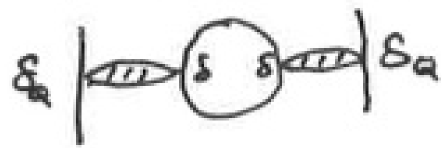
Now make a trivial change of variable: $\bar{\chi} = \delta^2\chi$; then the interaction amplitude reduces to

$$N_i' \int d^4\bar{\chi}_i \exp[\frac{i}{4}\bar{\chi}^2 + ig\int\cdots\int \delta^2(f\cdot\bar{\chi})^{-1}\cdots]$$

where N' is independent of δ.

It is at this point that we must define a Model Renormalization. Suppose we consider the scattering of two quarks, each part of a different hadron. Each GB contributes 2 factors of δ , and for clarity, move them to the ends of each GB, Because each quark represents the "physical particle" of QCD, we'll replace the δ at each quark site by δ_Q, a finite quantity. But where the δ touches the loop - which is a virtual and not a physical particle - it remains δ, and (very shortly) -> 0.

In this one-loop, 2 GB drawing, there is a net δ^2 multiplying the loop.



But the loop is proportional to an expected UV log divergence, which we'll call ℓ, $\ell = \ln(\Lambda/m_Q)$.

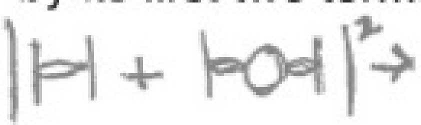
This loop - as well as every such loop in a chain of such GB loops - produces a factor of $(\ell \cdot \delta^2)\Big|_{\substack{\delta \to 0 \\ \ell \to \infty}} \equiv x$,

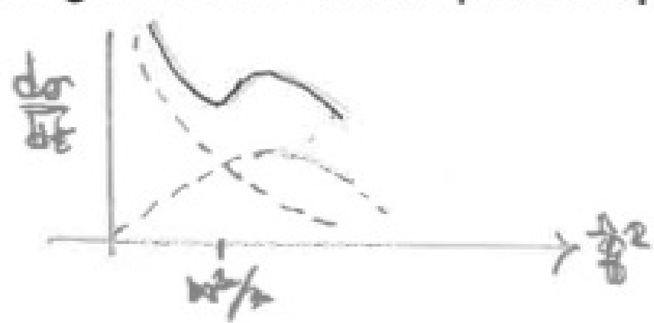
which we DEFINE to be a real, finite, positive number (subsequently determined by experiment).

What does this Model mean? That <u>only</u> the GB chain graphs are non-zero! <u>All other closed loops</u> entering into the Functional Cluster Expansion vanish.

AND, these chain graphs form a geometric series, which can be summed, is everywhere finite, and can be compared with HE pp scattering data. It can be used to define a renormalized charge $g_R(\vec{q}_\perp^{\,2})$, as combinations of diminishing dependence on $q_\perp$ arise from multiple factors of the Fourier transform of powers of $\varphi(b)$.

A preliminary, approximate analysis suggests a quite close resemblance to scattering data. For the amplitude approximated by its first two terms





which are the ones most relevant in the GeV momentum-transfe region of scattering, it is easy to produce the familiar "diffractio dip"; and we have every expectation of being able to produce at least qualitative fits at TeV energies and momentum-transfers.

Perhaps it is time to end this talk… but with the appreciation tha we now have at least one, finite, renormalized Model of non-perturbative QCD.

Comparisons with scattering data are now underway… and we hope for the best. Mais nous verrons…

My colleagues and I thank you for your attention!